Pedagogical Research
2020, 5(3), em0071
e-ISSN: 2468-4929
https://www.pedagogicalresearch.com

Research Article

MODESTUM
OPEN ACCESS

Knowledge of "Pre mathematics" in Times of the Modern Mathematics Movement (1960-1980)

Edilene Simões Costa dos Santos [1]*, Denise Medina França [2]

[1] UFMS, BRAZIL
[2] UERJ, BRAZIL
***Corresponding Author:** edilenesc@gmail.com

Citation: dos Santos, E. S. C., & França, D. M. (2020). Knowledge of "Pre mathematics" in Times of the Modern Mathematics Movement (1960-1980). *Pedagogical Research, 5*(3), em0071. https://doi.org/10.29333/pr/8282

ARTICLE INFO

Received: 29 Feb. 2020

Accepted: 10 Apr. 2020

ABSTRACT

Through the historical analysis of Dienes' works and through the suggestions of activities, the objective of this text is to understand the movement of systematization of knowledge for teaching and to teach classification, ranking, and ordination through the use of logical blocks during the Modern Mathematics Movement (1960-1980), a time of transformations in the structure and teachings of mathematics characterized by the circulation of educational experiences, which recommended the use of logical blocks as facilitators in the materialization of abstract ideas. To assist in this study, we bring references mobilized in works from the Research Team in the History of Educational Sciences (ERHISE), from the University of Geneva, Switzerland. Such references seek to historically understand the professional knowledge of teaching, objectified in legislation, programs, curriculum, decrees, among others with regard to the knowledge to teach and for teaching. The study suggests that one of the objectified knowledges in Brazilian programs, is the way to approach the logical-mathematical structures in a concrete way, that is, with the use of logical blocks to build and materialize knowledge referring to the logical structures of classification, ranking and ordination.

Keywords: logical blocks, knowledge to teach, objectified knowledges

INTRODUCTION

Manipulative materials have long been used to assist in the teaching of mathematics and, in its majority, are intended for the early school years. According to Vidal (2017), the culture of school materials has emerged as an object of investigation and source for understanding the history of school and schooling process.

Souza (2013) points out that, in a historical perspective, the relationship between school materials and pedagogical renovation has been consolidated in primary education since the 19th century in several Western countries, when new ways of organizing elementary school were experimented, aiming to universalize education. Additionally, Roberts (2014) states that there is a great number of researches on the use of manipulative materials in the classroom, which demonstrates their use since the 1960s to the present day. He explains this fact by stating that the United States experienced a Montessori (1870-1952) reconstruction beginning in the 1950s which, in the early 20th century, developed into several manipulative materials intended for learning, and this may have coincided with the interest in renewing mathematics education both in the United States and in Europe, using manipulative materials, combined with the development of Pedagogy and Educational Psychology, whose influence extended far beyond mathematics, with the Swiss Jean Piaget, among others.

In this text, we will consider manipulative, concrete, and structured materials, as "supports and utensils that, at different time and places, were invented, mobilized, transposed, disseminated to and by the school (ALVES, 2010, p. 103).

Among the educators who, in the 1960's, helped popularize materials for mathematics education, we can cite Emile-Georges Cuisenaire, Caleb Gattegno and Zoltan Dienes (1916-2014). The latter, is a Hungarian educator with PhDs in Mathematics and Psychology, who considered Mathematics a unique structure, however he used a more concrete methodology. He was one of the great pioneers of studies alluding to the methodology for teaching in the initial grades and was considered a reference in the field of Mathematics Education. During this period there were also curricular renovation movements in France, United States and other different countries.

Hence, we believe that the use of materials to assist in the teaching and learning of mathematics has, in fact, a history that precedes electronic technology, some of which were proclaimed as revolutionary, as is the case with logical blocks, due to its circulation and promises of modernization in the teaching of mathematics.

Valente (2007) supports the idea that educational knowledge is an element of school culture and within school mathematics history it is considered a product of that culture in the teaching of mathematics. Therefore, the research of the historical use of logical blocks can contribute to understand production and systematization of knowledge and objectification of school knowledge with the use of manipulative materials. As we understand it, systematization is a process of transforming knowledge into teachings, and objectification is the product of that process.

Thus, we can consider that the sixties and seventies of the twentieth century present certain particularities in mathematics education for the initial grades. The implementation of renovations in the educational systems of the Brazilian states, related to the deliberations of the Laws of Directives and Bases of the National Education, 4.024 / 1961 Law, and 5692/71 LDB) incorporated different strategies. Two of these strategies consist of training courses offered in many parts of Brazil and the distribution of publications for teachers, guaranteeing the circulation of methodological prescriptions and guidelines for schools in the new network organizational structure and guidelines related to the teaching profession (França, 2019).

In this study, we will focus on the prescriptions and guidelines related to the use of logical blocks as manipulative material during the term of the Modern Mathematics Movement (MMM), which, in general, aimed to "modernize" the teaching and learning of Mathematics, changing and updating its contents and methods, encouraging the participation of teachers in events in which issues related to the new teaching proposal were discussed. In this perspective, the national law of education, namely the 4024/61 Law, provided curricular flexibility to the educational systems of each Brazilian state, creating opportunities for the MMM ideas and new teaching/learning experiences to be adopted.

In the initial school grades, Curricula and Programs were influenced by the proposals of the mathematics educator Zoltan Dienes. In this article, we have focused on official sources, that is, educational programs prepared by official institutions. Dienes disseminated his work in Brazil through study groups, mainly by the Mathematics Education Study Group (GEEM), founded in 1961, under the presidency Oswaldo Sangiorgi and with George Springer as a collaborator. Fischer (2007) believes that the constitution and performance of GEEM were extremely important for the implementation and dissemination of MMM in Brazil.

At that time, some publications guaranteed the circulation of guidelines regarding both mathematics to teach and educational experiences carried out in experimental schools, which established changes, with a view to standardizing the actions of schools in the newly created education system, including new methodologies that used the logical blocks. We can infer that these guidelines were objectified in the programs of that period. The objectification of knowledge represents the last stage of the journey that transforms information about teaching experiences into professional knowledge of the teacher.

But what is this mathematics to teach, which we take in as a reference? To assist in the study, we bring references mobilized in works from the Research Team in the History of Educational Sciences (ERHISE), from the University of Geneva, Switzerland. Such references seek to understand historically the professional knowledge of teaching, objectified in legislation, programs, curriculum, decrees, among others with regard to the knowledge to teach and for teaching. According to Borer (2017), *Knowledge for teaching* is configured as professional knowledge, which is developed through the progressive constitution of the educational sciences disciplinary field; as for the *knowledge to teach*, it comes from the reference disciplinary fields, constituted by university disciplines. More specifically, the *knowledge to teach* is represented as an object of teaching and the *knowledge for teaching* is characterized as a professional tool for the teacher. For Valente (2017), the knowledge to teach is characterized as that which the teacher must use for the formative tasks (e.g., study plans, programs, and manuals) and the knowledge to teach as the one that should be mobilized in the teaching practice (e.g., forms of treating the knowledge to teach, ideas of how students should learn that knowledge, their ways of learning, the transformations that the knowledge to teach must undergo).

More specifically, we analyze in a historical perspective the systematization of methodological guidelines using logical blocks to teach elementary mathematical structures of classification, ranking and ordination, although this material presents different possibilities for pedagogical practice, throughout the whole schooling process. It contributes to the development of ideas in the multiplicative field (fundamental principle of counting).

According to Valente (2019), this knowledge is built upon new concepts, which take into account the "objectified knowledge", that is, knowledge institutionalized over time. This occurs due to a general consent over a knowledge that is explicit, formalized, transmitted and intentionally included in teacher education. To help us understand this movement, we bring Valente (2018, 381-382) to the dialogue that points out three steps for the investigation of the systematization of knowledge: recompilation of teaching experiences, comparative analysis of knowledge, systematization and analysis of knowledge, for the study of how scattered information becomes knowledge.

According to the Valente:

> The procedure of systematization and analysis of the use of knowledge as knowledge represents the last stage of the journey that transforms information about teaching experiences into professional knowledge of the teacher. Systematization and analysis of use are procedures performed concurrently. Thus, it is up to the researcher or group of researchers, to organize from the previous stage, an asepsis of subjective and conjunctural elements of the pedagogical consensus, so that the knowledge can be seen with a character that can be generalized and used, that is, as to know. On the other hand, the analysis includes, jointly, the verification in normative and / or pedagogical didactic instances of the occurrence of use of the elements systematized by the researcher. (Valente, 2018, p.381).

Thus, for the author, it is the process of systematization that leads us to the objectification of knowledge. As long as we have this systematization, we can make knowledge circulate and it only circulates if it is outside the subject, which is to say, objectified.

Therefore, the objective of the article is to seek, through the historical analysis of Dienes' works and the suggestion of activities, understand the movement of systematization of knowledge to teach and for teaching classification, ranking and ordination with the use of logical blocks, during the Modern Mathematics Movement (1960-1980).

MMM IN BRAZIL AND DIENES' CONTRIBUTION TO EDUCATION IN INITIAL GRADES

Times change, societies need new knowledge and many discussions on teaching and learning bring educators together around the MMM theme. According to Batista et al (2013), Franca (2016; 2019), Soares (2014) etc. the Movement began in the mid-1950s, a time when the requirements for professional qualification and scientific discoveries were associated with the growing appreciation for technology and science was considered a necessity in order to achieve economic growth.

The MMM, guided by cognitivism, that is, by Piaget's psychogenetic theory, defends the idea that the individual builds knowledge throughout his personal development. We can say that this Movement was constructed by several actions that occurred around the world, as a consequence of the misalignment that existed between Mathematics development and its teaching. Grounded on structuralism as a way of thinking about scientific production, analyzing the social reality based on the construction of models that explain how relationships occur from what they call structures. This movement is based on rationalism, that is, the source of knowledge is reason with an emphasis on abstraction. Dienes' proposals are considered appropriate for the initial grades, with an emphasis on his methodology and the introduction of the use of manipulative materials.

What are Dienes' conceptions on the teachings of elementary structures? Dienes proposes that these materials be mobilized so that abstract ideas can be materialized. How did he do it?

When reflecting on the importance of systematized knowledge guided by the use of logical blocks, we are obliged to consider the emblematic context of Dienes' pedagogical conceptions.

Many analyses have been carried out regarding this movement in Brazil, in terms of the country's political, economic, and social reality. In this text, we attempt to highlight how the movement of mathematics teaching renovation produced new knowledge, now considered requirements for pedagogical practice. It is possible to identify, at this moment, systematized and objectified knowledge for mathematics teaching in official public school programs for primary education, such as "pre-Mathematics" activities through an approach that uses logical blocks, that is, logical-mathematical activities for the construction of the concept of number by the child.

Pre-Mathematics activities, according to Denes, are those prior to the introduction of the concept of number. To him (1967b), "the concept of number is very complex". This can be explained by the fact that to him, grounded on Piaget (1984), number is a mentally constructed structure by the child, which involves three basic concepts: conservation (invariance of the number); ranking (order amongst its elements); and classification (the existence of one element inside the other). Therefore, such structures need to be built prior to the introduction of the concept of number.

Piaget (1975) believes that the construction of knowledge occurs through one's actions and by the coordination of these actions over objects that constitute themselves as the basis of learning and human development. However, Piaget does not use this belief on the pedagogical level, leaving it to the educators to experiment on new ways to offer knowledge to their students.

The development of the concept of number takes place in consonance with the development of the operations of quantity conservation, as well as other logical operations such as classification and serialization. An operational notion of number is only possible when the conservation of discontinuous quantities has been established, regardless of spatial arrangements. Numbers result from three fundamental notions: unity, class-inclusion and ranking (ordination). Number is a synthesis of serialization and inclusion, and therefore requires the following principles: commutativity, associativity, and reversibility. In the construction of numbers, the logical notions of space through spatial displacements are built in parallel to the elementary actions of gathering, separation, serialization or order changing, that is, making and undoing sets. This construction involves the development of the categories of thought: object, space, time and causality. Such constructions can only take place before the existence of implicative relations between assimilation schemes and explanatory relations between objects. Symmetric relations lead to the inclusion of classes; asymmetric relations lead to serialization. The synthesis of these two operations leads the concept of number. This synthesis is only possible because of the reversibility of thought with the grouping structure (Piaget, 1971; Rangel, 1992).

To exemplify: the basic concepts of number, measure, constants, lines, etc. are logical-mathematical knowledge (mental operations), made up of relationships that cannot be observed. These notions are the product of the construction and combinations of three mathematical structures, described by Bourbaki as mother-structures (algebraic, of order and topological), considered by mathematician as fundamental, primitive and irreducible.

Nicolas Bourbaki is the pseudonym under which a group of mathematicians, mostly French, have written a series of books, where they expose modern mathematics, which began to be published in 1935. The group disseminated, through books and articles, changes in the teaching of Mathematics, in a structuralist and abstract conception, preaching the use of a logical-deductive approach, and defended an internal revolution in Mathematics based on the development and study of the notion of structure. (Vitti, 1998, p. 55).

The invention of Logical Blocks is, to this day, reason for controversies. It is believed to have been appropriated by Maria Montessori (1870-1952), however in an interview conducted by Soares (2014), Dienes claims authorship to this material based on the ideas of William Hull (1924-2010).

Dienes' proposal for the logical blocks refers to a methodology that makes use of manipulative materials to carry out mathematical activities, predominantly in groups. So, what are logical blocks?

Figure 1. Logical Blocks

Source: UTFPR – Mathematics Laboratory (2019)

They are a structured material, or in other words, a material with established properties. They are a set consisting of 48 pieces of wood or plastic, which have the following features: color (red, blue, and yellow), size (large and small), shape (square, rectangle, triangle, and circle), and thickness (thin and thick). There is proportion among the pieces: the rectangle is half the size of the square; the triangle is equilateral and each side corresponds to the measurement of the side of the square; the side of the small square corresponds to a quarter of the side of the large square; and the thickness of the thick pieces must be twice the thickness of the thin pieces. With these variables, the concepts of set, universe, logical connectors (conjunction, disjunction, negative, and implication), study of groups, rings and bodies can be explored, and visual or auditory representations can also be encouraged.

Dienes compiles the previous teaching experiences with the use of logical blocks, to carry out activities with children and be able to verify their ideas on the materialization of abstract ideas through structured materials.

He believes that the process for building the child's thinking begins with the personification of the structures, then, when they become familiarized with them, they can combine them, transforming them into more complex ones and, later, easily apply them to numerical sets, thus, discovering, understanding and combining mathematical structures and the way they relate to one another. Dienes uses the expressions personify or materialize to identify activities in which mathematical properties are reproduced by means of structured material - in this case, logical blocks.

After a certain number of games with the same mathematical structure, of various forms, using different materials such as buttons, toys, sticks, etc., they become aware of the similarities, of the analogy between the elements, despite their different representations, that is, in the end it is all one single game. Dienes understands that the construction of graphical representations, such as decision trees, schema, Cartesian product, double entry table or enumeration of disjoint sets of clusters are activities for obtaining the solution to counting problems through these representations.

It can also be observed that Dienes guides himself by Piaget (1971), when arguing about the importance of exposing the child to increasingly challenging situations, appropriate for the development of the desired mathematical concepts. Thus, Dienes (1967a, p. 29) believes that: "There must be a rich variety of mathematical experiences, from which mathematical concepts can be constructed by the children themselves. Many experiences will be necessary for each concept".

Dienes' works, in Piaget's point of view, aim at a new methodology for teaching mathematics in the early school years. This new knowledge for teaching consists of activities such as logical blocks carried out in artificially built situations. These situations should illustrate the fundamental structures of mathematics that one wishes to explore, as well as the way they are related to each other, giving room to more complex structures in investigative activities, individually or in small groups. According to him: "It is from a rich environment that the child is able to build his knowledge, and we take as an example the learning of the mother tongue" (Dienes, 1967b, p. 1).

Dienes did not believe it was possible to begin the study of a structure by working with properties axioms. He considered it was important to familiarize the child with the structure through logical blocks, creating similar models. His idea was to play with the different models, making the child realize the differences and similarities of the analyzed structures. Afterwards, the game should become more difficult, including rules to restrict the student's logical movements, raising analytical questions to lead to axiomatic considerations. Dienes wanted the child to think until he reached a logical conclusion, using the reasoning he considered acceptable (Dienes, 1974).

POSSIBLE MOVEMENT OF KNOWLEDGE SYSTEMATIZATION

In this section, we propose to capture a possible movement of systematization of experiences, derived from Dienes' proposals regarding pre-mathematical activities using logical blocks, that is, the transformation of these experiences into knowledge. To achieve this, we will guide ourselves by the steps defined by Valente (2018), previously stated.

We begin the compilation procedure by investigating the process of constituting knowledge by Dienes, with the collection of information organized in books, manuals, etc., that can highlight Dienes' pedagogical work proposals. Then, we select the experiences that can be considered as a new knowledge. What did Dienes propose that set him apart from previous proposals?

We begin the study by recollecting teaching experiences, that is, experiences that resulted from applying the project developed by Dienes.

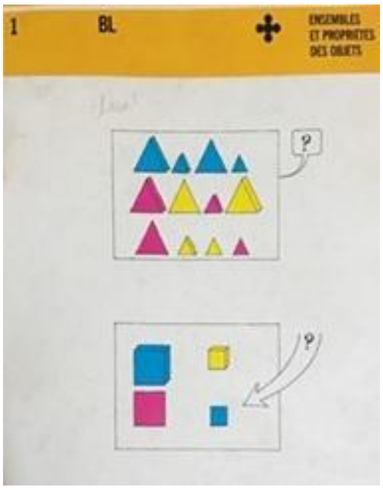


Figure 2. Example of Classification Activity

Source: APLBS (UFSC – Institutional Repository, 2019)

Figure 3. Activities of Class Formation

Source: APLBS (UFSC - Institutional Repository, 2019)

According to Franca (2019) Dienes compiled and systematized his experiences first in the article “The formation of mathematical concepts in children through experience”, in which he reported the new teaching methodologies, carried out in Leicester (1958-1959), published by *Educational Research*, London, England. The use of logic blocks to develop elementary mathematical structures has caught the attention of educators around the world. The idea was to begin school mathematics through these activities, because according to the new studies that involved pedagogy and learning psychology, the programs should be organized respecting the child’s cognitive development.

The complete results of this experience in Leicester, known as Project Leicestershire, were systematized into books, such as: Modern Learning of Mathematics (Dienes, 1967a), Logic and logical games (1974), among others. These books were widely circulated and published in Brazil, financed by the Ministry of Education and Culture (MEC) and widely advertised in teacher training courses.

When revisiting the examples of activities suggested by Dienes in some publications - see references, for the development of mathematical logical relationships (activities of classification, serialization, sequence, quantification, etc.) with the use of logical blocks, we can conclude that the material aims to approach the logical-mathematical structures in a concrete way, that is, with the use of material it is possible to construct and materialize the logical structures.

To better understand the use of manipulative material and the knowledge systematized by Dienes, we cite some activities suggested by Dienes and Tellier (1973) to be worked on individual files or in groups, with the use of logical blocks.

Initially, as shown in **Figure 2**, activities of comparison are activities that involve the identification and adoption of a preference criterion, groupings and games for organizing sets of objects.

In the process, it is essential to propose learning situations, as in **Figure 3**, in order to provide the acquisition of a language that provides support for abstraction and generalization of concepts, starting from the concrete ones.

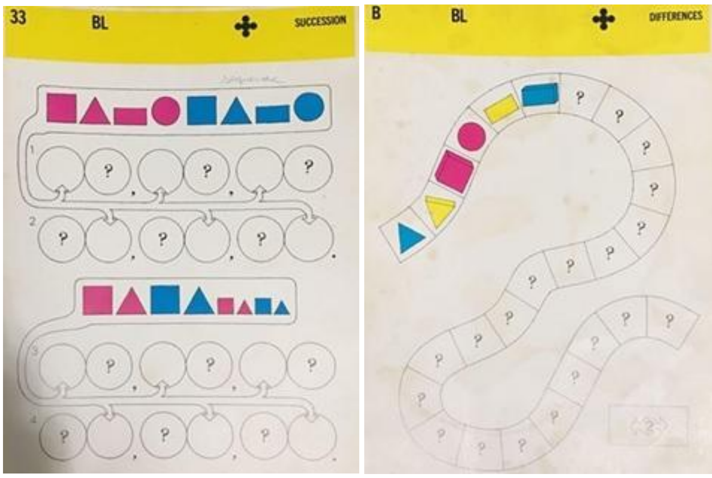


Figure 4. Activities of Ranking and Ordination

Source: APLBS (UFSC - Institutional Repository, 2019)

From this point forward, sets are ordered and, thus, one goes from the sets to simple and biunivocal correspondence, cardinal and ordinal numbers, entering the numbering system. Serialization activities, as shown in **Figure 4** - considered herein as: “organizing objects into sets so that they maintain the same relationship of difference with their neighbors” (São Paulo, 1976, p. 67) - imply in an arrangement or sets of objects. Oral language should be used in order to verify whether the child is able to understand the relationship between objects. In linear serialization, other criteria can be explored to ordinate features such as quantities, thickness, size and color, found in sets formed with the pieces of the logical blocks.

From the knowledge systematized by him and going back to the stages of analysis, we can say that in the last stage, systematization and analysis of the use of knowledge as teachings, we noticed the appropriations of knowledge systematized by Dienes materialized in the way the Programs prioritize and distribute their mathematical contents, addressing knowledge related to pre-Mathematical activities by the use of logical blocks. We can say, to a large extent, that after a knowledge has been developed, it becomes appropriate in different contexts, serving as a basis for new experiences.

Dienes’ collection *First Steps*, published in Brazil with funding from MEC, was considered at that time, by the developers of teaching programs, as the “Bible”, to be followed by everyone and was always recommended to teachers in public-school systems. In addition, research shows that this material was used by the vast majority of teachers and was considered, by many, as an informative of the practice models that the Education Departments expected from them, during the MMM period.

We can infer that the courses offered by the public-school systems in most Brazilian states were vehicles for appropriating and circulating the methodology developed and disseminated by Dienes for the use of logical blocks.

The systematization of knowledge by Dienes favored the production of new knowledge to teach and for teaching. Some of this new knowledge refers to the revelation of the need for “Mathematics prior” to school, from the pedagogical point of view - a “Pre-Mathematics”, which explores activities consistent with the period of psychological development. Thus, our theoretical hypothesis refers to pre-mathematics as knowledge to teach, and the use of logical blocks to teach and learn pre-mathematics as knowledge for teaching.

The knowledge *to teach and for teaching* disseminated by Dienes were appropriated by different publications in Brazil, according to references, which began to include the methodology of logical blocks in the development of mathematical structures in their programs. Therefore, we can infer that the activities with the blocks, proposed by Dienes for classification, ranking, and ordination, were appropriated and objectified in Brazilian Programs, taking as examples the Curriculum Guides of São Paulo (1975), Laboratory of Curricula in Rio de Janeiro (1977), Course Plan of Colégio Pedro II (1984), among others.

For example:

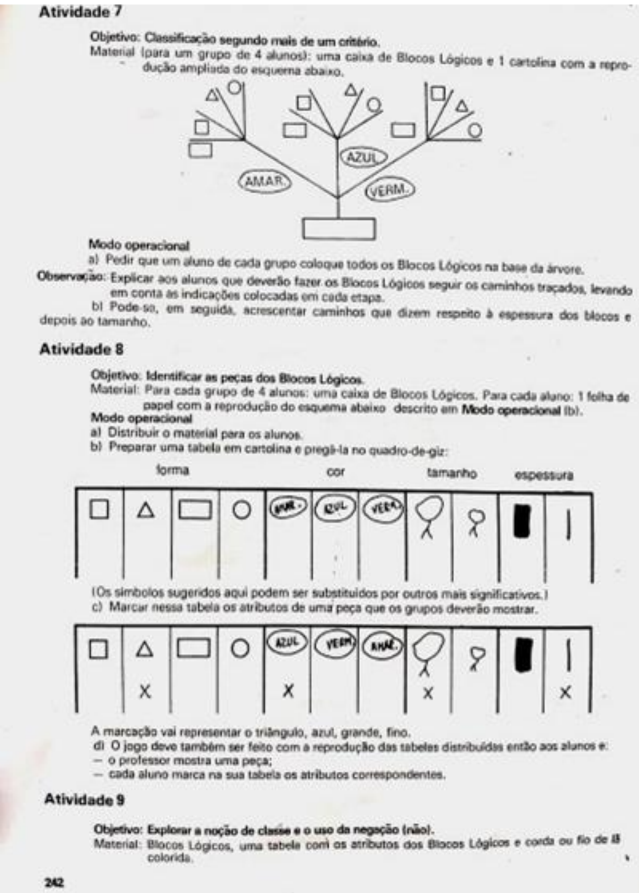
fl. 8

MODÊLO DE AULAS DE MATEMÁTICA

AULA	OBJETIVOS	ATIVIDADES	AVALIAÇÃO
1ª	Dado um material estruturado a criança deverá: -descrever com exatidão os atributos de uma peça.	MATERIAL: Blocos Lógicos (peças grandes) Crianças agrupadas de 4 em 4 1 caixa para cada grupo Cada criança escolhe uma forma - DESCRIÇÃO DA PEÇA Ex.1: O professor mostra uma peça e a criança diz seus atributos (cor, forma, espessura, tamanho) Ex.2: A criança mostra a peça e descreve seus atributos. Ex.3: O jogo da " peça escondida ".	Obsdervar se o aluno é capaz de: -descrever os atributos corretamente.
	-reconhecer ao menos um atributo entre: cor, tamanho, espessura em uma peça.	. Fazer uma construção com as peças. . Uma criança vira de costas enquanto os companheiros escondem uma peça sua(que está na construção). Esta criança terá que dizer qual a peça que foi escondida usando para isto ao menos um atributo a mais que o atributo forma (quadrado azul). . A seguir, esconde-se a peça de outra criança e repete-se o jogo com as 4 crianças do grupo. . As crianças trocam os lugares a fim de jogar com formas diferentes e faz-se novas construções. - Refazer o jogo até que cada criança tenha jogado com as 4 formas	-reconhecer pelo menos 1 dos atributos da peça escondida.
	Com um material estruturado à criança deverá: -identificar corretamente dois atributos das peças com que está trabalhando.	MATERIAL: Blocos lógicos (peças pequenas) Crianças agrupadas 1 caixa para cada grupo - Reconhecimento de uma peça escondida. . Jogo da " peça escondida "	Observar se os alunos são capazes de: - nomear com correção os atributos que podem identificar as peças.

Figure 5. PROGRAM MDC-SP, 1976

Source: SP, 1976 and RJ, 1976



Atividade 7

Objetivo: Classificação segundo mais de um critério.
Material (para um grupo de 4 alunos): uma caixa de Blocos Lógicos e 1 cartolina com a reprodução ampliada do esquema abaixo.

Modo operacional
a) Pedir que um aluno de cada grupo coloque todos os Blocos Lógicos na base da árvore.
Observação: Explicar aos alunos que deverão fazer os Blocos Lógicos seguir os caminhos traçados, levando em conta as indicações colocadas em cada etapa.
b) Pode-se, em seguida, acrescentar caminhos que dizem respeito à espessura dos blocos e depois ao tamanho.

Atividade 8

Objetivo: Identificar as peças dos Blocos Lógicos.
Material: Para cada grupo de 4 alunos: uma caixa de Blocos Lógicos. Para cada aluno: 1 folha de papel com a reprodução do esquema abaixo descrito em **Modo operacional** (b).
Modo operacional
a) Distribuir o material para os alunos.
b) Preparar uma tabela em cartolina e pregá-la no quadro-de-giz:

(Os símbolos sugeridos aqui podem ser substituídos por outros mais significativos.)
c) Marcar nessa tabela os atributos de uma peça que os grupos deverão mostrar.

A marcação vai representar o triângulo, azul, grande, fino.
d) O jogo deve também ser feito com a reprodução das tabelas distribuídas então aos alunos e:
— o professor mostra uma peça;
— cada aluno marca na sua tabela os atributos correspondentes.

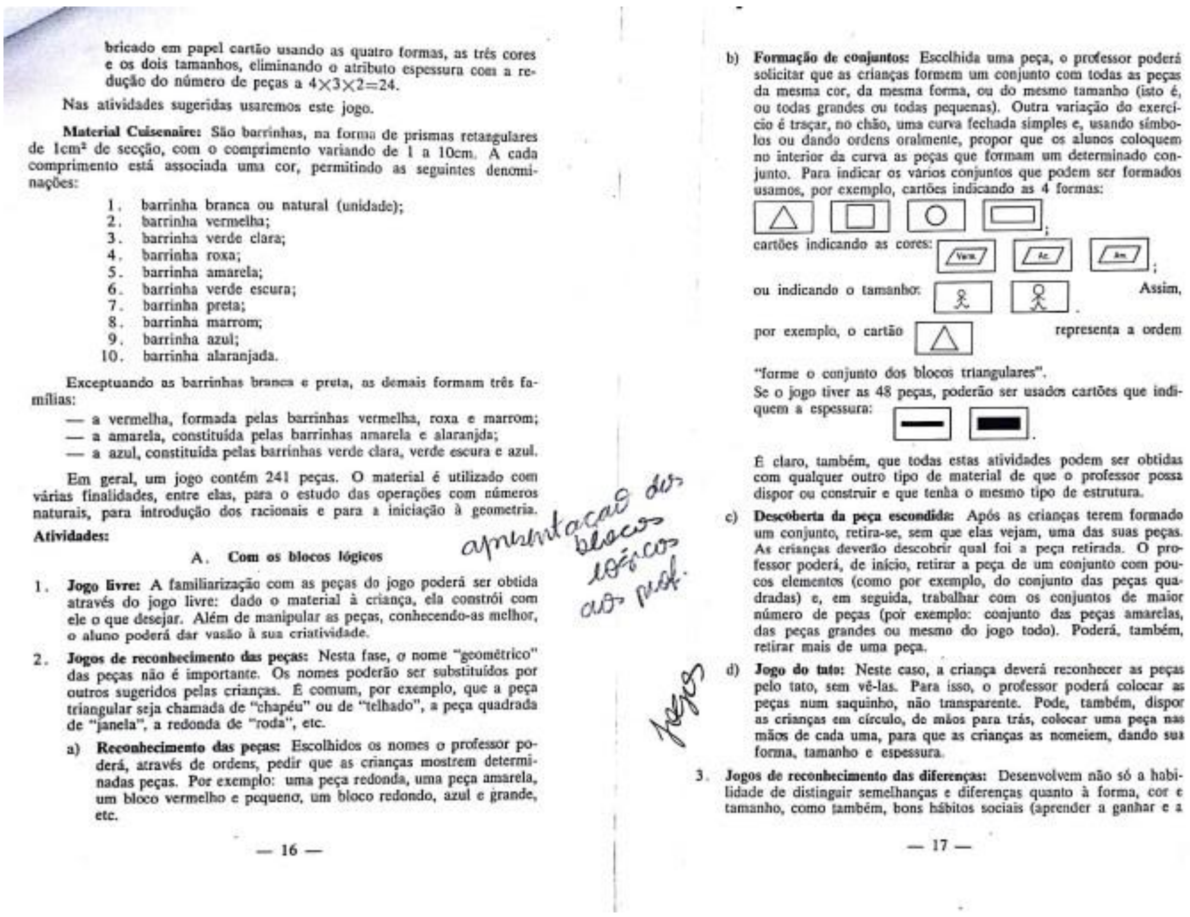
Atividade 9

Objetivo: Explorar a noção de classe e o uso da negação (não).
Material: Blocos Lógicos, uma tabela com os atributos dos Blocos Lógicos e corda ou fio de lã colorida.

242

Figure 6. Curricular Renovation in the State of Rio de Janeiro

Source: RJ, 1976



bricado em papel cartão usando as quatro formas, as três cores e os dois tamanhos, eliminando o atributo espessura com a redução do número de peças a 4×3×2=24.

Nas atividades sugeridas usaremos este jogo.

Material Cuisenaire: São barrinhas, na forma de prismas retangulares de 1cm² de secção, com o comprimento variando de 1 a 10cm. A cada comprimento está associada uma cor, permitindo as seguintes denominações:

1. barrinha branca ou natural (unidade);
2. barrinha vermelha;
3. barrinha verde clara;
4. barrinha roxa;
5. barrinha amarela;
6. barrinha verde escura;
7. barrinha preta;
8. barrinha marrom;
9. barrinha azul;
10. barrinha alaranjada.

Exceptuando as barrinhas branca e preta, as demais formam três famílias:

— a vermelha, formada pelas barrinhas vermelha, roxa e marrom;
— a amarela, constituída pelas barrinhas amarela e alaranjda;
— a azul, constituída pelas barrinhas verde clara, verde escura e azul.

Em geral, um jogo contém 241 peças. O material é utilizado com várias finalidades, entre elas, para o estudo das operações com números naturais, para introdução dos racionais e para a iniciação à geometria.

Atividades:

A. **Com os blocos lógicos**

1. **Jogo livre:** A familiarização com as peças do jogo poderá ser obtida através do jogo livre: dado o material à criança, ela constrói com ele o que desejar. Além de manipular as peças, conhecendo-as melhor, o aluno poderá dar vasão à sua criatividade.

2. **Jogos de reconhecimento das peças:** Nesta fase, o nome "geométrico" das peças não é importante. Os nomes poderão ser substituídos por outros sugeridos pelas crianças. É comum, por exemplo, que a peça triangular seja chamada de "chapéu" ou de "telhado", a peça quadrada de "janela", a redonda de "roda", etc.

   a) **Reconhecimento das peças:** Escolhidos os nomes o professor poderá, através de ordens, pedir que as crianças mostrem determinadas peças. Por exemplo: uma peça redonda, uma peça amarela, um bloco vermelho e pequeno, um bloco redondo, azul e grande, etc.

— 16 —

   b) **Formação de conjuntos:** Escolhida uma peça, o professor poderá solicitar que as crianças formem um conjunto com todas as peças da mesma cor, da mesma forma, ou do mesmo tamanho (isto é, ou todas grandes ou todas pequenas). Outra variação do exercício é traçar, no chão, uma curva fechada simples e, usando símbolos ou dando ordens oralmente, propor que os alunos coloquem no interior da curva as peças que formam um determinado conjunto. Para indicar os vários conjuntos que podem ser formados usamos, por exemplo, cartões indicando as 4 formas: ;
   cartões indicando as cores: ;
   ou indicando o tamanho: . Assim, por exemplo, o cartão representa a ordem "forme o conjunto dos blocos triangulares".
   Se o jogo tiver as 48 peças, poderão ser usados cartões que indiquem a espessura: .

   É claro, também, que todas estas atividades podem ser obtidas com qualquer outro tipo de material de que o professor possa dispor ou construir e que tenha o mesmo tipo de estrutura.

   c) **Descoberta da peça escondida:** Após as crianças terem formado um conjunto, retira-se, sem que elas vejam, uma das suas peças. As crianças deverão descobrir qual foi a peça retirada. O professor poderá, de início, retirar a peça de um conjunto com poucos elementos (como por exemplo, do conjunto das peças quadradas) e, em seguida, trabalhar com os conjuntos de maior número de peças (por exemplo: conjunto das peças amarelas, das peças grandes ou mesmo do jogo todo). Poderá, também, retirar mais de uma peça.

   d) **Jogo do tato:** Neste caso, a criança deverá reconhecer as peças pelo tato, sem vê-las. Para isso, o professor poderá colocar as peças num saquinho, não transparente. Pode, também, dispor as crianças em círculo, de mãos para trás, colocar uma peça nas mãos de cada uma, para que as crianças as nomeiem, dando sua forma, tamanho e espessura.

3. **Jogos de reconhecimento das diferenças:** Desenvolvem não só a habilidade de distinguir semelhanças e diferenças quanto à forma, cor e tamanho, como também, bons hábitos sociais (aprender a ganhar e a

— 17 —

Figure 7. Subsidy for the Implementation of the Curricular Guide for Mathematics - Algebra for the 1st to 4th grade

Source: São Paulo, 1976

The images shown above, of Programs in São Paulo and Rio de Janeiro, respectively, depict guidelines for teachers during the MMM period. We recall that in these periods, the official publications, in addition to norms and directives referring to content, graduation, evaluation, among others, assumed a didactic characteristic, for the immediate application of methodologies appropriate to the new structural approach to Mathematics. As for the ideas of Zoltan Dienes, they presented the representation that they would be easily applicable and achievable in the classroom with the use of logical blocks.

The curricular reformulation, proposed by the governments of many Brazilian states, triggered numerous other publications, aimed at initial grade teachers, containing methodological guidelines, suggestion of activities and theoretical training to support their practice, based on the systematization of Dienes' proposals.

CONCLUSION

Returning to our guiding question: What knowledge to teach and for teaching were systematized in the teaching of pre-mathematics in the early school years during the MMM?

Through the reflections presented, we can consider that one of the objectified and institutionalized knowledge in Brazilian programs, were the approaches to logical-mathematical structures in a concrete manner, that is, with the use of logical blocks to build and materialize knowledge referring to the logical structures classification, ranking and ordination.

We understand that the proposals in programs, courses for teachers and books, among others, objectified knowledge for the structuralist approach to Mathematics, that proposed practices through manipulative activities, in this case by using logical blocks and that, according to the conception of mathematical learning, contributed to the construction of the elementary notions mentioned above. Such activities offer several examples of how to materialize mathematical structures through situations in which the child participates in experiences, artificially constructed, using logical blocks. Thus, there is the possibility to build logical-mathematical reasoning, from the elaboration of relations between the pieces, starting from the concrete to the abstract.

In summary, the knowledge objectified in the studied educational programs, by the use of logical blocks in the pre-Mathematics activities approach, were circulated during the term of the MMM. Some of this new knowledge revealed the need for "pre-mathematics", that is, before the introduction of the concept of number, work on classification, ranking and ordination activities.

REFERENCES

Almeida, W. B. (1999). Simetria e entropia: sobre a noção de estrutura de Lévi-Strauss. *Revista de Antropologia (São Paulo), 42*(1-2), 163-197. https://doi.org/10.1590/S0034-77011999000100010

Alves, C. (2010) Educação, memória e identidade: dimensões imateriais da cultura material escolar. *História da Educação (Porto Alegre), 14*(30), 101-125.

Batista, C. O., Santos, E. S. C., Souza, M. M., & Carvalho, R. P. F. (2013). O Movimento da Matemática Moderna em Brasília-DF: Indícios de sua implementação a partir do relato de dois professores pioneiros. In VII Congreso Iberoamericano de Educación Matematica. (4037-4044) Montevideo: Universidad de Montevideo. Retrieved from http://www.cibem7.semur.edu.uy/7/actas/pdfs/435

Borer, V. L. (2017). Os saberes: uma questão crucial para a institucionalização da formação de professores. *In*: R. Hofstetter & W. R. Valente (orgs.). *Saberes em (trans)formação*: tema central da formação de professores. (pp. 173-199). São Paulo: Livraria da Física.

Brasil. Ministério da Educação e Cultura (1961). *Lei 4.024, de 20 de dezembro de 1961*. Lei de Diretrizes e Bases da Educação Nacional, 1961. Retrieved from http://www.jusbrasil.com.br/legislacao/129047/lei-de-diretrizes-e-base-de-1961-lei-4024-61

Brasil. Ministério da Educação e Cultura (1971). *Lei no 5.692, de 11 de agosto de 1971*.Lei de Diretrizes e Base, 1971. Retrieved on 20 March 2020 from https://presrepublica.jusbrasil.com.br/legislacao/128525/lei-de-diretrizes-e-base-de-1971-lei-5692-71

Colégio Pedro II (1985). PGE-Plano Geral de Ensino do 1º segmento do Ensino Fundamental. Rio de Janeiro, RJ.

Dienes, Z. A. (1967a). *Matemática Moderna no ensino primário.* São Paulo, SP: Editora Fundo de Cultura.

Dienes, Z. A. (1967b). *Conjunto, números e potências*. São Paulo: Herder.

Dienes, Z. A. (1967c). *Aprendizado moderno da Matemática.* Rio de Janeiro: Zahar.

Dienes, Z. A. (1969). *O poder da Matemática.* São Paulo: Herder.

Dienes, Z. A. (1974). *Lógica e jogos lógicos*. (E. José Dotto, Trad.). (2. ed. rev.). São Paulo: EPU.

Dienes, Z. A., & Tellier, A. (1973). *Mathématique vivante.* Canadá: Éditions Hurtubispe. Arquivo Pessoal Lucília Bechara. DOC. 374.

Fischer, M. C. B. (2007). Formação de professores em tempos de Matemática Moderna: uma proposta de investigação histórica**.** *Revista Diálogo Educacional, 8*(25), 663-678. https://doi.org/10.7213/rde.v8i25.3745

França, D, M. (2016) A educação elementar pela pedagogia de Zoltan Dienes. In: SBHMat (Eds). Anais do 3 Congresso Iberoamericano de História da educação matemática. Belém, Brasil: SBHMat, 229-241. Retrieved from https://repositorio.ufsc.br/handle/123456789/169988

França, D, M. (2019). *A Matemática nas séries iniciais:* o que mudou (1960-1980)? Curitiba: Appris.

Piaget, J. (1971). *A formação do símbolo na criança. Imitação, jogo e sonho, imagem e representação.* (A. Cabral. Trad.). Rio de Janeiro: Zahar.

Piaget, J. (1975). A equilibração das estruturas cognitivas. Rio de Janeiro: Zahar.

Piaget, J. (1984). *A Gênese das Estruturas Lógicas e Elementares*. Rio de Janeiro: Zahar.

Rangel, A. C. (1992). *Educação matemática e a construção do número pela criança:* uma experiência em diferentes contextos sócio-econômicos*.* Porto Alegre: Artes Médicas.

Roberts, D. L. (2014). History of Tools and Technologies in Mathematics Education. *In*: A. Karpa & G. Schubring (eds.). *Handbook on the History of Mathematics Education* (pp. 565-577). NY: Springer. https://doi.org/10.1007/978-1-4614-9155-2_28

São Paulo (Estado). Secretaria do Estado da Educação. (1976) *Subsídio para a Implementação do Guia Curricular de Matemática* - Álgebra para o 1º grau de 1ª a 4ª série. São Paulo: SEE-SP. Retrieved on 20 February 2020 from https://repositorio.ufsc.br/handle/123456789/201998?show=full

São Paulo. Estado (1975). Secretaria de Educação do Estado de São Paulo. *Guias Curriculares*. São Paulo.

São Paulo. Município (1976). Secretaria de Educação do Município de São Paulo. *Modelo de desenvolvimento do currículo-1ª série-MDC*. São Paulo.

Soares, E. (2014). *Zoltan Paul Dienes e o Sistema de Numeração Decimal na cultura escolar paranaense (1960-1989).* (Tese Doutorado em Educação). Pontifícia Universidade Católica - PUC, Curitiba, PR.

Souza, R. F. (2013). Objetos de ensino: a renovação pedagógica e material da escola primária no Brasil, no século XX. *Educar em Revista*, *49*, (jul/set), 103-120. https://doi.org/10.1590/S0104-40602013000300007

Universidade Federal de Santa Catarina. Repositório Institucional. *APLBS* - Arquivo Pessoal Lucília Bechara Sanchez. Retrieved from https://repositorio.ufsc.br/handle/123456789/1769

Universidade Tecnológica Federal do Paraná. Laboratório de Matemática. *Blocos Lógicos.* Retrieved from http://www.utfpr.edu.br/cornelioprocopio/cursos/licenciaturas/Ofertados-neste-Campus/matematica/laboratorios/material-didatico/blocos-logicos

Valente, R. A. (2017). A matemática a ensinar e a matemática para ensinar: os saberes para a Formação do educador matemático. In: R. Hofstetter & W. R. Valente (org.). *Saberes em (trans)formação*: tema central da formação de professores. (pp. 201-228). São Paulo: Livraria da Física.

Valente, W. R. (2007). História da Educação Matemática: interrogações metodológicas. *Revista Eletrônica de Educação Matemática, 2*(1) 28-49.

Valente, W. R. (2018). Processos de Investigação Histórica da Constituição do Saber Profissional do Professor que Ensina Matemática. *Revista Acta Scientiae, 20*(3), 1-9. https://doi.org/10.17648/acta.scientiae.v20iss3id3906

Valente, W. R. (2019). Saber objetivado e formação de professores: reflexões pedagógico-epistemológicas. *Revista História da Educação (Online)*, *23*(1), 1-22. https://doi.org/10.1590/2236-3459/77747

Vidal, D. G. (2017). História da Educação como Arqueologia: cultura material escolar e escolarização. *Revista Linhas, 18*(36), 251-272. https://doi.org/10.5965/1984723818362017251

Vitti, C. (1998). *Movimento da Matemática Moderna*: memória, vaias e aplausos. (Tese Doutorado em Educação). Programa de Pós-Graduação em Educação. Universidade Metodista de Piracicaba, Piracicaba, SP.