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History for the Teaching of Mathematics: Transformation and Mobilization of Mathematical Knowledge for School

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ABSTRACT

This article aims to discuss processes of transformation and mobilization of knowledge related to the history of Mathematics as a teaching approach for mathematics at the Basic Education level. The core of the discussion is to highlight ways to explore this theme and point out guidelines to integrate information related the historical development of mathematics and its didactic uses in the classroom, associated with the rigor and naturalness appropriate to the treatment of the mathematical content stipulated in school curricula. In this sense, I highlight some possibilities of offering mathematics teachers in Basic Education and undergraduate courses didactic guidelines that contribute to their teaching practice. I also indicate central aspects to be focused on when inserting the historical dimension in mathematics classes as a thematic and material presentation, a conceptual development built from the exploration of primary or secondary sources in the form of didactic activities that can be used by the teacher to introduce, illustrate, or deepen a concept that is to be taught.

Keywords: mathematical histories, teaching knowledge, mathematics teaching

INTRODUCTION

The need to discuss the training of teachers in their current situations has worried me very much in the context of my actions as a professor of teachers who teach mathematics, especially with regards to the professional knowledge concerning mathematics' formation and teaching action. Investigating the historical constitution of this knowledge can offer elements for understanding what is taught and the ways in which mathematics is taught or can be taught nowadays. It is about reflecting on the implications of the epistemological constitution of what is taught and how it is taught, in view of the type of school, students and society one has and those that one wants for the future.

Do the teaching methods currently adopted in Mathematics Undergraduate courses and in Basic Education meet the interests of students at these levels of education? What do students learn from the way we teach or the way we believe we teach? I certainly have no answer to such questions, but ever since I was a student in a Mathematics undergraduate course, I have questioned myself and worried about the attempt to find answers to these questions. Because I consider that such responses are not definitive or absolute, but always in the process of enlargement. These are some of the questions raised when we deal with the teaching of Mathematics and to think about such issues includes thinking about multiple approaches to accomplish student learning.

Thinking about the transformations of this knowledge in a continuous process has led me to ask, initially, where do the types of mathematical school knowledge incorporated into historically established teaching models, in different social contexts, arise from? How were these types of knowledge transformed, in the teaching process of these space-time trajectories? What do we inherit and what do we bequeath as a school cultural heritage for today's society and the future generations? How the stories of school mathematics can be used to explain that historically *the mathematics that is taught and the mathematics that we must know in order to be able to teach*[1], partly reflect some of the models of representation of mathematics and its teaching model, which throughout of the times were incorporated into the school culture? These are some questions that demand to be investigated and discussed due to their relevance to the transformations that happen to different types of mathematical knowledge in undergraduate courses for teachers who teach mathematics.

Throughout my teaching experience, I realized that using research in mathematics' teaching provides students with an opportunity to exercise reading, writing and to discuss mathematical ideas, as well as their relationship with other areas of

[1] In this article, the expression stories of mathematics to be taught and mathematics to be taught were taken in the sense of historical knowledge related to mathematics to be taught at school and to mathematics that the teacher must know to properly teach this mathematics to students.

knowledge. In the last two decades (1995-2018) I noticed that such an exercise can be enriched when it is associated to historical aspects that involve the production of mathematical knowledge in the time, space and the socio-cultural contexts in which this knowledge was produced and used. For this reason, I consider this to be one of the productive ways which one can use in order to achieve a teaching of mathematics that promotes an education that is autonomous, creative and enlarges human cognition.

Currently, the amount of studies on possible didactic approaches that can be proposed for mathematics' teaching based on the history of this discipline has increased. One of the ways indicated to put this pedagogical perspective into practice is to revisit, in the best possible way, the historical moments that involve the notions, concepts and mathematical properties that one intends to teach. This approach provides students with the occasion of practicing studies, research and problematizations that stimulate their thinking strategies and, consequently, produce knowledge during their school activities. Such a didactic approach assumes that students can have the enriching opportunity of inserting themselves as much as possible in the mathematician's historical context, the context in which the mathematical text was written, in the community in which the mathematician lived, worked and produced such mathematical knowledge, with the aim of establishing a multiple explanation for the mathematical notions they will need to learn.

Thus, in this article I consider it important to reflect on processes of transformation and mobilization of mathematical knowledge to be taught and knowledge to teach mathematics based on the mathematical knowledge identified in the historical development of the different types of mathematics. I mainly reflect from my work as a researcher and professor in a Mathematics undergraduate course, for three decades (1990-2017), as well as in the continuing education of Mathematics teachers, in search of overcoming the conceptual and didactic difficulties of teachers in order to contribute to students' mathematical learning.

However, it is not just a matter of showing that the concepts covered by academic mathematics have a history, but it also reminds us that it often dates back to the birth of history itself, and that this view can be interpreted as if saying that everything that is taught nowadays has already been thought out and practiced by others for a long time. We cannot limit ourselves to the present time or to the first occurrences of a mathematical concept or topic without following procedural developments that involve practices and theories without a hierarchical order, which students will be submitted to when we chose how to deal with mathematical themes at school, as we often tend to do, in our courses.

It is important to recognize, therefore, that this manner of proposing the insertion of history in the mathematical explanations that are given in the classroom is composed of other aspects that may show the different ways through which a subject related to mathematics developed in time and space, and how this subject became a theory in the academic field through the elaboration of questions, answers, new questions and problematizations, which consequently raised the need for an axiomatization of such a subject (concept, notion and theory).

In order to take the first step towards understanding this process and establishing actions and connections between mathematics, its history and its teaching, it is necessary to clarify a few things about the meanings attributed to the word *history*, such as: the place of mathematics in this history, so as to provide informative material for the accomplishment of transformations that contribute to the exercise of meaningful teaching and learning of mathematics. The history of mathematics is not only a history of the definition of mathematical objects, but that of a creative process that involves society, culture and cognition.

So that we can materialize our guidelines in the search for the meaning of these histories towards a transformation of the knowledge related to school mathematics, we will focus, hereafter, on some questions that will take us further. The first question pertains to the explanation of which history do we deal with in our proposal and the place of mathematics in this approach, that is, which histories of mathematics can we taken, transformed and mobilized for the teaching of mathematics in the form of school knowledge to be taught and professional knowledge to teach mathematics?

Likewise, to enter the paths that can enlighten the topic, which I propose to establish, I choose to emphasize beforehand the meanings and senses of the words *knowledge*, *professional knowledge*, *professional knowledge of the teacher* and *professional knowledge of the mathematics' teacher*, for these preliminary explanations must necessarily come before I clarify my understanding of the *knowledge to be taught at school and professional knowledge to be able to teach at school* in the their relationship to history applied to the teaching of Mathematics and how such knowledge can be included in the activities of school mathematics.

Thus, I admitted the need to assume a central question as a guideline: how were mathematics instituted and how did they establish themselves as disciplinary fields and disciplines, at local, regional or national levels? How did the history of mathematics institute and establish itself as a discipline in Mathematics undergraduate courses in Brazil, especially when it comes to shaping itself as different kinds of knowledge in constant transformation, as they have been understood in the approach called history of Mathematics for teaching. Such an approach understands the history of Mathematics as a process of mobilizing disciplinary knowledge to be taught in mathematics and professional knowledge to teach mathematics, that is, to understand the history of mathematics as a research method, a discipline and an instrumental basis for teaching.

HISTORY OF MATHEMATICS AS RESEARCH METHOD, DISCIPLINE AND INSTRUMENT FOR TEACHING

Regarding this epistemological construction, I deepened my reflections on the historical dimension to be included in didactic activities focused on school mathematics. I outlined, tested, reformulated activities, and pointed out theories related to this subject based on my reflections and on dynamic exercises of reorganization and reinvention. These exercises were expanded to include principles related to the epistemologies of mathematics. Such principles directed my focus to the process of establishing

a historical-cultural dimension that deals with the history of socially produced ideas and characterizes the mathematics produced by societies over time and in multiple socio-cultural spaces, unfolding in three major axes: research, discipline and the mathematics teaching instrument.

The history of mathematics as a discipline has a natural vocation to use the knowledge produced by research and recorded in socially disseminated documentary sources. It underpins the subjects that deal with conceptual and epistemological training in Mathematics undergraduate courses and is characterized by its organization under three approaches: history of mathematical topics; history of mathematics to be used in the classroom; history of Mathematical Education. It also acts directly in the configuration of the different kinds of knowledge produced over the centuries which are normally registered in information sources such as written documents, material artifacts, works of art, architectural works or any other type of material that indicates the production of ideas intended to make sense of human existence on the planet. Thus, all objects and instruments that refer directly or indirectly to this kind of activity can become vehicles for the representation of historically produced mathematical knowledge.

The exercise to be practiced means research as an experience that makes it possible to recreate the history of mathematics to make students reflect on the socio-cognitive strategies created throughout our history to understand the same mathematical facts practiced or created by society. It is essential, therefore, to reflect on the possibilities of using in mathematics classes, in a didactic way, the information one finds in documentary sources, artifacts, works of art, and in architectural constructions, among other material objects, so as to contribute to the effectiveness of a meaningful teaching of mathematics. It is a perspective of conceptual and didactic training of mathematics teachers that benefits students at the Basic Education level. This owes to the fact that if the mathematics teacher has a conceptual and didactic training based on the epistemology of mathematics, they may possibly be able to unfold this construction of knowledge with their students in the classroom. This is a central justification for the didactic and conceptual use of mathematics' historical information in order to teach the discipline, that is, the use of the history of mathematics as a didactic and conceptual mediator in its teaching (Cf. Mendes, 2015).

WHAT HISTORY AND WHAT MATHEMATICS AM I TALKING ABOUT ANYWAY?

Given that human societies produce culture, I believe that it is possible to extract histories of human ideas and practices from within that culture, that is, from our attempts to respond to the challenges that arise in time and space, and which we try to move away from in order to overcome difficulties and thus find the means to survive on the planet, always with the goal of finding better possibilities for maintaining life.

The history for which I argue favorably is a history of the explanations and comprehensions about existing objects in the world and the construction of realities that can be structured and restructured to the extent that society reinvents itself, reflects on, and redirects its way of being, that is, a cultural dynamic that requires this reality-building movement.

I would like to clarify, however, that the story I deal with in this article, as well as in my studies and researches, is focused on the cultural aspect on which society is founded so as to establish itself, think and produce ideas in order to take them as guidelines of order and power in the social construction of reality, based on the knowledge established in everyday life in search of understanding and explaining social practices as a dialectical process between objective and subjective reality, as highlighted by Berger and Luckmann (2012).

It is important to keep in mind that the objectivity of the institutional world, however massive it may seem to a single individual, is an objectivity produced and built by man. The process by which externalized products of human activity acquire the character of objectivity is objectivation itself. The institutional world is human activity objectivated, and this happens in each particular institution (Berger and Luckmann, 2012, p. 84).

In this sense, mathematics is a social production built on this objective reality, but one which also receives a subjective burden to the extent that it establishes itself between that which is individual and that which is collective, in search of solving highly diverse problems at all times throughout human history. It is in this objective-subjective duality that we understand the socially established historical construction, that is, the construction of a social, or even sociocultural, history, for it is necessary to take into consideration the relationship between society and culture fully evidenced in the historical constructions of reality, among which we find mathematics. These discussions about the social construction of reality, to be observed historically, were renewed in the works of Claude Lévi-Strauss, when dealing with the relationship between the concrete and the abstract (1989), the raw and the cooked (2004), as a strategy of human thought, in its discussions about the relationships between nature and culture.

Likewise, such discussions echoed in the propositions about the history of science that were released by Thomas Kuhn (2011), in which he dealt with the concepts of structure, scientific revolutions and paradigm to explain this social construction, and, finally, in Michel Foucault's propositions (2000), in philosophy, with the reinvention of the concepts of archeology, genealogy and regime in order to address the ways of thinking and acting in the social construction of reality. Furthermore, to these three thinkers, there are many others who in the course of the 20th century dealt with this subject, such as Ludwik Fleck (2010), among others who instituted the studies and research in the social history of science, which includes mathematics.

In this dynamic, several philosophers framed their reflections on mathematics as a way of explaining and understanding social reality in its macroscopic and microscopic dimensions, one which includes different social groups, including the school, universities, science academies and other institutions through which mathematics can be understood as human culture. These philosophers also stated their propositions in this regard. We can mention, for example, the discussions and arguments established by Imre Lakatos (1998) and Kitcher (1984). It is in this perspective that I favor the insertion of the history of mathematics as a basis for the transformation of knowledge to be taught and *knowledge to be taught at school and professional*

knowledge to be able to teach a discipline in Basic Education, in search of the construction of meaning for mathematical objects in the classroom.

> If the use of real history has the merit of bringing itself mathematical procedures to itself, it also has other questionable effects, namely transposing the search for origin, thought out by Husserl in the register, a transcendental field to research historically real beginnings. It is the fact that the transition to real history does not necessarily deliver the assumption that the historicity of a sense of formation like mathematics structurally involves the idea of a starting point. For a long time, this starting point was in ancient Greece: what does not proceed and was rejected in non-mathematics, empiricism, the essay involving correctness and error proposed by Kant that was borrowed to explain the utilitarianism of the Egyptians' mathematics. Twentieth century historiography recognizes that we cannot simply identify the operative with the empirical and that, beyond the Greek mathematical theoretical form, there were forms of different rationalities, on the basis of which genuine knowledge was developed (CAVEING, 2004, p.55).

In this sense, Jean-Pierre Vernant (2002), in the book *Entre mito e política (Between myth and politics)*, dedicates an entire chapter to discussions on Greek reason and rationalities. In that chapter, the author argues about the historical and social importance of the emergence of a plurality of rationalities in search of explanations for phenomena of all kinds, be they physical, chemical, biological, mathematical and pertaining to cultures in general. To clarify his assumptions Vernant takes civilization as an example and mentions aspects related to reason in the past and nowadays, the forms of belief and rationalities in ancient Greece and the historical development of that civilization until the advent of rational thought as a politically established modality used to explain facts and phenomena involving Greek society.

From the models of Greek rationalities preserved over the centuries, what we today call western thought was gradually designed, and it serves as an anchor for the hegemonic model of admitting the ways in which mathematics is constituted. However, it is necessary to better understand what mathematics we are dealing with. We reiterate, therefore, that the mathematics we refer to is in fact mathematical culture, that is, the mathematics constructed in a sociocultural way. It is a culture of practices that are thought out, reflected on, and experienced socially and that, consequently, give rise to explanatory models of such mathematics, among which the models that are incorporated into academic mathematics and transported to the educational system.

Regarding the creation of these models by science in its historical-epistemological development, Bunge (2013) mentions that the conceptual conquest of reality begins through idealizations, which seems paradoxical. It is the schematization, followed by the formulation of a theoretical image of the schematic model and the subsequent processes of operationalization, analysis and reformulation of the explanations related to the connections between the model object and the theoretical model. This is an exercise that constitutes one of the practical ways of thinking, verifying and explaining the creation of mathematical theories in time and space.

According to Caveing (2004, p. 55-56), the field of historical research receives an extension of information from the past in an attempt to understand the beginning of Greek relations to the current model of western thought. A pilgrimage to the sources is being carried out in a search for justifications that might clarify how the content of this knowledge was created and accumulated, in order to obtain subsidies that contribute to the criticism of the purposes of this mathematical creation. In this sense, Caveing (2004) mentions that the comprehension process of the emergence of these Western ways of doing mathematics becomes more complex when endeavors to understand the historical development of mathematics in India, China and Japan, for example. In this process, it was necessary to admit the existence of cultural variables in the way of doing mathematics, not only the various operational algorithms and irreducible concepts, but also the modes of justification procedures, that are distinct from the Greek demonstrative form.

> Furthermore, it became clear that written mathematics appear unless the Greek cultural exception in the great administrative states or empires: in fact writing extends considerably the scope and complexity of calculus, beyond mental calculus itself. Then, they are introduced in the symbol numbering systems for whole numbers and fractions that involve certain calculation rules and are provided with certain properties. This symbolism predates any form of mathematical activity, but the systems are in many civilizations as a whole and know the different principles, including how to record basic powers. So, where does the beginning still hide? (CAVEING, 2004, p. 56).

Such ways of understanding and explaining these practices give rise to descriptive mathematics of natural, cultural and social phenomena, which through a process of transformation of knowledge have led to the emergence of school mathematics, in view of the fact that systematized practices had to be incorporated into a model of social education and, in this dynamic, mathematical practices also started to be seen as one of the axes of such an education. It is from this time that the mathematics produced by professional mathematicians are also made official (in the sense of a craft or work activity, occupation or profession). These different kinds of mathematics are also the ones we are dealing with when we undertake an investigation through a historical lens in order to understand and explain the ways of thinking about and doing mathematics by society throughout history.

AND WHAT IS THE HISTORY OF MATHEMATICS FOR THE TEACHING OF MATHEMATICS?

Based on the questions and arguments presented since the beginning of this article, I reiterate, then, that the history whose insertion in the classroom I favor refers to *histories*, in its plural form, because they are connected, integrated or in fact woven in the midst of other histories with the most diverse socio-cultural qualities. Therefore, it is possible to consider that I refer to

histories that deal with the production of mathematical ideas and their materialization in multiple representative languages. Perhaps it is from this selfsame multiplicity that the plural character of these histories emerges. If we forget or despise this plurality, we tend to impoverish any approach for the mathematics we teach that is said to be or conceived as transversal, integrated or even contextualized.

These histories focus much more on the systematization of mathematical content in time and space, without losing sight of the characters, political and philosophical systems that originated these systematized productions, as well as the ways in which these histories gradually became decisive in the transformation and institutionalization of types of knowledge which are currently mobilized to teach mathematics in Basic Education schools. When it comes to Mathematics undergraduate courses, for example, these histories play a decisive role in understanding the epistemological relationships established by the different kinds of mathematics with regards to their social dimensions, which are inserted in the various academic and school environments and, consequently, result in transformations and the mobilization of mathematical knowledge for the disciplinary mathematics of Basic Education.

It is up to the teacher to think carefully about what this history of mathematics is for and to whom is it aimed at. In my actions and reflections on the training of mathematics teachers, this is the history that I consider important for the transformation and mobilization of knowledge in the development of students' mathematical learning in the classroom; is a history that should explain the conceptual organization of the different kinds of mathematics produced in time and space. It should also explain the ways in which the understanding of different kinds of mathematical knowledge was processed both in the past and in the present time, in order to transform and mobilize them to compose school textbooks to be used in professional teaching practices.

Thus, this history can be understood as a contribution towards an explanation that is epistemological and didactic in nature, and may help the teacher to explain and guide the organization of school mathematics. In this sense, historical information can be used to help mathematics teachers to improve the planning and execution of their presentations during mathematics classes, as well as to justify the modes of mathematical production in time and in space. It is a history that should address students and teachers from Mathematics undergraduate courses, Basic Education teachers and, though indirectly, Basic Education students in the way of organizing the knowledge covered in school mathematics.

WHY AND WHAT HISTORY FOR THE TEACHING OF MATHEMATICS?

One of the main justifications regarding the recommendation of the didactic or pedagogical use of historical information in mathematics teaching activities, frames it as a contribution to the expansion of students' understanding of the conceptual dimensions of mathematics, as well as a didactic contribution to teachers' work that serves to strengthen their educational skills for the teaching practice.

Furthermore, several experts on the subject have pointed out that this way of conducting mathematics teaching activities is important to clarify the formative, informative and utilitarian aspects of mathematics, especially in that it leads students to the cultural heritage of mathematics, with the purpose of developing their interest in the subject and encourage the preservation of this human intellectual memory (Cf. Mendes, 2015).

Likewise, there are other indications that the inclusion of discussions about the historical development of mathematics with regards to the teaching of this discipline becomes extremely important in that it imparts meaning to the mathematical knowledge that is both taught and learned by students at Basic and Higher Education levels. In order to better understand the arguments that are intended to strengthen the justification for the use of historical information in mathematics classes, it is necessary to illuminate which histories I refer to and how are they directly and indirectly linked to the mathematics that is to be taught and to what extent can they be used in a pedagogical context.

To decide which history should be deemed appropriate or not to be applied to mathematics' teaching is a very complex issue, but one that provokes questions from teachers, be they experts on the subject or not, always with the intention of exposing both their arguments for and against the use of such information in the development of students' mathematical learning. I emphasize, however, that it is not only a matter of promoting learning, but also of establishing training principles related to research practice, to the autonomy of studies and to the scientific spirit, as proposed by Mendes (2015), when he argues about historical investigation as a principle of teaching and learning mathematics.

In view of what was mentioned earlier, I can assert that the history of mathematics adequate to be inserted in the conceptual development of students refers directly to the epistemological development of mathematical ideas, concepts and relationships taught and learned in Basic Education and in Higher Education, provided that the different kinds of mathematical knowledge emerging from these histories are transformed and mobilized pedagogically in the exercise of teaching. More specifically, I mean the histories related to the mathematical aspects in their process of creation, reinvention and logical organization, established in time and space in order to systematize solutions to sociocultural, scientific and technological problems, at all time periods and places.

This is how I consider the historically established mathematical culture to be a potentially enriching and viable agent to enlighten students about the ways in which mathematics has developed both temporally and spatially. It is necessary, therefore, to clarify for readers that not all historical information may contain a potential that contributes sufficiently to teach mathematics. Let us see a little more about this aspect of these histories. Histories that deal exclusively with the lives of mathematicians or mathematics teachers, and which have a strong biographical appeal, can contribute only in an illustrative way to the teaching and learning of mathematical concepts, properties and relationships, if they are explored only in the context of these biographies.

An alternative for overcoming limitations regarding biographies is for the teacher to plan, execute and evaluate the development of historical research projects that advance the objective of connecting the life, the work and the mathematical practice of the persons who are the object of such an investigation. In this way, the teachers will be able to go beyond the biography itself, illuminating the professional teaching knowledge produced by the mathematicians or mathematics teachers on whose lives the biographies focus, as well as the knowledge transformed by them, in order to facilitate the students' learning in their own time period.

The use of legends and mythologies related to the history of mathematics is another aspect that demands great caution from teachers, because they are often found in literature books or even in history of mathematics books or in other books that are used to aid the teaching practice. Sometimes, the elaboration of such books is based on historical information that comes from uncertain sources or accept general assumptions on the subject. The teachers are free to use such material, as long as they are able to explore its imaginative potential and stimulate the students to exercise problematization, as well as the creative ability to elaborate different types of mathematics and connect them to the contents provided in their teaching plans. Novelized histories present this potential in their elaboration and can often be harmful if the teachers do not explore, transform and mobilize them properly in the exercise of teaching.

Other histories of Mathematics that present slightly unsuitable characteristics for pedagogical use in the classroom are those that present themselves as synonyms of historical narratives about names, dates and places, without fundamentally establishing the development of mathematical concepts, properties and relationships. In this regard, I reiterate again that teachers needs to redirect the use of these stories so as to promote the exercise of a broader historical investigation based on these histories and guide the composition of a scenario where the histories of conceptual development are added to already existing information. Hence, it will be possible to systematize the mathematical ideas that need to be formalized in students' learning. To accomplish this exercise, it is necessary to understand that:

> the history of human cultural objects is very similar to the history of species, which can be modeled, with reasonable precision, after the mathematics typical of dynamic systems (that is, the mathematics of chaos theory). The history of mathematics, therefore, is less like the history of a linear march and more like the history of molecules that collide with each other in a pressure cooker: it is obvious that the current state of molecules can be explained by the succession of previous states, but it is impossible to say, except in statistical terms, where each molecule will be after another minute of boiling - because the history of the molecules in the pan is of a random nature. (REVISTA CÁLCULO, 2013, p. 40).

To conclude my reflections and suggestions about which history should be used in the teaching of mathematics, I reaffirm that to teach mathematics with the support of the history of the development of mathematical ideas does not mean the same as teaching history of mathematics. In this sense, it will fall to the mathematics teacher to exercise the transformation of knowledge that is to be mobilized in the classroom, based on the knowledge derived from the history of mathematics. For a better explanation, I will present hereafter some reflections on the process of transformation and mobilization of knowledge.

TOWARDS THE TRANSFORMATION AND MOBILIZATION OF HISTORICAL KNOWLEDGE FOR THE TEACHING PRACTICE

Regarding the process of transformation and mobilization of knowledge from the history of Mathematics for the teaching practice, I notice the need to problematize school disciplines and academic disciplines. From the multiple meanings attributed and/or conferred to the notion of discipline, Hofstetter and Schneuwly (2017a, p. 23, quoting Bourdieu, 2001 footnote) emphasize that "domains, in the disciplinary system, are characterized by the fact that they bring together different disciplines within one single discipline". Perhaps this is the case of the history of Mathematics as a research field that was established as a discipline, one which involves multiple knowledge pertaining to other disciplinary fields such as Education, History, Mathematics, the sciences, generally speaking, etc.; besides, it is also structured in three basic dimensions: the epistemological one, the pedagogical one and the memorial one[2] of Mathematics and its teaching, as highlighted by Mendes (2017).

My inquiries have the purpose of distinguishing the fields in which this history for the teaching of Mathematics is located, aiming at an expanded discussion about the institution and constitution of mathematical knowledge to be taught to students and professional knowledge of the teacher to teach this mathematics, to be inserted in the training and practice of teachers who teach mathematics. Thus, the history applied to the teaching of Mathematics has to have its philosophical affiliations and epistemic intentions clearly defined, in order to be able to configure this knowledge for the training of Mathematics teachers and, so it can be incorporated later in the teaching practice. These are some of the concerns that drive me to mobilize information produced through researches on the history of mathematics, history for the teaching of mathematics and the history of mathematical education.

According to what Hofstetter and Schneuwly (2017a, p. 25) affirm, the disciplines are frequently constituted in relation to each other, or even against each other, in a constant reconfiguration movement that includes, essentially and since the beginning, interdisciplinarity [...], conceived as any and all connection between disciplines, be it competition, conflict, alliance or cooperation, that denote the continuous displacement of disciplinary boundaries as processes of fissure, fusion and extension of disciplinary fields.

[2] Related to the practices and concepts of Heritage Education as an academic field.

In the case of the disciplinary movement originated and produced through researching in the history of Mathematics, it is possible to identify, characterize and analyze the processes of institution of knowledge that derived from research in this field of knowledge and to explain how did the cracks, mergers, extensions and bifurcations occurred within them in the past and continue to occur in the present. The purpose is to meet recent demands of school culture, especially in the case of the pedagogical dimension mobilized by knowledge related to history when it is applied to the teaching of Mathematics.

The results of the research carried out by Mendes (2008; 2010; 2013; 2015; 2017) pointed out there was an advance in the production of epistemologies related to the history of Mathematics applied to teaching and the history of Mathematical Education. This movement was strengthened by the work of Brazilian research groups dedicated to these areas. However, regarding of the knowledge to be taught, and of knowledge so that one can teach, that is, the kinds of knowledge that bring about the mathematics' teacher didactic practice the classroom, I identified that these are characterized as knowledge in transformation, which need to be established as a discipline in order to be included in the pedagogical dimension of the history of mathematics.

In this perspective, the process of knowledge mobilization, established in the social and scientific context to favor teaching and learning activities, that is, the transformation of an established knowledge into a new knowledge yet to be defined, can become more dynamic through the transformations of knowledge to be taught, in knowledge so that one can teach. Thus, this process can strengthen the knowledge that is to be taught by the teacher and make it possible to be learned by the students. It is in this sense that different types of mathematics explored through historical research can be mobilized for the classroom in order to constitute a didactic apparatus that enables the learning of mathematical concepts, properties and theories.

Thus, historical information, therefore, comes to be understood as knowledge that is already socially established which can be taken as raw material to be conducted so as to transform the knowledge to be learned into something closer to the learner. It is, in fact, a didactic reinvention that should be better suited to the objectives defined by mathematics teachers and the level of depth that the student needs in order to learn.

Based on what was presented in this section, in the following I present aspects that I believe to be essential in the creative process that characterizes the construction of meanings in the mathematics that was produced over the centuries. It is the reorganization of these meanings for a didactic approach of the mathematics contents taught in Basic Education and in the training of mathematics teachers by way of exercises of cognitive connections whose synapses must converge for the understanding and practice of mathematical creation in the classroom.

The incorporation of heuristics as an element of school culture materialized through reinventions of the mathematical production process will also contribute to stimulate the investigation of mathematical practices and elaborations in history, in their experimental and formal levels. These are aspects that define the outline of the challenges that have led to the production of mathematical themes currently addressed in elementary, high school and higher education. In this sense, I believe it to be extremely important that the mathematics undergraduate courses propose a mathematics curriculum that has some central purposes, such as establishing and analyzing the didactic and epistemological connections of the construction of a pedagogical work mediated by researcher-teachers, by post-graduate students, mathematics undergraduate students, and Basic Education teachers.

In this curricular organization, it is important to leave a place for teachers in training to exercise the investigation of mathematical aspects in the histories of social and scientific practices, aiming to enable them to build other epistemological foundations for the mathematical topics they learn and that, later, they will have to teach in Basic Education. Furthermore, this curricular reformulation should promote discussions about the didactic and conceptual possibilities of historical research in the classroom, during the training of mathematics teachers, with possible implications for future educational processes in Basic Education.

Likewise, it means stimulating mathematics teachers to improve their investigative and reflective skills while studying the conceptual development of mathematics from a historical and epistemological perspective, something they will come to learn in their future teaching practice. This may result, in the future, in a scenario where we have students who are more autonomous regarding the search for their own learning about the mathematical knowledge that is required of them in any instance of life.

This curricular change presupposes the development of research attitudes and habits in the socio-historical and cultural context, based on the field of study of each professional involved in such a context, in order to contribute to the formation of professionals who are committed to the quality of the educational work that is to be developed in the socio-cultural context in which they inhabit. Perhaps this change thus allows the achievement of actual interaction among the school contents addressed in the classrooms and the sociocultural and scientific practices established in the past and in the present, through means of a process that stimulates the exercise of the teachers' mathematical creativity in their relationship with the students, in order to enable the incorporation of this exercise by the students themselves.

When we talk about creativity, we understand it as a socio-cultural phenomenon. Therefore, we must understand that this is not an individual phenomenon, but a collective and systemic process that contributes to the expansion of social cognition, for, to be creative means to practice divergent ways of thinking. Thinking creatively means being able to be provocative, paradoxical, metaphorical, playful with one's own thinking, thus exercising one's flexibility to always find better options and better paths for any and all life situations, both personal and professional. Perhaps this is one of the ways of including the histories of mathematics into classroom practice.

To advance the reflections on this section, I return to some points of the discussions established by Hofstetter and Schneuwly (2017a, p.32) when they assert that there is a tension "related to the approach or withdrawal maintained by the researcher and the disciplinary field as an entity with socio-professional and political-administrative expectations, that is, the imperatives linked to educational action". In this sense, the results obtained in my studies and research show pointed differences in appraisals,

analyzes and evaluation between researchers directly linked to researches on history applied to the teaching of Mathematics and those who have never worked in this research field in the history of mathematics related to teaching. These differences appear when they express their opinions regarding the validation, effectiveness or adequacy of historical information in the development of knowledge related to mathematics, which has been produced disciplinarily throughout history (mathematical knowledge), and the teacher's professional knowledge of the mathematics he will teach, in the training of mathematics teachers and in the implications of this information for the teaching practice in Basic Education.

As Hofstetter and Schneuwly (2017a) point out,

> (...) by emancipating themselves from the disciplines sometimes referred to as "mothers", particularly philosophy, psychology, sociology, (...) the educational sciences integrate contributions by renewing them in a flow of knowledge in reciprocal interactions allowing the emergence of new domains and problems, some of which are specific to educational sciences (Hofstetter and Schneuwly, 2017, p.35).

Perhaps this is the case of history applied to the teaching of mathematics in its disciplinary constitution. In the process of establishing this subject as a discipline, both knowledge to be taught and knowledge to teach were produced. Likewise, in a similar process, my teaching experiences in the formation of mathematics teachers, my studies and my research have pointed out how appropriate the incorporation, transformation and mobilization of this knowledge can be in the act of teaching mathematics, both in training courses for mathematics teachers and in Basic Education, because,

> in the process of establishing a discipline, researchers sometimes claim, sometimes refuse disciplinary loans and affiliations, since such loans can reveal a complex, hybrid and abundant set that evolves in close relationship with the social demands and advances of their contributing disciplines, which renew each other (Hofstetter and Schneuwly, 2017a, p.35-36).

Such hybridization highlights the intersections between disciplinary fields that result in multidisciplinary, interdisciplinary or transdisciplinary processes, which often overlap, and are a part of different disciplines. In this respect, the meaning connections that exist between the verbs *to discipline (to disciplinate, disciplination)* and *to disciplinarize*, according to Hofstetter and Schneuwly (2017a) establish that these verbs are united by teaching, since

> it is a systematic transmission of cultural and institutional knowledge, because teaching constitutes a part of the scientific disciplines, partly in the context of university education and partly through socialization – the disciplination. Therefore, discipline is a way of transmitting knowledge and, in contemporary societies, is a way of producing new knowledge. The relationship between the process of transmission of cultural knowledge and the production of institutional knowledge is at the heart of the scientific disciplines themselves (Hofstetter and Schneuwly, 2017a, p. 47).

From this same point of view, Hofstetter and Schneuwly (2017) clarify that in the process of establishing a discipline and constituting the professional knowledge to be taught and the knowledge needed to teach, the words disciplinary and disciplinarize,

> thus appear as two dimensions of a long-lasting dialectical process, in which reason, thought, proof, data, argumentation, study of knowledge have a central role and are opposed, from any point of view, to obedience (Hofstetter and Schneuwly, 2017a, p 49).

It is, therefore, about understanding the epistemological place in the institutionalization process of knowledge in this movement of establishing a formal discipline and its importance at every moment of the teacher training and teacher practice, that is, it is necessary to understand its historical configuration. Therefore, Hofstetter and Schneuwly (2017), assert that

> to inscribe the history of intellectual productions and of the functioning of a disciplinary field within its social, political, cultural context also implies taking into account the multiplicity of actors and factors that contribute to the institutionalization and professionalization of educational research, as well as the epistemological and socio-professional transformations and controversies that accompany this process (Hofstetter and Schneuwly, 2017a, p.26-27).

It is in this sense that, in order to understand what the knowledge needed to teach mathematics in Basic Education and in the training of mathematics teachers is about, it is necessary to describe the epistemological trajectory of this knowledge, in order to clarify some questions. For example, when someone first encounters the expression *the use of history applied to the teaching of mathematics*, one is almost always confused in thinking that it means only using narratives that refer to dates, names, places and heroic deeds related to mathematics, often disconnected from the content that teachers intend to teach their students. However, what I always try to clarify is that, when I talk about using history to teach Mathematics, I refer to didactic explorations of the history of ideas that were produced in time and space and how, nowadays, we can find them reflected in the Mathematics we teach. Therefore, this understanding is very frequent in Brazilian research on the subject.

For such a cognitive exercise of mathematical creation to take place in the perspective presented here, it is necessary for the teacher to propose, continuously, challenging practices in the classroom, in which the students venture out in search of support or revalidation of established truths through historical research, with the purpose of increasing their educational domain in mathematics.

To support the validity of the principle stated above, I rely on the activities developed during my teaching experience both in Basic and Higher Education, on my personal and professional education, and on the results of researches I have accomplished since 1995. Such researches have focused on the relationships between History and Mathematical Education. In this regard, I highlight that during my teaching and research practice, my studies were aimed at building an epistemology that could explain the didactic value of a teaching approach based on the use of conceptual information derived from the history of Mathematics, so that it could be possible to use them in pedagogical approaches to be implemented in mathematics classes.

In this regard, I reiterate that the history of mathematics needs to be understood as knowledge that is always in transformation and that can operationalize a transformation in students' creative learning. An example of this are the results of Brazilian studies and research published in Brazil in the 1990s, such as those of Miguel (1993) and Mendes (1997), concerning the use of the history of mathematics in the classroom. Reflections such as these were encouraged in the National Curriculum Parameters (Brasil, 1998), so that some of this knowledge about the history of mathematics could be systematized in textbooks in the 1990s, as well as in teaching activities in schools.

However, in the current National Common Curricular Base - BNCC (Brasil, 2018), I identified a minimum of guidance or suggestion for using the history of Mathematics as an integrating element in the transformation of knowledge to be taught by the teacher in Mathematics classes.

HOW TO PUT THESE HISTORIES OF MATHEMATICS INTO PRACTICE IN THE CLASSROOM?

To discuss a little further the insertion of historical information as a cognitive agent in the development of students' classroom learning (Mendes, 2006, 2015, 2017), we first need to consider that when a student asks any questions about mathematical topics in the classroom, they are not interested in practical applications. Perhaps they might believe they are, that they would like to know the practical applications, but, in fact, they would be satisfied with another kind of answers. One of them is to explain that the knowledge yet to be learned will contribute to the expansion of their thinking strategies and, consequently, will help them in their knowledge production, that is, it will increase their learning capacity.

In another case, the teachers must explain to the students that certain subjects in mathematics are taught because they are very useful for certain professions. Therefore, learning such a subject can expand the possibilities in their future career and will give them a sense of security regarding the mathematics they will have to learn in the future.

Finally, the teacher may be able to extract epistemological aspects from historical information which favor their explanation of mathematical questions and which often favor the expansion and enrichment of the students' learning, generating interest for future studies on the themes presented by the teacher. Such historical information may include problems extracted from primary sources or mathematical models created or reformulated at certain time periods, as well as different ways of demonstrating a theorem or justifying the existence of a mathematical property.

In this future perspective, I reiterate that any solution found and officially proposed to answer a problem is a solution that was particularly validated at a certain historical moment. Mendes (2015, p. 100) calls this a solved question, which, when codified and reused in a given process, may give rise to new open questions. It is important that teachers try to put themselves in the place of the person who created these solutions so that they can incorporate, in the best possible way, the justifications and arguments for the students to understand and accept this solution. Furthermore, this position will give teachers the possibility of establishing creative dialogues that support the incorporation of new aggregating elements to the reformulation of mathematical theories that have been perfected throughout the historical development of mathematics. This way, students can expand their understanding of the formulation of the concept they are learning in the classroom.

However, based on the experiences that I developed with basic education students and with teachers at the initial or continuing education level, regarding the use of history to teach mathematics, I admit more and more that it is possible to use historical investigation in mathematics classes, after having noticed that when students encounter this methodological proposal, they develop a meaningful process of understanding reality and establish relationships with the mathematical aspects involved in it. Therefore, I consider the use of historical research as a didactic strategy fundamentally important for mathematical learning, with the support of the socio-historical and cultural problematizations on which mathematics was built.

Furthermore, when using historical research projects in the classroom, teachers can and should stimulate the ability to investigate and understand the reality that surrounds the mathematical knowledge that is to be pedagogically established in the classroom. This process may lead students and teachers to build new ways of representing mathematics, mankind and the world, because when we give way to new perspectives on things under an investigative lens, it is possible to perceive new information that are transmitted by them.

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