




Problems and the History of Mathematics Education: A Proposal for the Continued Teacher Training

Andreia Fernandes de Souza ^a
 Luciane de Fatima Bertini ^b
 Alan Marcos Silva de Rezende ^a

^a Universidade Federal de São Paulo (UNIFESP) – Campus Guarulhos, Programa de Educação e Saúde na Infância e na Adolescência, Guarulhos, SP, Brasil.

^b Universidade Federal de São Paulo (UNIFESP) – Campus Diadema, Programa de Educação e Saúde na Infância e na Adolescência, Diadema, SP, Brasil.

ABSTRACT

This text is about a continued teacher training which aim was to use the history of mathematics education like a leading to problematize the use of problems to teach mathematics and the relation with the proposes of teach in different moments (historical and actual) and the changes and continuities over time. For this, as a start point, articles of pedagogical journal and textbooks from the last and present century were used to talking about the problems. After the meetings, we can conclude that the continued teacher training was a favourable environment to discuss and to think about the role of the teacher. The problematization in the perspective of the history of mathematical education was a possibility found to approximate the academic knowledge and the school culture; this contributed to the denaturalisation of the daily practices of the teacher, understanding that is not something immediate but a process. If we think in a metaphor, the idea is that paying attention to how the trains arrive at station gives us the possibility to see the different paths it has travelled and obtain a better understanding of its arrival.

Keywords: Mathematical problems; Teacher training; History of mathematics education.

Problemas e a História da Educação Matemática: uma Possibilidade para a Formação Continuada de Professores

RESUMO

Este texto é um relato sobre uma formação continuada de professores cujo objetivo foi fazer uso da História da educação matemática como norteadora para problematizar o papel dos problemas no ensino de matemática, a sua relação com as propostas de ensino em diferentes momentos (históricos e atual) e as mudanças e continuidades ao longo do tempo. Para isso, como ponto de partida foram utilizados manuais de ensino e artigos de revistas pedagógicas do século passado e atual para tratar de elementos acerca dos problemas. Após os encontros podemos concluir que a formação continuada foi um ambiente favorável para discussões, reflexões e troca de conhecimento acerca do papel do professor. Partir da problematização na perspectiva da História da educação matemática foi uma possibilidade encontrada de aproximar o conhecimento dito acadêmico e o ambiente da cultura escolar, e que contribuiu para a desnaturalização das práticas cotidianas do

Corresponding author: Luciane de Fatima Bertini. E-mail: lfbertini@gmail.com

professor, entendendo que não se trata de algo imediato e sim de um processo. Se pensarmos em uma metáfora, a defesa é a de que se atentar para o funcionamento de como o trem chega na estação nos dá a possibilidade de olhar para os diferentes caminhos percorridos por ele e melhor entender a sua chegada.

Palavras-chave: Problemas de matemática. Formação de professores. História da Educação Matemática.

INTRODUCTION

“The arrival of the train at the station” film of the Lumière brothers, exhibited in Paris in 1895, caused the public certain awe to see moving images projected on a screen. Some people say that the panic took over the people they were attending, to the point of motivating them to leave the session.

Taking the train at the iconic station of Luz, built at the end of the 19th century in São Paulo, 123 years after the historical exhibition, reminded us that if the previous shock was the projection of the train, today it is due to the number of people who cross the city to fulfil their commitments. Many of us did not, despite living here, known part of the suburb of São Paulo.

The train begins the approximately 40-minute journey that separates the centre of São Paulo from the Guaianases neighbourhood in the East Zone.

The landscape changes over 38 kilometres, the Tamanduateí river surrounded by older buildings, the Brás district, the train line following the Radial Leste highway, the new and pompous buildings, the traffic stopped, the bars, the narrowing of the avenue, the buildings with less floors, the appearance of houses, vacant lots, full bus stops, soccer fields, everyday commerce, people walking on foot and, finally, “the arrival of the train in the station”.

The train is still full and for many of those passers-by that had just been part of the trip, for us, it ended there. Informal commerce takes care of the surroundings of the station: fruit, bread, manioc, flour, popcorn, potato, or even cold water, and we go ahead for a few meters, seeing political opinions plastered on walls people walking to their homes. We arrived.

However, what would a group of researchers in History of Mathematics education be doing in the Pedagogical Board belonging to the Municipal Secretariat of Education of São Paulo? Were they looking for new sources? Perhaps new sources for the training of teachers who teach mathematics in the early years.

The initiative for the planning of this training came from the participation of the authors of this report in the Research Group on History of Mathematics Education (GHEMAT). In the year 2018, GHEMAT became part of the network of Institutes of research on the teaching of mathematics (*Instituts de recherche sur l'enseignement des*

mathématiques – IREM)¹ already present in 31 French cities. The network of institutes focuses on the teaching of mathematics as well as the concern with teacher training that relies on research.

Thus, the intention arose to carry out a project of continuous formation of teachers of the initial years from the researches carried out by the group that involve the History of mathematical education. Considering that

[...] a mathematics teacher who maintains an a-historical relationship with his professional ancestors may, with the appropriation of this history, relate less fancifully and more scientifically to that past. This tends to change their everyday practices, which are more consistently realized. (Valente, 2013, p.26)

Specifically, we chose to take the problems in mathematics classes as the central theme of the training making use of the History of Mathematics Education as an element of formation, considering that “the study of the history of mathematical education creates the possibility of distorting curricula, to the teaching and learning of Mathematics” (Oliveira, 2017, p.658).

It is worth considering that this possibility is reinforced by the fact that the problems are present in the guidelines and proposals for the teaching of mathematics in different historical moments, including nowadays.

The training was held in the second half of 2018, totalling 24 hours, including five face-to-face meetings and the preparation, application and final presentation of an activity carried out in the classroom. Of the thirty-six teachers, most of them worked in primary education (1st to 5th year) and some in early childhood education (0 to 5 years). All with full superior education and holders of position, that is, passed in public competitions. The course² was offered outside working hours and therefore was optional. The professors were entitled to a certificate that could be used for purposes of functional evolution in the teaching career.

Thus, in this scenario, the objective of the training was to discuss the purposes of the problems in the mathematics classes starting from the historical perspective and the articulation with the new curriculum for mathematics teaching, launched in the final year of 2017 by the Municipal Secretary of Education of São Paulo.

¹ More information at: <http://www.univ-irem.fr>

² Registered as an Extension Course at the Federal University of São Paulo under the coordination of Professor Luciane de Fatima Bertini.

PROBLEMS AND THE HISTORY OF MATHEMATICAL EDUCATION

The proposal for training was to make use of the historical perspective in continuing education in order to promote discussions about the role of problems in mathematics teaching, its relation with teaching proposals in different historical moments (including the current one) and on the changes and permanence over time.

Such initial reflections were triggered by questioning; some of the answers will be presented throughout this text as a way to exemplify part of the discussions in the training course. For example, when asked how they relate to discipline, we get an answer like: “I like it when I understand”; “Rules I do not like”; “I’ve been more aristocratic”; “If I had learned otherwise I might have liked it more”; “I only like mathematics in elementary school”; “I have skills for myself, but sometimes I cannot explain to the students” and so on.

In these lines, it is possible to perceive aspects related to mathematics as a disciplinary knowledge and to the ways of learning and teaching mathematics, teachers show carrying with them their experiences as students. However, is the mathematics of school culture the same as mathematics? Is there a single math? Is it necessary to know much more mathematical content to teach the students or would it be enough to know the contents pertinent to the age group of whom I am going to teach? Is it important to know, beyond the content, ways to teach them? What do I need to know to teach math?

In order to trigger and consolidate such reflections we present some elements of texts that involve discussions about school culture (Julia, 2001) and on the relations between academic mathematics and school mathematics (David, Moreira, Tomaz, 2013).

The readings lead to more conversations: is there only mathematics or math? Some scholars argued that perhaps in some times this single mathematics was “plastered”, it has now become “lighter”. How many people were “like my father who did so many different calculations and had bad grades at school”, could be because “the student understands otherwise” this “science that imprisons you”.

An element of school culture that helps us perceive these transformations in the teaching of mathematics is the textbook. Official documents such as PCN³ or a BNCC⁴ were taken as the starting point for authors and publishers of textbooks. However, has this influence come recently? Has the textbook always been present in schools? Moreover, has your format been modified? What changes and permanencies do we observe in this journey? Moreover, the problems as they were taught?

³ Parâmetros Curriculares Nacionais. Available in <http://portal.mec.gov.br/seb/arquivos/pdf/livro03.pdf>

⁴ Base Nacional Comum Curricular. Available in <http://basenacionalcomum.mec.gov.br/wp-content/uploads/2018/02/bncc-20dez-site.pdf>

The discussion started and the teachers were able to say what they thought about what they are and why they use problems. Many have defined problems as “something that needs to be solved”, “imitation of reality”, and their use in class said, for example, that it would be important to “strategize”, “understand the context of a problem”, “or “solving in different ways”.

After this initial survey, teachers were invited to explore some textbooks available at the Digital Content Repository of the Federal University of Santa Catarina. Among those suggested were “Primary Arithmetica” (18--–) – Antonio Trajano, “Manual of Primary Education for the 4th. Year” (1939)– Miguel Milano, “Our Mathematics Exercises – 2nd Elementary Year” (1960) – Noelly Sagebin and Margarida S. Sirangelo (Figure 1).

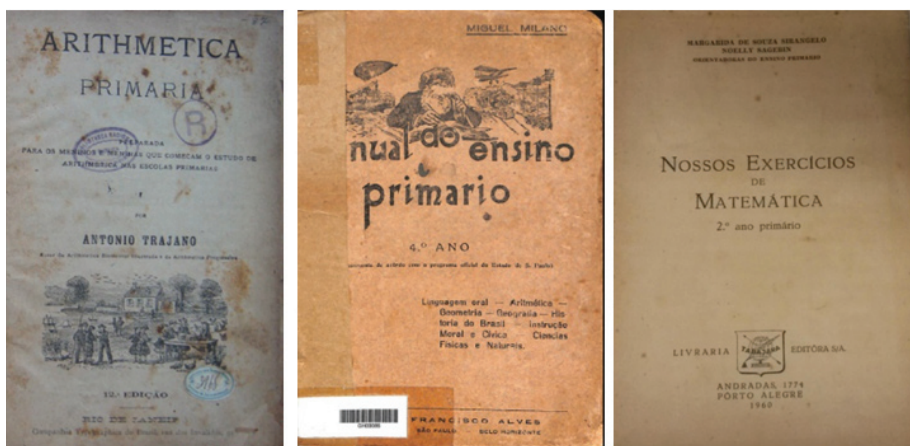


Figure 1. Covers of books observed by teachers.

We noticed that the teachers were mostly curious to see these copies. They found interesting the organization of the books and made some notes, such as the type of questions, the value of the products, the endless list of exercises, the repetition, and the ethical / moral values conveyed from problems such as “she gave your poor friend”.

We reflected on how problems appeared, initially nothing changed beyond vocabulary, seals by cards, fabric for toys, among other examples, but when we approach the lens of research in History of mathematics education we can see changes, some conceptions have changed and the ways of presenting and solving a problem as well.

It is important to emphasize that our intention was not only to use historical documents in an illustrative way, but to make use of results of researches in History of mathematical education that have used these documents, in this perspective the results of these researches offer us elements to think the teaching of math. Thus, in addition to exploring textbooks from other historical moments, teachers were able to take contact with research results and used them as source (Bertini, 2016; Bertini, 2018; Santos, Búrigo, 2016; Mendes, Valente, 2017). From the results of these researches, aspects such as the

presence of psychology in the orientations of how one teaches and how one learns and the relation of the teaching of mathematics with pedagogical proposals in circulation for teaching in general, which allowed elaborations on changes and permanence in teaching over time and also school mathematics and academic mathematics.

At the next meeting, we return to the definitions of problems and observe how they are proposed in contemporary textbooks for each elementary year.

In this analysis teachers observed the problems arranged in different forms and purposes: from everyday situations (cinema, bakery, fair, school, shopping, zoo etc.), in games such as using tracks, data, letters and etc. to evaluate the contents proposed in the units, in “problematecas” in order to offer students different types of problems.

Teachers also analysed support materials developed by municipal public networks (Student Notebook – City Curriculum – Mathematics)⁵ and State (EMAI – Mathematical Education in the Early)⁶ In the first, they perceived the interdisciplinarity as guiding the material, being that the objectives of each unit were not so clear, already in the second the objectives are more focused, even leaving aside interdisciplinary aspects.

The “Curriculum of the City – Mathematics” addresses the contents and objectives for the nine years of elementary school divided into three cycles of three years each: literacy, interdisciplinary and author. Throughout this document, it is possible to perceive the emphasis given to subjects such as games, modelling, algebra, statistics and problems. Regarding the problems, the municipal network this year began a follow-up of students’ probes of both numbers and problem solving. The survey is an individual assessment carried out by the teacher, in which he synthesizes the information regarding numerical writing and resolution strategies in a digital system monitored by the municipal education department. Since problem assessment is embedded in an environment in which coordinators, principals, and teaching supervisors have access, there is some interest in discussing strategies for how to solve them and their different typologies.

Teachers, observing these works, mentioned the “old” books presented at the first meeting. Comparisons about what was in vogue before and currently permeated the discussions, as a way of approaching and distancing between the proposals.

In exploring all the documents, both historical and current, the objective was not only to identify thematic and types of problems, but mainly to identify the role of the problems in the teaching of mathematics.

Moreover, how not to problematize the teaching practice? The teachers were invited, after these analyses, to elaborate, an activity using problems and to justify this presence, placing what would be the purpose of these activities.

Infancy teachers proposed a circuit activity in which infants (0 to 3 years old) would be challenged to devise strategies to overcome various obstacles. Those in the first year

⁵ Available in <http://portal.sme.prefeitura.sp.gov.br/Portals/1/Files/44132.pdf>

⁶ Available in <http://lereescrever.fde.sp.gov.br/SysPublic/InternaMaterial.aspx?alkfjlkjaslkA=301&manudjns=2&tpMat=0&FiltroDeNoticias=3>.

thought of taking the students in the video room and organizing them from the numbers that would be drawn and each child would go to the raffled chair. By the 2nd year, teachers thought of problems they would deal with on the days of the week. The 3rd and 4th year group thought about measures of length and the problems would be the starting point to think about some standardized measures. The fifth year teachers thought of using logic problems with different themes, which appeared even in textbooks, or planning a recipe collectively using concepts such as proportion.

At the third meeting, the teachers socialized the planned activities. The problems appeared in different situations: as trigger of an activity in the activities on the measures of length; open problems; problem as strategy in circuit activities and in the elaboration of a recipe; as a check on the activities involving the days of the week.

From the analyses and the proposals, we ask, “only in textbooks can you find indications of how to teach problems? What other pedagogical materials could contribute to a reflection on the practice?”

Thus, we present the pedagogical journals (Figure 2) as a trigger for reflections at this point in formation. The challenge was to read early 20th century articles in groups and to understand what the authors indicated for teaching problems.

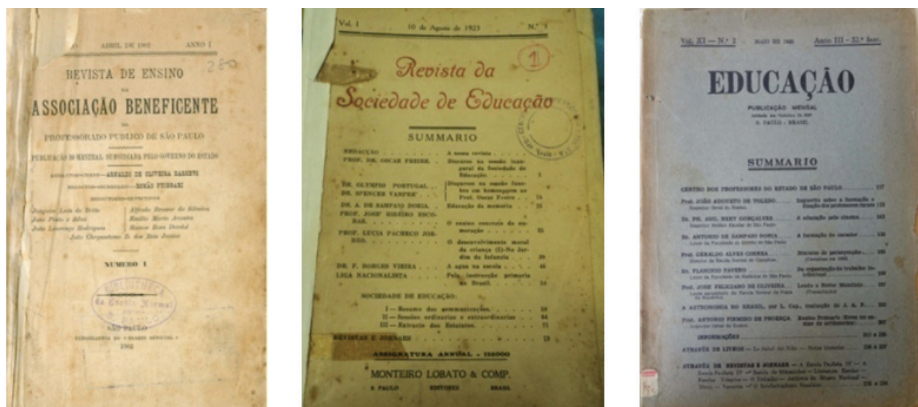


Figure 2. Covers of pedagogical journals observed by teachers.

We distributed two different articles for the six groups. In the article by Flávio Monteiro, published in the *Revista de Ensino* in 1916, the students observed with their current work: “from the easy to the difficult”; “Repeat operations”; “Need for constant revisions”; “Need to use problems”.

The article written by José Escobar, published in the Journal of the Society of Education in 1923, had a lesson plan on number six. The teachers were surprised by the propositions: “a whole class for the number 6?”; “Start with the concrete, we already do it”; “It’s a recipe”; “If all activities have the result six students would already know the answer”; “First the objects, then the operations and finally the problems”.

In the words of the teachers, we perceive two aspects: identification and judgment. The identification of current procedures in these articles and the discovery that this teaching methodology has been present in the discussions for nearly a century, but is not yet fully appropriated by teachers. The judgment when reading the articles and fail to conceive that the authors underestimated the teachers by giving ready recipes and the students by the questions so simplistic.

To conclude these reflections, we present to teachers an article written by Anna Nogueira Ferraz, published in 1929 in the Education Magazine, in which the author brings a step by step on how to teach problems, details of how context, interpretation, resolution, socialization response and correction were important for learning. The students were surprised to discover that the article published in 1929 referred to a 1905 book in which Victor Mercante, an Argentine pedagogue, suggests a sequence to teach problems in his book "*Cultivo e desarrollo del niño*".

As in the case of books, the discussion about the presence of problems in journals also involved the results of researches in History of Mathematics Education that carried out analyses of articles in journals seeking articulation with the teaching proposals of each epoch (Souza, 2017; , Santos, 2015, Valente, Pinto, 2016). The option for the articles to be read was made from these studies.

The fourth meeting started with the book "Problems Boborildos" by Eva Furnari, in which problems are presented with different characters in unusual contexts, with the aim of arousing students' interest in solving. The book was suggested by one of the students.

After the nomination, we asked teachers what materials they used in their classes. For counting, some cited the abacus, the overlapping chips for the conventional writing of the numbers and golden material for acquiring the concept of the decimal number system. We questioned whether the teachers had already used some concrete material to solve problems; there were no demonstrations.

We introduced the Tangran and proposed that the teachers assemble in pairs a square using the seven pieces. As most teachers already knew, they completed the task more quickly. We propose that they reassemble a square using two pieces only. The resolution was also quick, however when asking to make a square using three pieces the problems began to appear. In addition, when they were solved we would add another piece to get to the square again. The teachers tried and we realized that there was more difficulty.

During the proposal to assemble squares with the pieces of the Tangran, mathematical contents that could be worked out, as, for example, equivalence relations between the pieces and geometric forms were presented. Other possibilities of activities were also presented, such as colour relations, figure set-up and story creation with material pieces.

In socialization, the teachers said how they had set up the squares and we presented other possible solutions. With this activity the idea was the discussion of the equivalence

relations between the pieces, which can all be decomposed into small triangles (smaller piece of Tangran). The goal was to explore a way to use problems as a starting point and to discuss possibilities from this option. One of the possibilities explored was to use Tangran to propose problems involving equivalence as a starting point for the teaching of fractions. These discussions somehow disturb teachers who make statements such as: *“I do not know the content of all forms and so I cling to what I’m sure”*; *“When I’m going to teach 5th grade I have to study”*; *“I always ask for math teachers”*; *“Whoever did different goes to the blackboard to explain. If you miss, let’s think together”*; *“A teacher called me on the blackboard and it scared me”*.

In the perspective of discussing also current proposals for the use of problems in mathematics classes, we have brought to the meetings the proposal of problem solving as a teaching methodology, defended in Onuchic’s work (2014) and the proposals presented in the work of Smole and Diniz (2001) to work with problems in the initial years of elementary education. Such perspectives were not addressed in the sense of pointing out a correct path, a way to be followed, but in order to discuss the role of problems in each proposal and its articulation with current guidelines for teaching mathematics, including the official proposals already explored.

Again, we take up with the teachers concepts of what are problems, how they can be inserted in math classes and their objectives. We proposed a new round of planning of collective activities, separating teachers by year of performance.

The time between this and the last meeting was approximately 45 days, in which they would have to apply the activity, socialize among the group, and prepare a collective presentation and an individual report. The last meeting allowed the teachers to socialize their activities planned in-group and applied in their respective classes.

Teachers of Infant Education used circuits with higher levels of difficulty for infants; the challenge of shoes, in which the children have their shoes mixed and need to find and put on; and even the Tangran to assemble figures and superimpose pieces. In the classes of the 1st and 2nd year, the teachers chose games, playing with bowling and track, marking the points and from these they proposed problems relating the context of the game and arriving at the calculations. In the 3rd year in the activity developed from the length measurements, a distance jump dispute was made in which the measurements were used to create problems. The group of teachers of the 4th year brought the mathematical games made to the Science Fair of the school, in this event the students could choose a game of logical reasoning and test several possibilities of resolution. For the 5th year students, the activity performed was the “Slime” recipe, a type of a modelling mass, in which students calculated the value of the prices of the items, the proportion in the increase of the revenue and from these situations were solved problems.

In the presentations, the discussions about the problems, about their role in the presented proposal, did not gain prominence. They were centred on resources and playfulness, which are important elements in teaching practice, but which in themselves do not play the role proposed by the problems.

Finally, we conducted a questionnaire about the importance of training for them. On the use of the historical perspective in teacher training, we obtained answers such as *“It is always important to bring historical perspective so that we can understand how it all began, what was thought and how it evolved over time, whether or not there were changes to teacher practice and student development. In this way, we can reflect on our practice, bringing new ways of thinking”*.

Teachers also wrote about how this might contribute to changing / complementing the understanding of problem use in math classrooms: *“It contributes a lot, I learned that it is important to start from a problem and systematize so that the student learns in a significant way. Before I followed the contents of the textbook without caring about the context or difficulty of the student”*; *“The training contributes much to the deconstruction of paradigms of how the problem is presented to the students with different objectives, not only as the last strategy and consolidation of concepts. The problem situation as construction of a logical, procedural and challenging logical reasoning”*.

Assertions like these give us elements to consider that the use of the History of Mathematics Education offers potentialities in the formation, in this specific case, having offered opportunities for discussions and reflections about the use of problems for the teaching of mathematics, as already pointed out by Valente (2013) and by Oliveira (2017).

SOME CONSIDERATIONS

After the meetings, when reading the written evaluations and observing the presentations of the developed activities, we can conclude that the continuous formation is a favourable environment for discussions, reflections and exchange of knowledge about the role of the teacher. Elements that had as guiding point the History of the mathematical education for the problematization of the use of the problems of mathematics in historical and current perspective.

We consider that the work in this perspective is a possible way of approaching the so-called academic knowledge and the environment of the school culture through a perspective other than that of the curriculum already systematized. In the sense that from the use of history it is possible to identify elements of changes and continuities in the process of elaborating official guidelines for teachers, in particular, for teaching problems.

We are aware that few training meetings are not enough to exhaust discussions on a theme and that, possibly, are not sufficient to cause effective and immediate changes in teachers' practices. However, we consider that discussions based on the questions about why problems in math classes and their role in teaching proposals were triggered by many reflections that may accompany teachers in their classes, in their use of guiding material (books, articles, curricula, teaching programs, etc.).

With regard to the presence of the History of Mathematics Education in continuing formations we can think

[...] the history of mathematical education is important to understand the problems of the present ... Such assertions are no less a siren song, in an area where the need to solve immediate problems tends to absorb proposals of an extremely pragmatic nature. (Valente, 2013, p.26)

Perceiving transformations in the use of problems in mathematics classes throughout history still seems to be more related to the search for answers in theoretical researches in the area of History of mathematical education. The demand of teachers who teach math in the early years seems to be practical. How to work out problems? What are conceptual fields? Are there different types of problems?

However, the contact and the discussions from school textbooks and magazine articles was an opportunity to denaturalize looks that were constituted about the problems throughout the teaching career of each teacher.

We wish that the professors in other moments of reflection, conversations, formations, or even in their teaching practice, could resume discussions carried out in order to problematize the current conceptions. We consider that the denaturation of daily practices occurs in a continuous process of movement, of comings and goings, of retakes, of looking differently at the same landscape (practice), knowing where we left off and where we want to arrive, as in the route travelled before “Train Arrival at the Station”.

ACKNOWLEDGEMENTS

Special thanks to Ester Marques Dionísio, director of DIPED / DRE Guaianases, teacher of the late CEFAM, for having granted the space and Bianca Freire, educational assistant of DIPED / DRE Guaianases, a teacher of the municipal public network, who welcomed us and accompanied us during the formations.

AUTHORS CONTRIBUTIONS STATEMENTS

L.F.B. was the responsible for the idealization of the teacher training course and responsible for conducting the meetings. A.F.S. and A.M.S.R. accompanied the entire training and were responsible for planning and developing one of the activities of the course, together with the students. A.F.S. was responsible for the idealization of the format of the proposed text. All authors discussed the results and contributed to the final version of the manuscript.

REFERENCES

- Bertini, L. F. (2016). O Manual do Ensino Primário, de Miguel Milano: que problemas? *Revista de História da Educação Matemática*, 2, 117-129. <https://repositorio.ufsc.br/handle/123456789/177618>.
- Bertini, L. F. (2018). Problemas de aritmética na escola primária no final do século XIX: aplicação, ilustração ou introdução dos estudos? *Jornal Internacional de Estudos em Educação Matemática*, 11(1), 72-79.
- Búrigo, E. Z. & Santos, J. G. (2015). Os problemas de aritmética na Revista do Ensino dos anos 1950. In: *Anais do XII Seminário Temático – Saberes elementares matemáticos do ensino primário (1890-1970): o que dizem as Revistas Pedagógicas?* (pp.1-9), Curitiba. Recuperado em 20 de novembro, 2018, de http://www2.td.utfpr.edu.br/seminario_tematico/artigos/54.pdf.
- David, M. M., Moreira, P.C., & Tomaz, V. S. (2013). Matemática escolar, matemática acadêmica e matemática do cotidiano: uma teia de relações sob investigação. *Acta Scientiae*, 15(1), 42-60.
- Julia, D. (2001). A cultura escolar como objeto histórico. *Revista brasileira de história da educação*, 1.
- Mendes, I. A. M. & Valente, W. R. (Orgs.) (2017). *A matemática dos manuais escolares: curso primário, 1890-1970*. São Paulo: Livraria da Física.
- Oliveira, M. C. A. (2017). História da educação matemática como disciplina na formação de professores que ensinam matemática. *Cadernos de História da Educação*, 16(3), 653-665.
- Onuchic, L. R. (Org.) (2014). *Resolução de problemas: teoria e prática*. Jundiaí: Paco Editorial.
- Santos, J. G. & Búrigo, E. Z. (2016). Representações do cotidiano em livros de exercícios de matemática do Rio Grande do Sul nos anos 1950. In: *Anais do XIV Seminário Temático: Saberes Elementares Matemáticos do Ensino Primário (1890-1970): Sobre o que tratam os Manuais Escolares?* (pp.1-15). Florianópolis: UFSC. Recuperado em 20 de novembro, 2018, de http://xivseminariotematico.paginas.ufsc.br/files/2016/08/SANTOS_BURIGO_T2vf.pdf.
- Smole, K. S. & Diniz, M. I. (Org.) (2001). *Ler, escrever e resolver problemas: habilidades básicas para aprender matemática*. Porto Alegre: Artmed.
- Souza, A. F. (2017). *Discursos sobre problemas aritméticos (São Paulo, 1890-1930)*. Dissertação (mestrado). Universidade Federal de São Paulo, São Paulo. Disponível em: <https://repositorio.ufsc.br/handle/123456789/178612>.
- Valente, W. R. & Pinto, N. B. (Orgs.) (2016). *Saberes elementares matemáticos em circulação no Brasil: dos documentos oficiais às revistas pedagógicas, 1890-1970*. 1. ed. São Paulo: Livraria e Editora da Física.
- Valente, W. R. (2013). Oito temas sobre história da educação matemática. *Revista de Matemática, Ensino e Cultura*. Natal, RN. 8(12), 22-50.