Metodologias alternativas no ensino de física

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Minicurso 2 – Parte 3

UNIVERSITY OF COPENHAGEN

BLUMENAU - SANTA CATARINA
V ESCOLA BRASILEIRA DE ENSINO DE FÍSICA
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Universidade Federal do ABC

Valores pedagógicos do uso de originais no ensino de Física

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Pedagogical value of the history of physics (Cajori, 1898)

- A knowledge of the struggles which original investigators have undergone leads the teacher to a deeper appreciation of the difficulties which pupils encounter;
- The difficulties which students encounter are often real difficulties such as the builders of the science succeeded in overcoming only after prolonged thought and discussion;
- To the instructor the history of science teaches patience, to the pupil it shows the necessity of persistent effort;
- The necessity of checking speculation and correcting our judgment by continual appeal to the facts, as determined by experiment.
- The history of science demonstrates the futility of the pedagogical theory, according to which the pupils in the laboratory should be made to re-discover the laws of nature

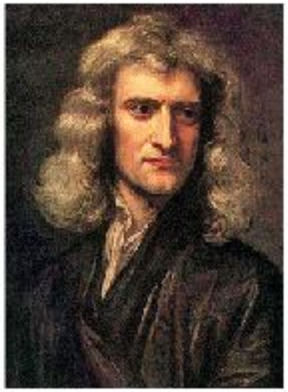
Plan of the talk:
Specific lessons from 3 episodes

1) Newton's PQRST force

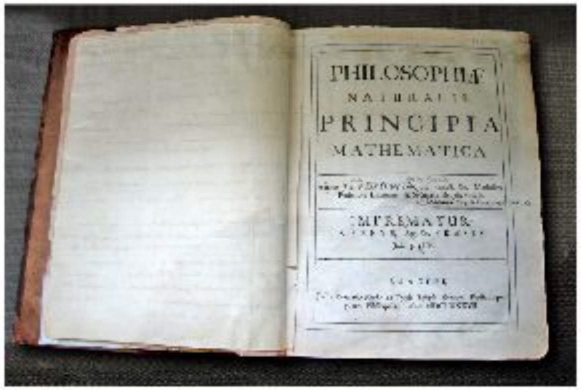
2) Faraday's and Maxwell's lines of force

3) Schrödinger's ontology on wave mechanics

Philosophiæ Naturalis Principia Mathematica (1687)



Newton (1687)



Dec 14, 2016 theguardian

Isaac Newton masterwork becomes most expensive science book sold

First edition of Principia Mathematica, which was published in 1687 and sets out Newton's laws of motion, raises £3m at auction

“The *Principia* is perhaps the greatest intellectual stride that it has ever been granted to any man to make” (Einstein)

“The *Principia* marked the epoch of a great revolution in physics. The method followed by its illustrious author Sir Newton … spread the light of mathematics on a science which up to then had remained in the darkness of conjectures and hypotheses” (Clairaut)

“The *Principia* is one of the most influential works in Western culture, but it is a work more revered than read” (Brackenridge)

Motivation to write the *Principia*

January 1684

Hooke Wren Halley

How to derive the laws of planetary motion?

Hooke claims to have derived that an inverse square law leads to an ellipse, but shows no evidence.

August 1684

Months passed and Hooke had yet to produce his evidence. Edmund Halley traveled to Cambridge to find out what Isaac Newton had to say on the matter.

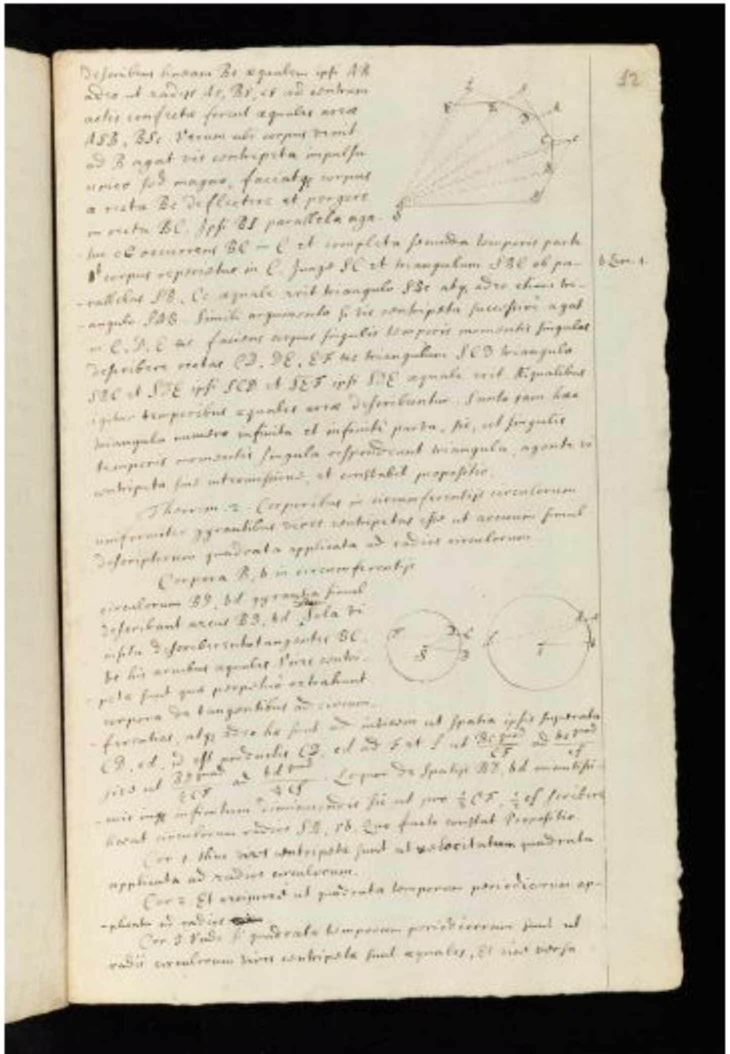
When Halley put the question to Newton, Newton surprised him by saying that he had already made the derivations some time ago; but that he could not find the papers…

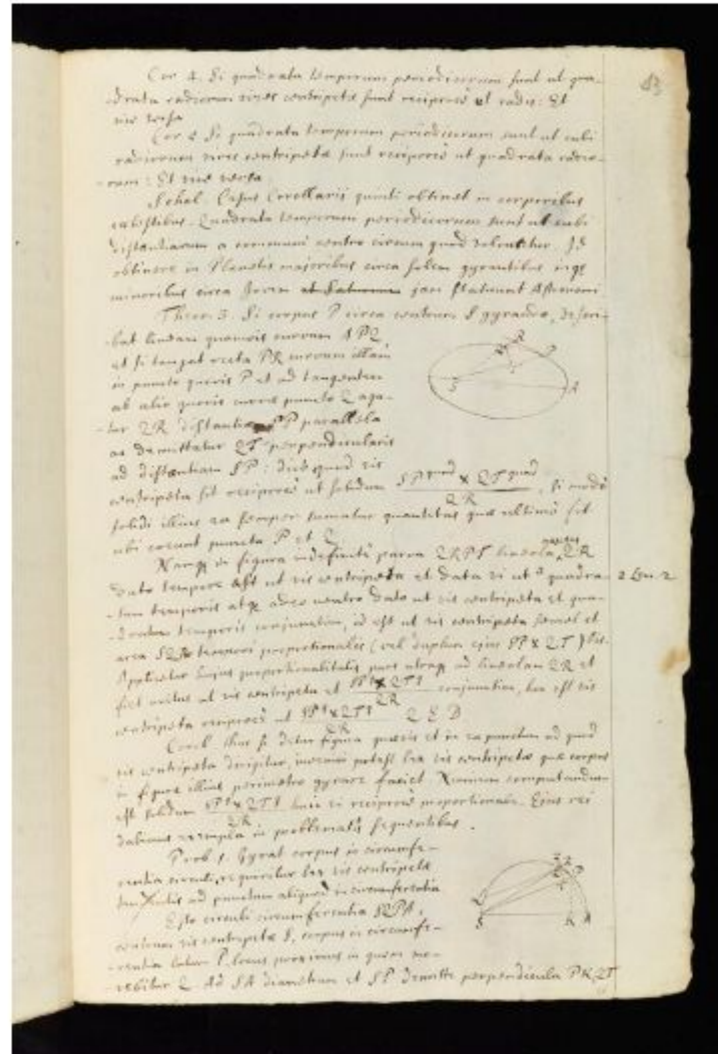
November 1684

Newton sent Halley a nine-page manuscript titled *De Motu Corporum in Gyrum* (On the Motion of Orbiting Bodies).

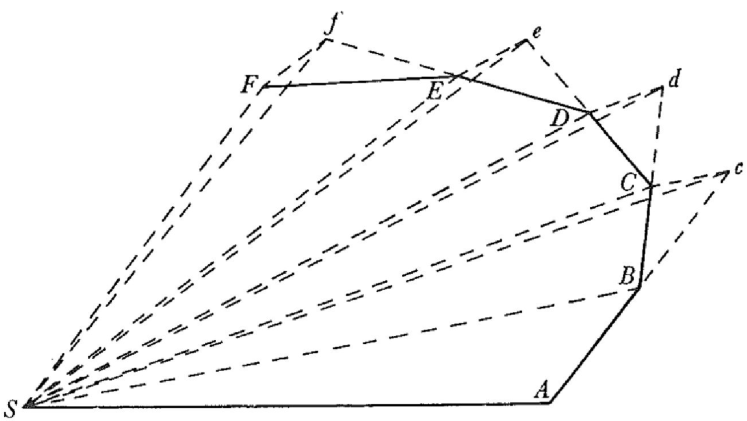
Halley is so fascinated by its content and method that he demands Newton to send more of his work to the Royal Society – which leads to the *Principia* (1687)

De Motu Corporum in Gyrum



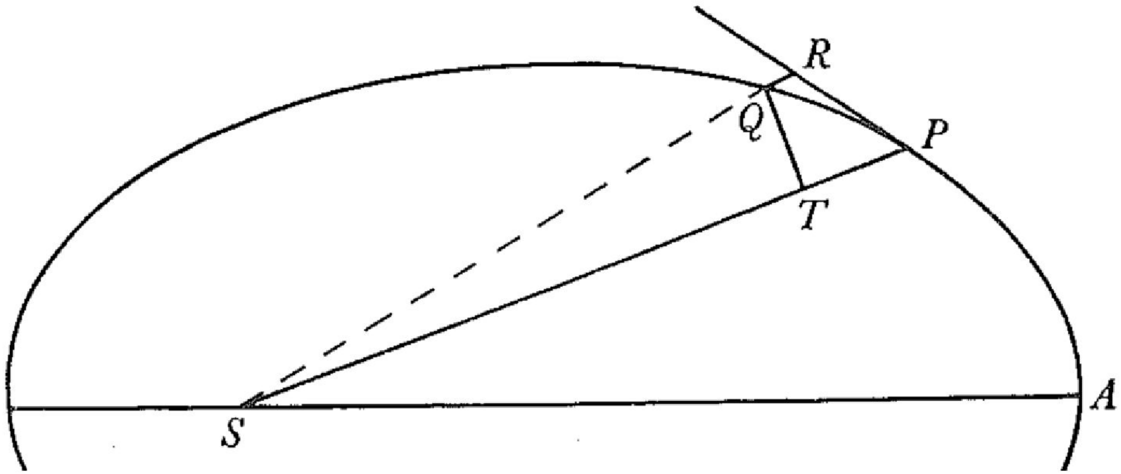


Theorem 1: Central force ➔ Equal areas



Wikipedia: Newtons proof of Keplers second law.gif

Theorem 3: Force proportional to QR/(SP² x QT²)



Kepler problem

Shape of orbit ⟷ Force law

Problems 1, 2 and 3: Force laws and orbits

$F \propto QR/(SP^2 \times QT^2)$

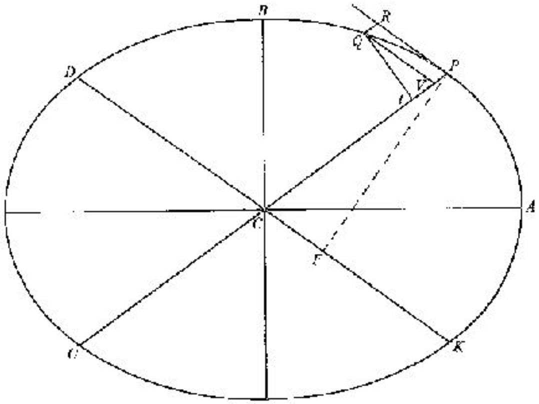
ꝑ ad lineolam QR et fiet unitas ut vis centripeta

st vis centripeta reciprocè ut $\frac{SP^q \times QT^q}{QR}$. (30)

vis et in ea punctum ad quod vis centripeta
ntripetæ quæ corpus in figuræ illius perimetro
dum est solidum $\frac{SP^q \times QT^q}{QR}$ huic vi reciprocè
xempla in problematîs sequentibus.

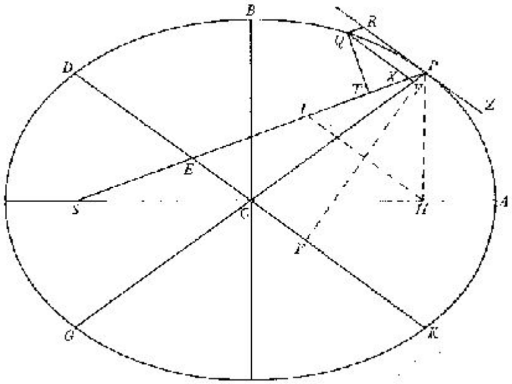
Prob. 1

$F(r) \propto 1/r^5$



Prob. 2

$F(r) \propto r$



Prob. 3

$F(r) \propto 1/r^2$

Problem 1: Center in the circumference

$F \propto QR/(SP^2 \times QT^2)$

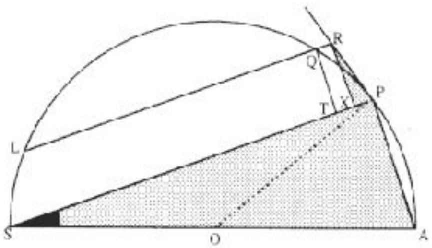


Figure 5.3A
A revised diagram f r Problem 1. The perpendicular *RX* n d the radius *OP* r e a ded.

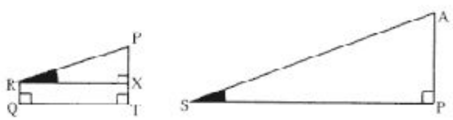


Figure 5.3B
The triangle *RPX* s similar to the triangle *AS P* .

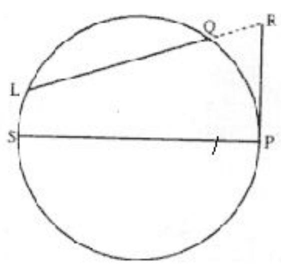
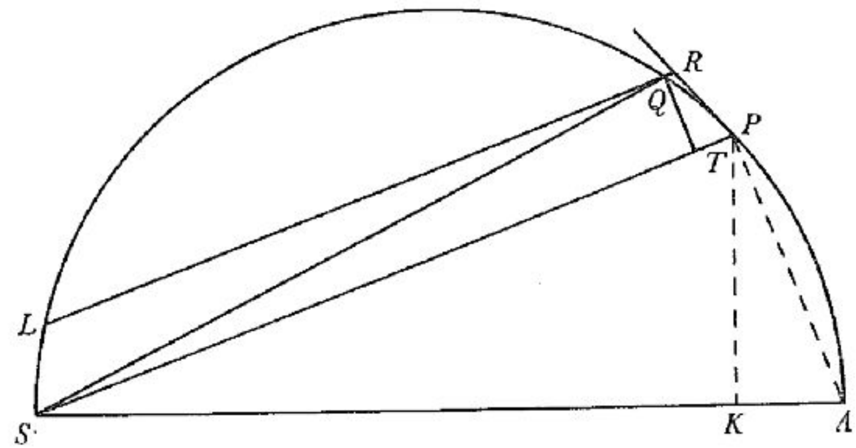


Figure 5.4B
Thus, *RL* = *RP* / *RP* *QR* r
*RP*² *QR* × *RL* s required in Problem 1



1: $\Delta SAP \sim \Delta RPX \;\therefore\; (SA/SP)^2 = (RP/QT)^2$

2: $RP^2 = (QR).(LR)$

3: R▯ P LR▯ SP

$QR/QT^2 = SA^2/SP^3$

$F(r) \propto 1/r^5$

A proposal for high school

Elliptical Orbit ⇨ $1/r^2$ Force

Newton's Recipe

Given only two ingredients— the *shape of the orbit* and the *center of the force*— "Newton's Recipe" allows one to calculate the relative force at any orbital point. The recipe consists of the following steps:

1. *The inertial path:* Draw the tangent line to the orbit curve at the point P where the force is to be calculated.
2. *The future point:* Locate any future point Q on the orbit that is close[6] to the initial point P.
3. *The deviation line:* Draw the line segment from Q to R, where R is a point on the tangent, such that QR (line of deviation) is parallel to SP (line of force).
4. *The time line:* Draw the line segment from Q to T, where T is a point on the radial line SP, such that QT (height of "time triangle") is perpendicular to SP (base of triangle).
5. *The force measure:* Measure the shape parameters QR, SP, and QT, and calculate the force measure $QR/(SP \times QT)^2$.
6. *The calculus limit:* Repeat steps two to five for several future points Q around P to obtain several force measures. Take the limit $Q \to P$ of the sequence of force measures to find the exact value of the force measure at P.[7]

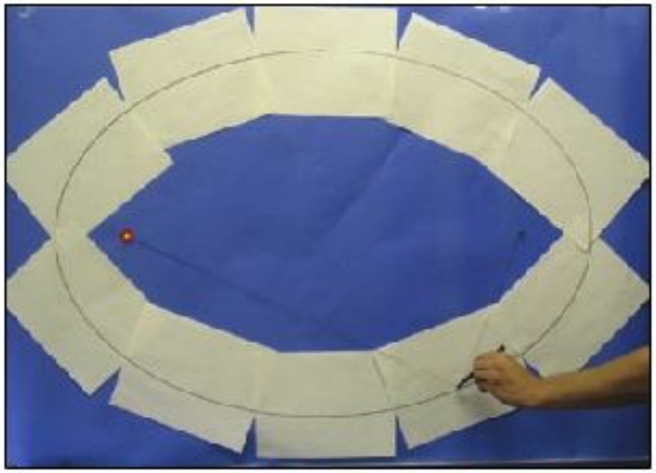


Fig. 5. The class constructs an elliptical orbit. Each student gets a small piece (arc) of the whole ellipse and measures the force responsible for the shape of his or her arc.

Table II. Values of the force F measured by a team of students at nine different radii r along their elliptical orbit. The team uncovers a simple pattern in the data: $F = 1.23/r^{2.12}$.

r (m)	F (m^{-3})
0.324	14.0
0.359	10.0
0.419	8.60
0.460	6.00
0.560	4.00
0.607	3.66
0.625	3.42
0.644	3.46
0.647	2.80

Prentis et al. (2007)

Some lessons from Episode 1

- Force and time are geometrical entities
- Inertial path and deviation are made visual
- Force was assumed constant for a small Δt (linear approximation)
- Geometrical calculus ("ultimate ratio")
- PQRST formula is the general recipe;
- Nature tells us the orbit shape and we determine the force law (PQRST)
- Pros and Cons of drawing (infinitesimal) triangles (mind and eye)

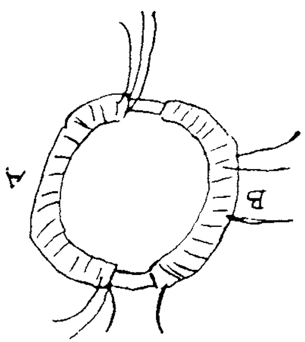
Perguntas/Comentários?

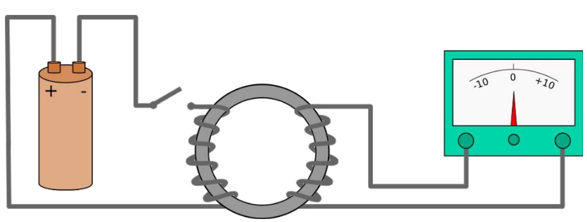
Faraday's discovery of induction

Faraday's diary (1822)

Convert magnetism into electricity

Induction ring





29.8.1831

3. [...] Then connected the ends of one of the pieces on A side with battery; immediately a sensible effect on needle. It oscillated and settled at last in original position. On *breaking* connection of A side with Battery again a disturbance of the needle.

4. Made all the wires on A side one coil and sent current from battery through the whole. Effect on needle much stronger than before.

5. The effect on the needle then but a very small part of that which the wire communicating directly with the battery could produce.

8. Hence effect evident but transient; but its recurrence on breaking the connection shows an equilibrium somewhere that must be capable of being rendered more distinct (electro-tonic state).

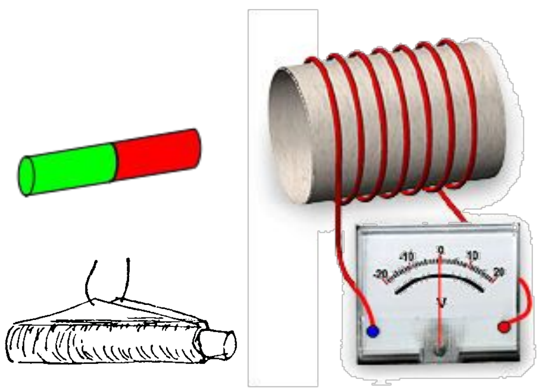
Faraday's discovery of induction

Faraday's diary (1822)

Convert magnetism into electricity

17.10.1831

Moving a magnet through a coil



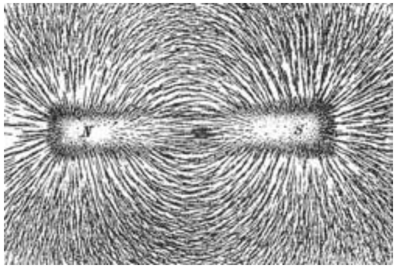
57. The 8 ends of the helices at one end of the cylinder were cleaned and fastened together as a bundle. These compound ends were then connected with the Galvanometer by long copper wires then a cylindrical bar magnet 3/4 inch in diameter and 81⁄2 inches in length had one end just inserted into the end of the helix cylinder—then it was quickly thrust in the whole length and the galvanometer needle moved—then pulled out and again the needle moved but in the opposite direction. This effect was repeated every time the magnet was put in or out and therefore a wave of Electricity was so produced from mere approximation of a magnet and not from its formation *in situ*.

Magnetic lines of force

IMAGInation: Continuous curved patterns

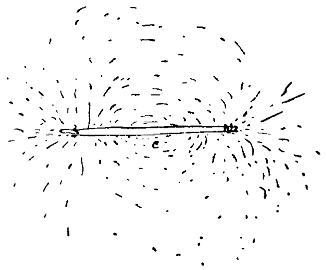
- Place a bar magnet beneath a sheet of paper

- Spread iron fillings



- Continuous curves from pole to pole

Diary (1851)



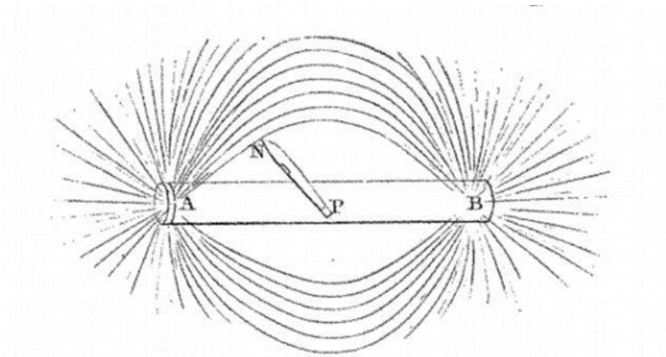
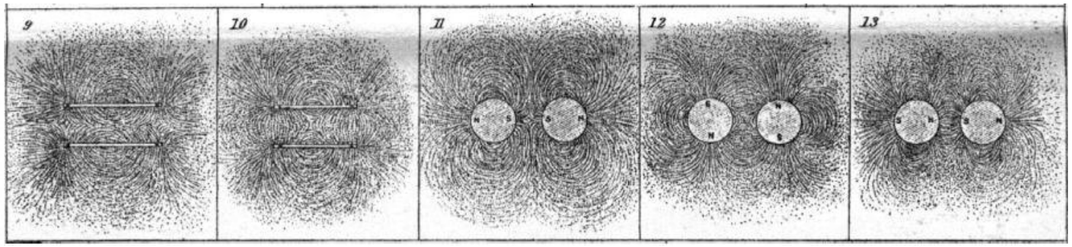


Fig. 25.

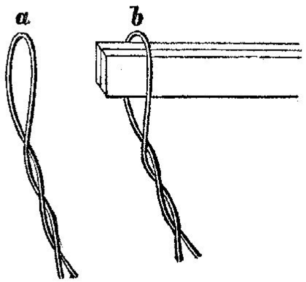
1st series (1831)



29th series (1852)

Magnetic lines of force

Moving wire
28th series (1852)

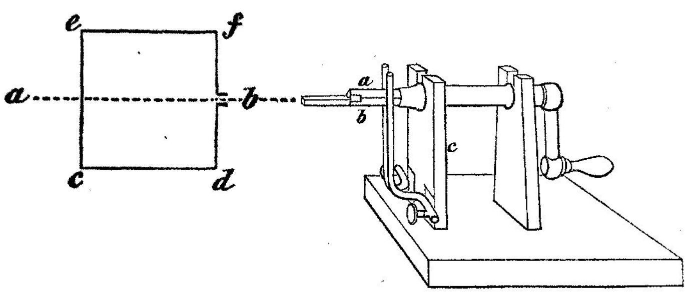


When the bend of the wires was formed into a loop and carried from *a* to *b*, the galvanometer needle was deflected two degrees or more. The vibration of the needle was slow, and it was easy to reiterate this action five or six times, breaking and making contact with the galvanometer at right intervals, so as to combine the effect of induced currents; and then a deflection of 10° or 15° could be readily obtained.

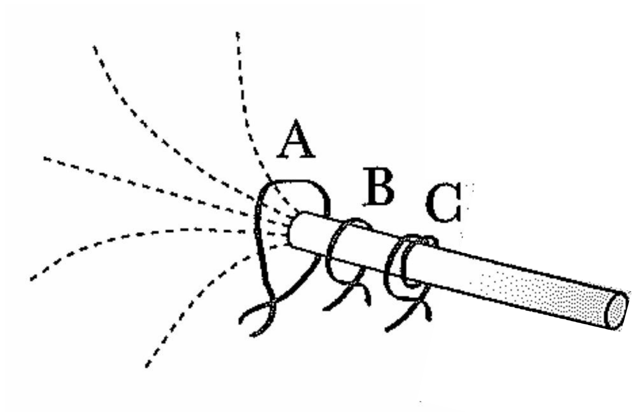
- Deflection is proportional to number of times, i.e. "number of lines of force that cut the loop" (Counting principle)
- The "moving wire" undergoes a profound transformation: from a *phenomenon* to a [reasoning?] *instrument* to interpret other phenomena (Fisher, 2001)

Magnetic lines of force

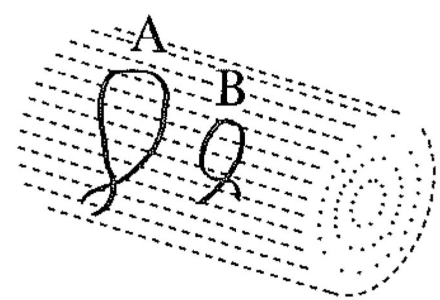
Revolving rectangles
29th series (1852)



3195. When a given length of wire is to be disposed of in the form best suited to produce the maximum effect, then the circumstances to be considered are contrary for the case of a loop to be employed with a small magnet (39. 3184.), and a rectangle or other formed loop to be employed with the lines of terrestrial force. In the case of the small magnet, *all* the lines of force belonging to it are inclosed by the loop; and if the wire is so long that it can be formed into a loop of two or more convolutions, and yet pass over the pole, then twice or many times the electricity will be evolved that a single loop can produce (36.). In the case of the earth's force, the contrary result is true; for as in circles, squares, similar rectangles, &c. the areas inclosed are as the squares of the periphery, and the lines of force intersected are as the areas, it is much better to arrange a given wire in one simple circuit than in two or more convolutions. Twelve feet of wire in one square inter-



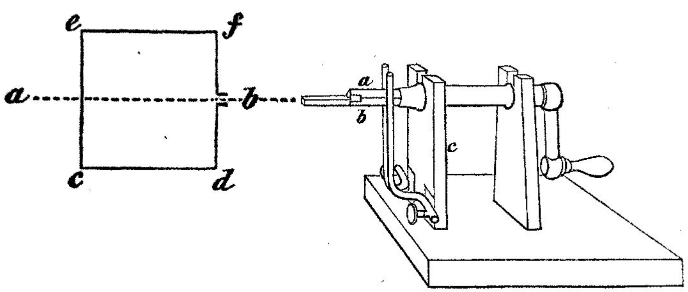
Loop in a small magnet



Loop in "lines of terrestrial force"

Magnetic lines of force

Revolving rectangles
29th series (1852)



Now 144 square inches is to 128 square inches as 2,61° is to 2,32° proving that the electric current induced is directly as the lines of magnetic force intersected by the moving wire [...] no alterations are caused by changing the velocity of motion, provided the amount of lines of force intersected remains the same. […] "thrice as advantageous to intersect the lines within nine square feet once, as to intersect those of one square foot three times"

On the physical character of the lines of magnetic force (Faraday, 1852, Philosophical Magazine)

Maxwell's subsequent papers:

- On Faraday's Lines of Force (Maxwell 1855)
- On Physical Lines of Force (Maxwell 1862)
- A Treatise on Electricity and Magnetism (Maxwell 1873)

On Faraday's Lines of Force (Maxwell, 1855)

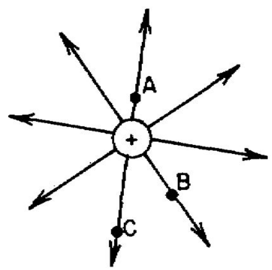
No experiment / Place before the mathematical mind

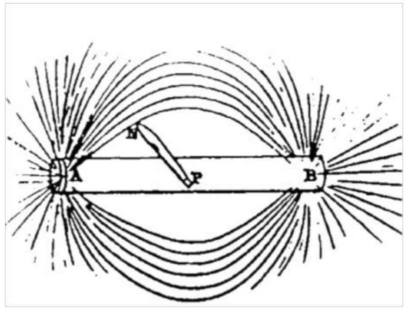
By the method which I adopt, I hope to render it evident that I am not attempting to establish any physical theory of a science in which I have hardly made a single experiment, and that the limit of my design is to show how, by a strict application of the ideas and methods of Faraday, the connexion of the very different orders of phenomena which he has discovered may be clearly placed before the mathematical mind.

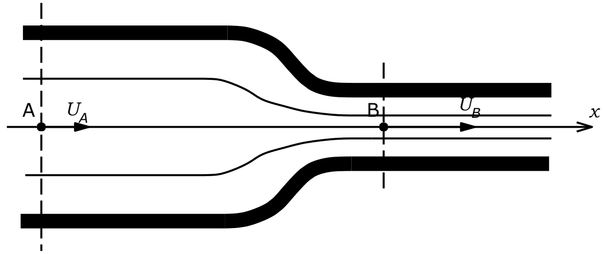
On Faraday's Lines of Force (Maxwell, 1855)

Intensity of the force

[…] we might find a line passing through any point of space representing the *direction* of the force acting on a positively electrified particle or on an elementary north pole.







$F \propto v$

$F \propto 1/S$

$$v_A \cdot S_A = v_B \cdot S_B = const.$$

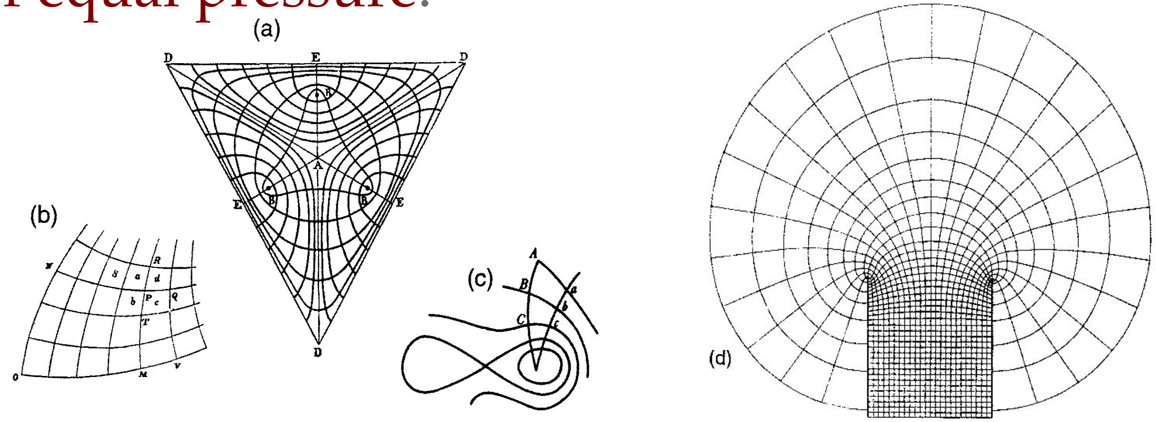
[…] but we should still require some method of indicating the *intensity* of the force at any point. If we consider these curves not as mere lines, but as fine tubes of variable section carrying an incompressible fluid, then we may make the velocity vary according to any given law, by regulating the section of the tube, and in this way we might represent the intensity of the force.

On Faraday's Lines of Force (Maxwell, 1855)

Fluid motion in resisting medium

Any portion of the fluid moving through the resisting medium is directly opposed by a retarding force proportional to its velocity.

[…] all the points at which the pressure is equal to a given pressure p will lie on a certain surface which we may call the surface (p) of equal pressure.



Maxwell's geometrical representations in Darrigol (2000, p. 140)

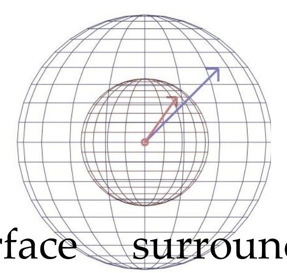
On Faraday’s Lines of Force (Maxwell, 1855)

Fluid motion in resisting medium

If the velocity be represented by *v*, then the resistance will be a force equal to *kv* acting on unit of volume of the fluid in a direction contrary to that of motion. In order, therefore, that the velocity may be kept up, there must be a greater pressure behind any portion of the fluid than there is in front of it, so that the difference of pressures may neutralise the effect of the resistance.

Unit point source

Unity of volume flows out of every spherical surface surrounding the point in unit of time



$$v = \frac{1}{4\pi r^2}$$

Decrease of pressure (force)

$$\frac{k}{4\pi r^2}$$

Pressure at point *r* (0 at ∞)

$$p = \frac{k}{4\pi r} \qquad p = -\frac{k}{4\pi r}$$

source sink

For S unit sources

$$p = \frac{kS}{4\pi r}$$

Does this seem familiar to you?

On Faraday's Lines of Force (Maxwell, 1855)

Analogy between imaginary fluid and electrostatics

$$v = \frac{1}{4\pi r^2}$$

Velocity is analogous to *E field*

$$p = \frac{k}{4\pi r}$$

Pressure is analogous to *Potential*

Number of unit sources (+ sources, – sinks) is analogous to *Charge*

On Faraday's Lines of Force (Maxwell, 1855)

It is not a fluid!

The substance here treated must **not be assumed to possess any of the properties of ordinary fluids** except those of freedom of motion and resistance to compression. It is **not even a hypothetical fluid which is introduced to explain actual phenomena**. It is **merely a collection of imaginary properties** which may be employed for establishing certain theorems in pure mathematics in a more intelligible way to many minds and more applicable to physical problems than that in which algebraic symbols alone are used.

Some lessons Episode 2

- From discovery to conceptualization of induction (30 years)
- Wire loop/rev. rectangles: from experiments to reasoning instruments
- Number of lines of force as crucial quantity
- Maxwell´s fluid analogy: velocity as analogous to force
- Where does the term flux of E or B field come from?

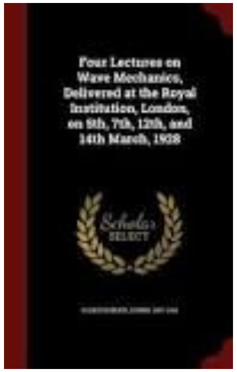
Perguntas/Comentários?

Where does the Schrödinger equation (SE) come from?

- "Where did we get that [SE] from? **Nowhere!** It is not possible to derive from anything you know. It came out of the mind of Schrödinger, invented in his struggle to find an understanding of the experimental observation of the real world." (Feynman)

- Griffiths: SE is presented on p. 2 "falling from the sky". It is said to be analogous to Newton 2nd law in classical mechanics.

- Schrödinger DOES present derivations of his equations (even if *a posteriori*) and has clear ontological commitments.



Four Lectures on Wave Mechanics

delivered at the Royal institution, London,
on 5th, 7th, 12th, and 14th March, 1928

Schrödinger's analogy/completion

Mechanics

Least action (Euler, 1744)

$$\delta \int_A^B 2T dt = 0$$

$$\delta \int_A^B \sqrt{2m(E-V)}\, ds = 0$$

Optics

Least time (Fermat, 1662)

$$\delta \int_A^B \frac{ds}{u} = 0$$

$$u = \frac{C}{\sqrt{2m(E-V)}}$$

$$E = h\nu$$

This enables us to push the analogy a step farther by picturing the dependence on E as dispersion, i.e. as a dependence on frequency.

Can we make a small "point-like" light-signal move exactly like our mass-point?

Schrödinger's analogy/completion

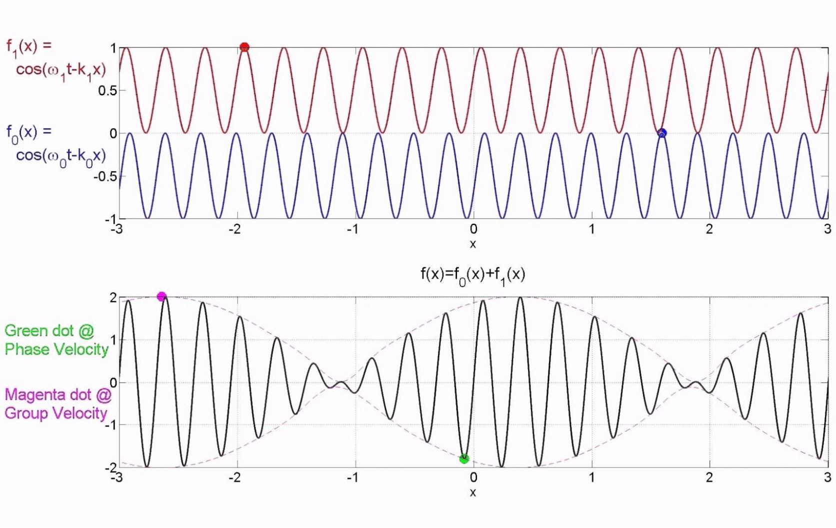
Can we make a small "point-like" light-signal move exactly like our mass-point?

At first sight this seems impossible

$$w = \frac{1}{m}\sqrt{2m(E-V)}$$

$$u = \frac{C}{\sqrt{2m(E-V)}}$$

But *u* is *phase-velocity*. A small light-signal moves with the so-called *group-velocity g*



http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/wavepacket/wavepacket.html

$$\frac{1}{g} = \frac{d}{d\nu}\left(\frac{\nu}{u}\right)$$

$$\frac{1}{g} = \frac{d}{dE}\left(\frac{E}{u}\right)$$

We will try to make $g = w$

$$u = \frac{E}{\sqrt{2m(E-V)}}$$

Schrödinger's analogy/completion

$$\nabla^2 p - \frac{1}{u^2}\ddot{p} = 0$$

Wave equation

$$p(x, y, z, t) = \psi(x, y, z)\, e^{2\pi i \nu t}$$

Separation of variables

$$\nabla^2 \psi + \frac{4\pi^2 \nu^2}{u^2}\psi = 0$$

Amplitude equation

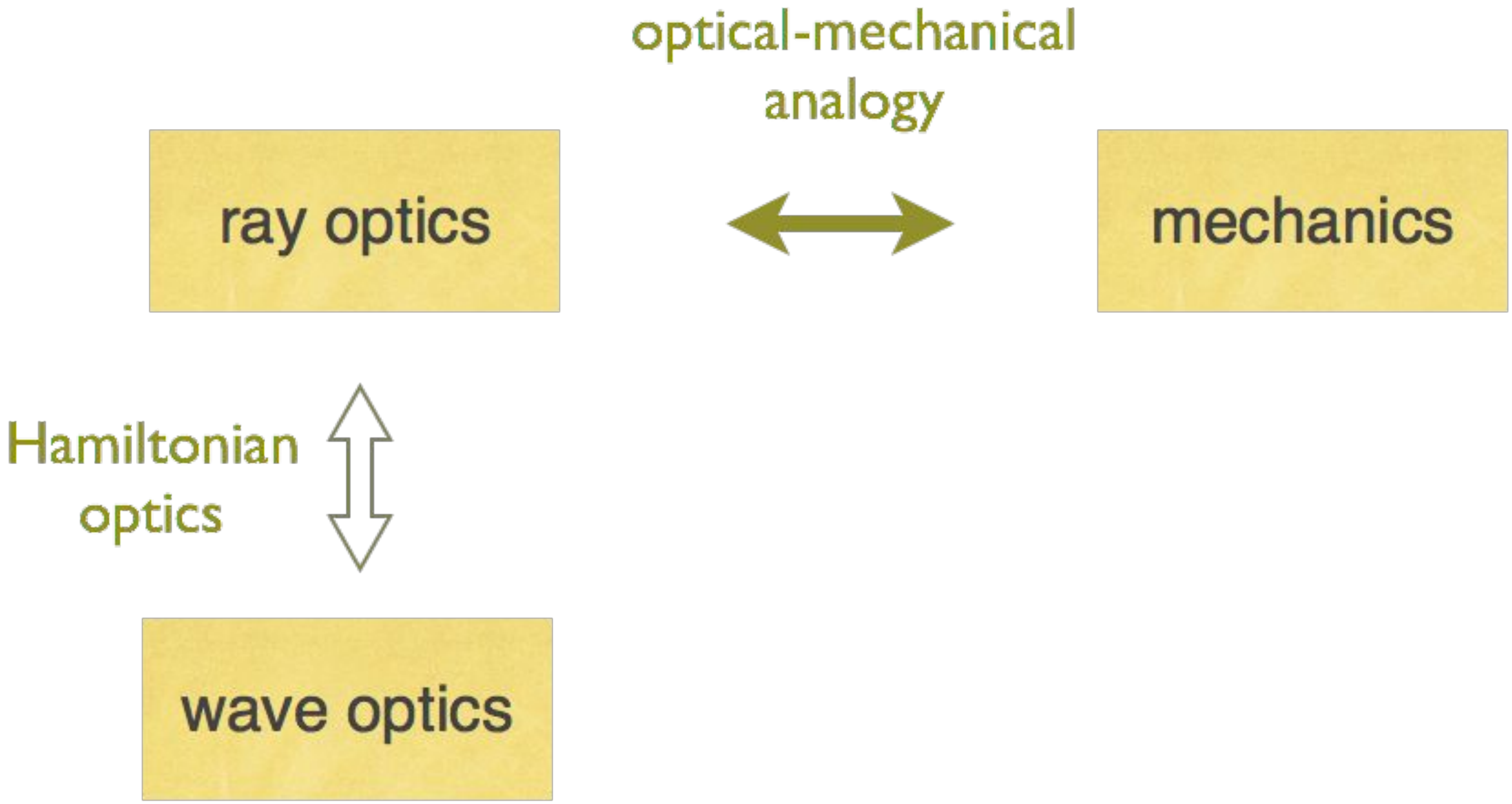
$$u = \frac{E}{\sqrt{2m(E - V)}}$$

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

Time-independent SE

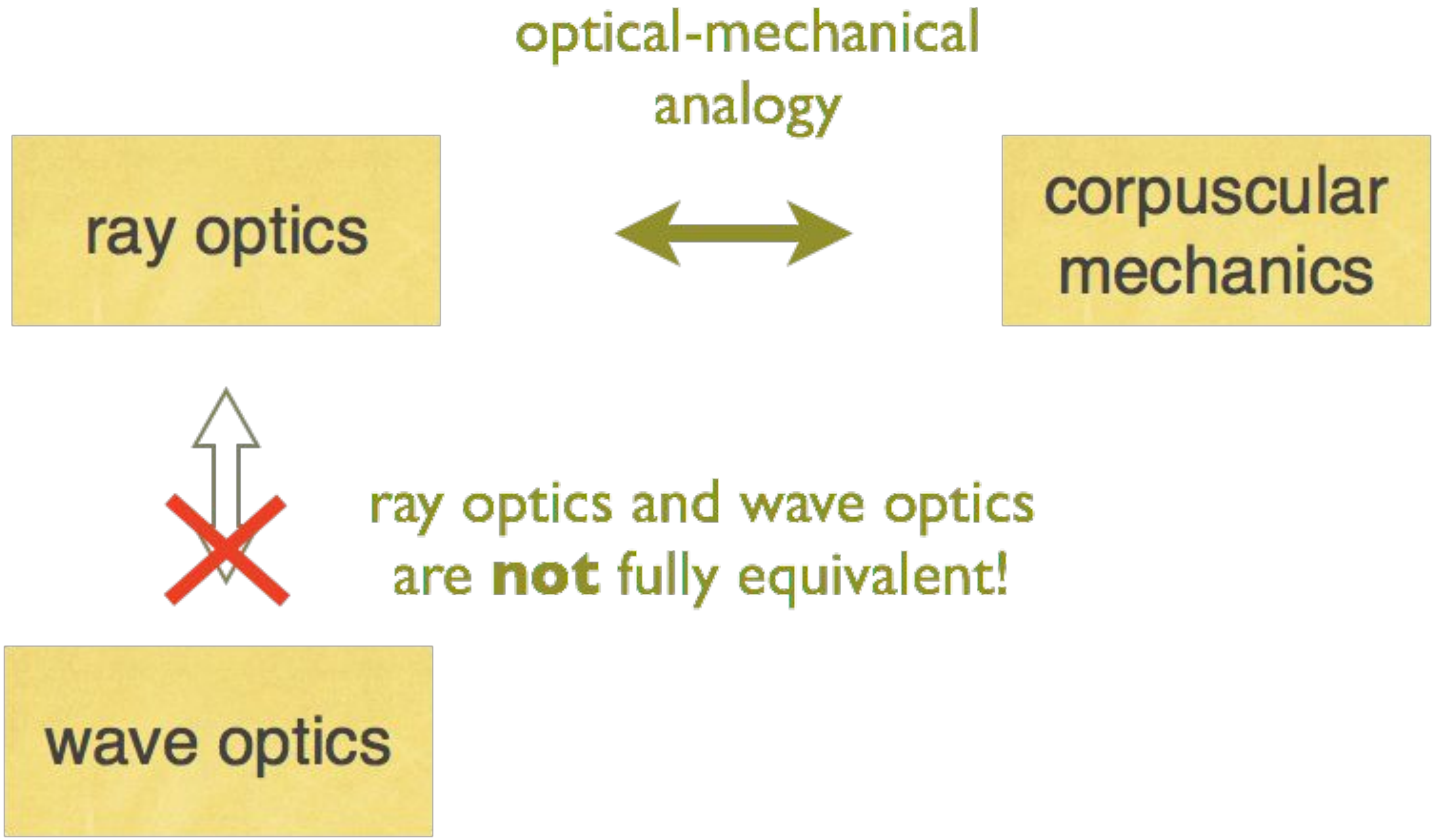
$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\boldsymbol{x})\psi = E\psi$$

Schrödinger's analogy/completion



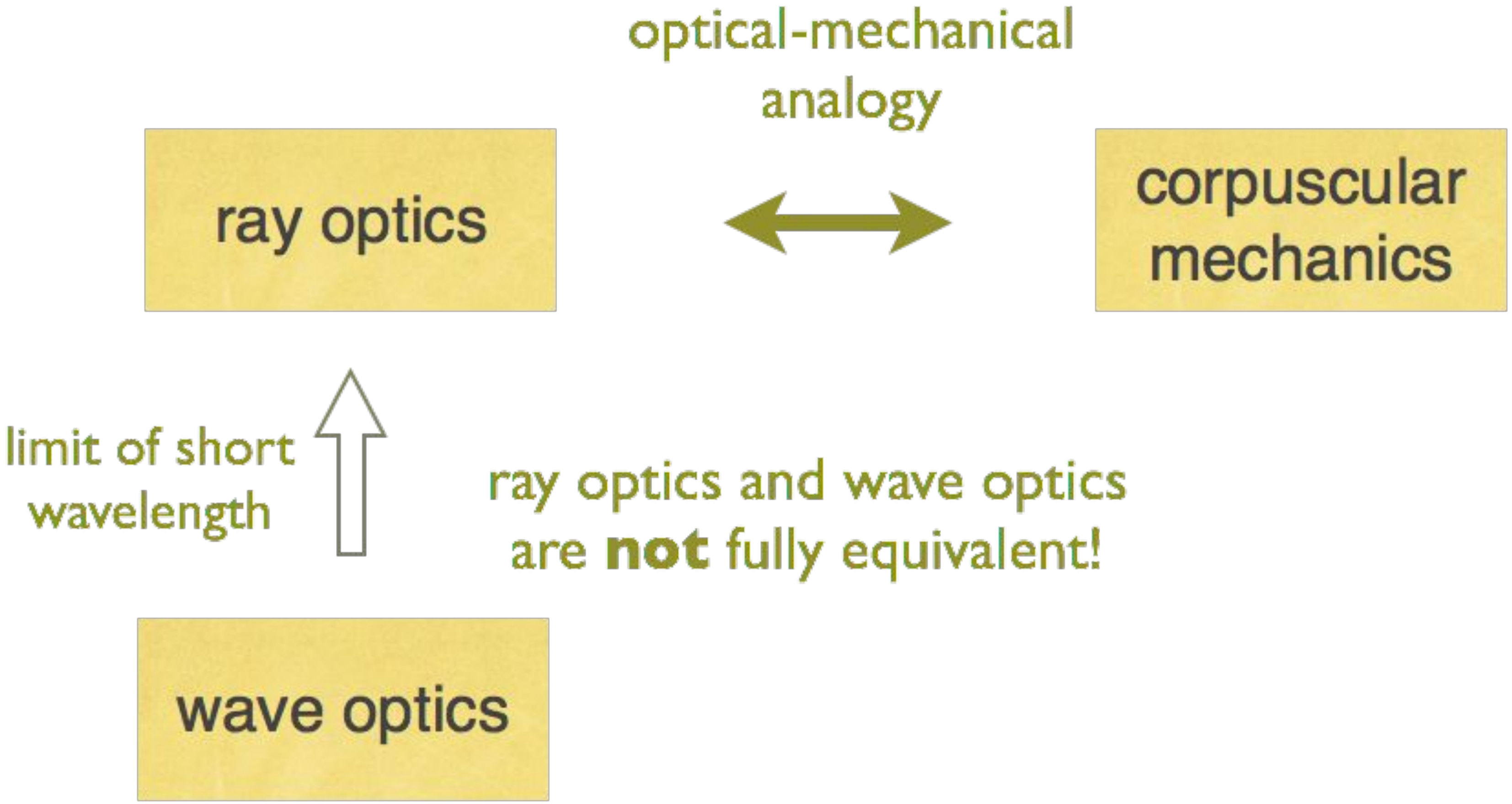
Thanks to Christian Joas

Schrödinger’s analogy/completion



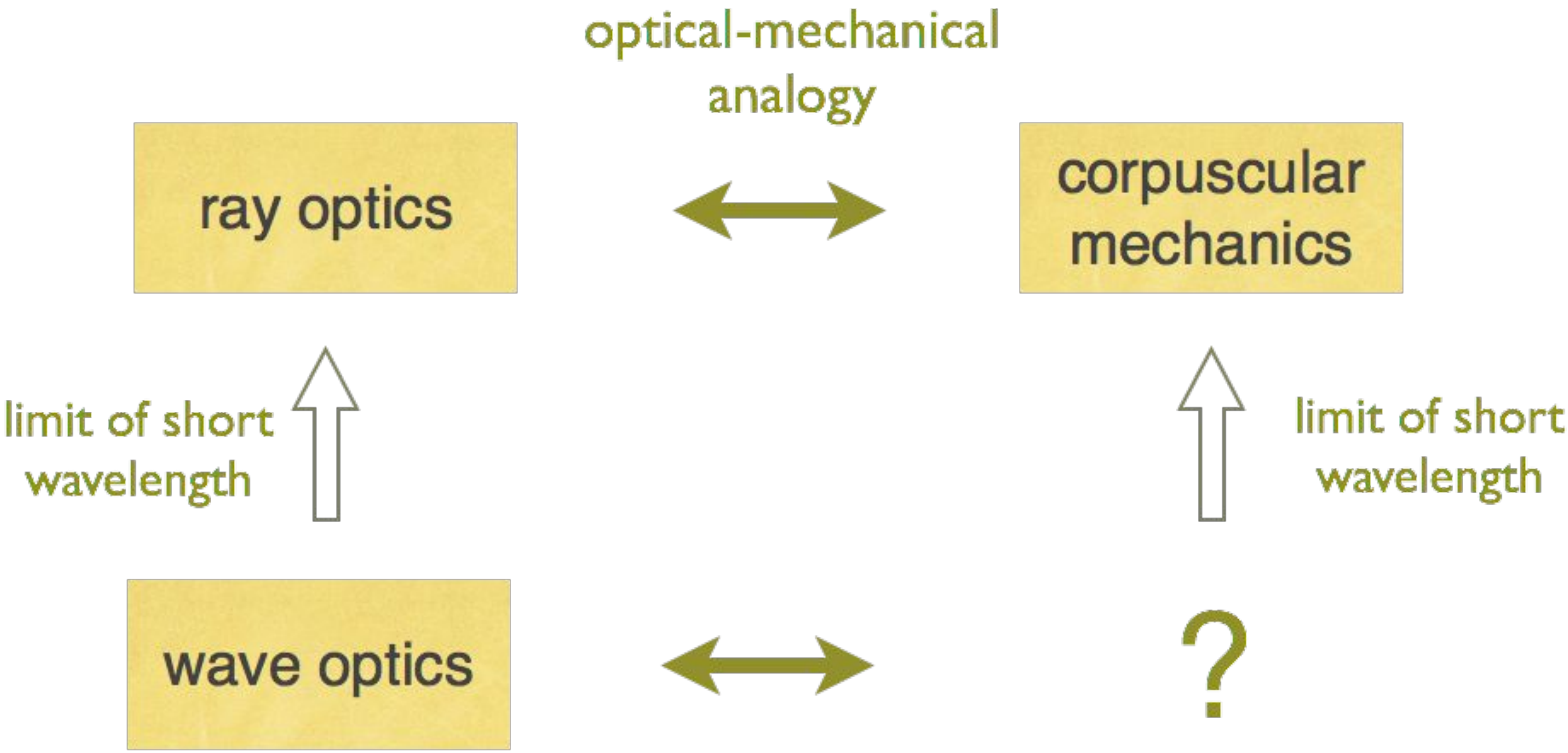
Thanks to Christian Joas

Schrödinger's analogy/completion



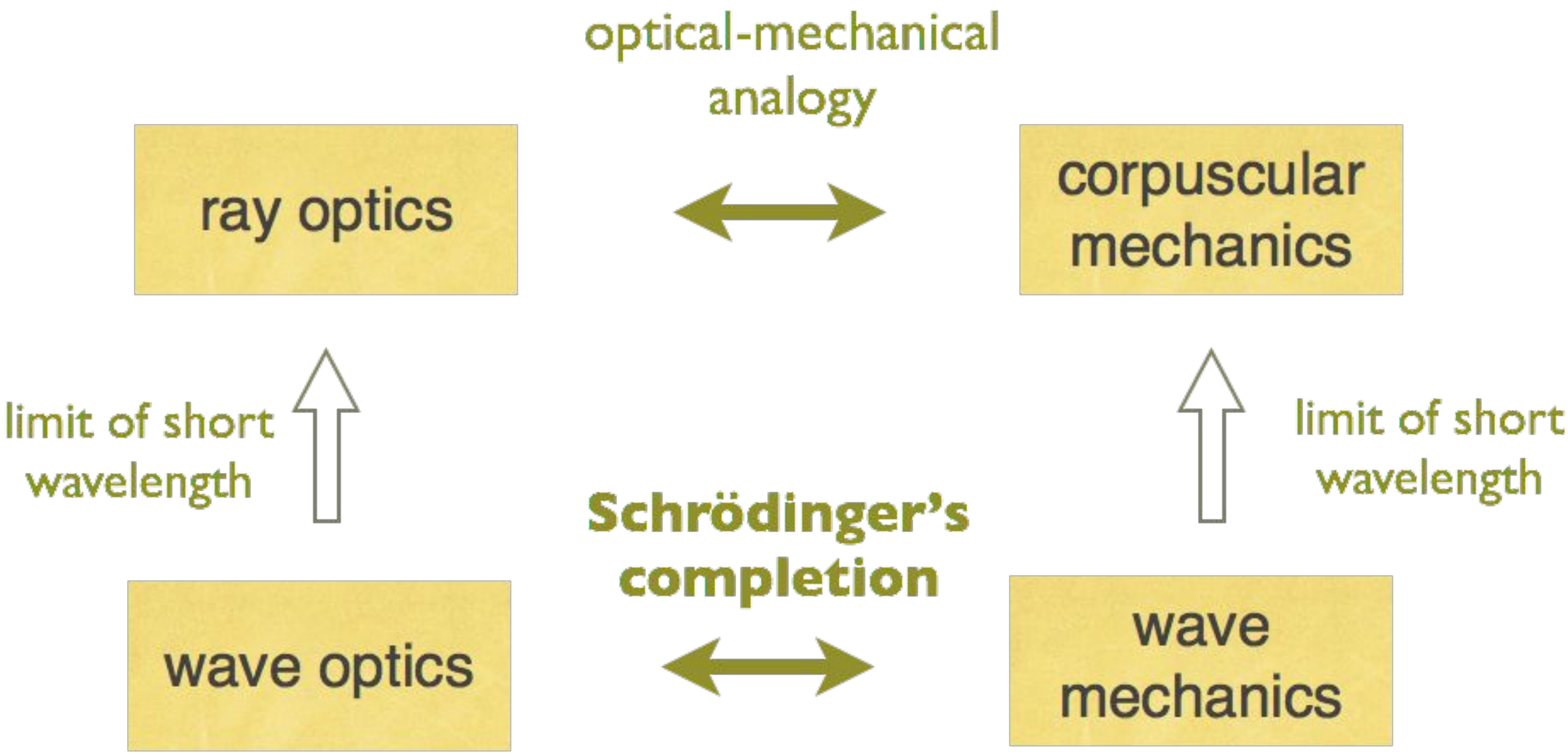
Thanks to Christian Joas

Schrödinger’s analogy/completion



Thanks to Christian Joas

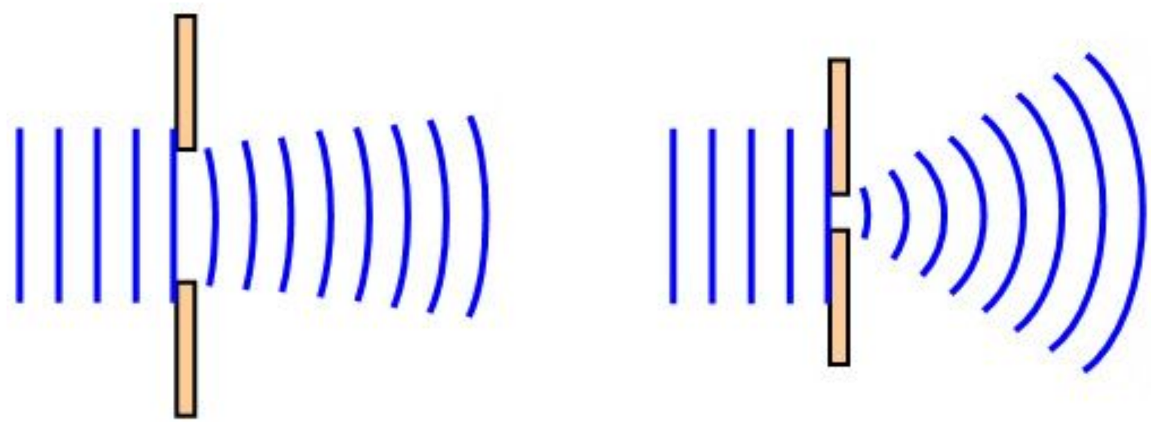
Schrödinger's analogy/completion



Corpuscular mechanics is merely a limiting case of a more general wave mechanics!

Thanks to Christian Joas

Schrödinger's analogy/completion



The step which leads from ordinary mechanics to wave mechanics is an advance similar in kind to Huygens' theory of light, which replaced Newton's theory.

Ordinary mechanics : Wave mechanics = Geometrical optics : Undulatory optics.

Typical quantum phenomena are analogous to typical wave phenomena like diffraction and interference.

Some lessons Episode 3

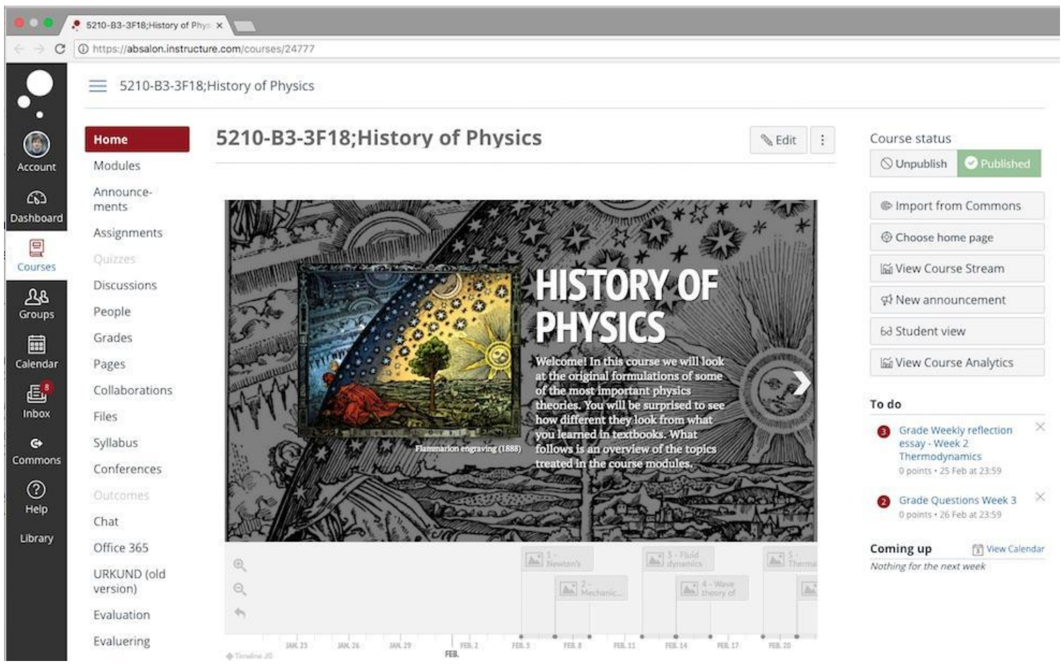
- Where does the Schrödinger equation come from?
- Ontological commitments matter!
- The real meaning of wave mechanics
- For Schrödinger, there are NO particles, only waves (in "config. space")
- Schrödinger <u>never</u> accepted the probabilistic (Born) interpretation

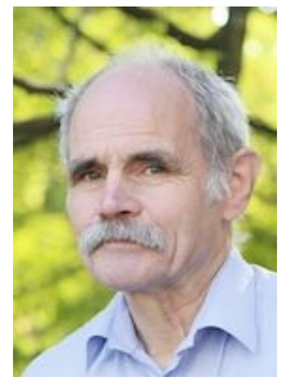
Perguntas/Comentários?

Interested in original texts/reasoning?

- History of Physics course

https://absalon.ku.dk/courses/24777





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Muito obrigado!

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