

# UNIVERSIDADE FEDERAL DE SANTA CATARINA CENTRO TECNOLÓGICO PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA DE AUTOMAÇÃO E SISTEMAS

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# DECENTRALIZED SYNCHRONOUS DIAGNOSIS WITH COORDINATION

Florianópolis 2022 Patrícia Mônica Campos Mayer Vicente

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Dissertação submetida ao Programa de Pós-Graduação em Engenharia de Automação e Sistemas da Universidade Federal de Santa Catarina para a obtenção do título de mestre em Engenharia em Automação e Sistemas.

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## Patrícia Mônica Campos Mayer Vicente

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O presente trabalho em nível de mestrado foi avaliado e aprovado por banca examinadora composta pelos seguintes membros:

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Certificamos que esta é a **versão original e final** do trabalho de conclusão que foi julgado adequado para obtenção do título de mestre em Engenharia em Automação e Sistemas.

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"What is behind the visible?" (PINKOLA, 1992)

### RESUMO

O diagnóstico de falhas é uma tarefa fundamental em sistemas de engenharia com o intuito de evitar comportamentos indesejados que podem afetar o funcionamento de equipamentos ou a segurança humana. Neste trabalho, somente o diagnóstico de falhas em Sistemas a Eventos Discretos (DESs) modelados como autômatos são considerados. Recentemente, uma nova arquitetura para diagnóstico descentralizado, denominada Diagnóstico Síncrono Descentralizado, foi proposta. No Diagnóstico Síncrono Descentralizado, os diagnosticadores locais são calculados com base no comportamento livre de falha dos componentes do sistema, o que reduz o tamanho dos diagnosticadores locais ao ser implementado. Embora esse método tenha sido implementado com sucesso, sua principal desvantagem é o crescimento da linguagem livre de falha do sistema para o diagnóstico, reduzindo a sua eficiência. A fim de contornar este problema, é proposto um novo método de diagnóstico síncrono descentralizado (DSD) refinando o status do diagnóstico por meio de *cluster* autômatos dos componentes locais. Para tanto, um protocolo de comunicação entre os estimadores locais e um diagnosticador, descrito em um algoritmo de diagnóstico é proposto. O método elimina o crescimento da linguagem livre de falha para o diagnóstico, garantindo o mesmo desempenho de diagnóstico do método centralizado tradicional. Além disso, uma implementação prática do método em um sistema didático de manufatura é apresentada.

Palavras-chave: Sistemas a eventos discretos. Autômatos. Diagnóstico de falhas. Diagnóstico síncrono.

## **RESUMO EXPANDIDO**

## Introdução

O desenvolvimento da Indústria 4.0 aumenta a implementação de sistemas automatizados em inúmeras aplicações, como Internet das Coisas Industrial, Robôs Autônomos, Integração de Sistemas e Segurança Cibernética. O poder de processamento desses sistemas está aumentando, enquanto que o tamanho de seus componentes está diminuindo, e há mais capacidade de comunicação entre os diferentes sistemas e seus componentes. Além disso, esses sistemas podem estar fisicamente distribuídos ou até mesmo construídos de forma descentralizada. Esses recursos levam a sistemas integrados e cada vez mais complexos, conhecidos como Sistemas Ciber-Físicos (SCFs), que incluem ambientes virtuais e físicos.

A modelagem de SCF como um Sistema a Evento Discreto (SED) é útil para tais fins. Um SED é um sistema dinâmico cujo espaço de estados é um conjunto discreto, e cuja evolução é provocada pela ocorrência de eventos que representam mudanças instantâneas no sistema que podem modificar seu estado atual. Portanto, uma vez que um grande número de processos dos SCFs não depende diretamente da passagem do tempo, mas dessa abstração de eventos, um SED pode ser usado para modelar esses sistemas. A chegada ou partida de cargas em um armazém, uma mudança de estado do sensor, a conclusão de uma tarefa ou uma falha mecânica são exemplos de eventos. Podem ser classificados como observáveis quando sua ocorrência puder ser identificada por um sensor ou não-observável se sua ocorrência não estiver associada a um sensor.

Como a evolução de estados de um SED é dada pela ocorrência de eventos e não pela passagem do tempo, equações diferenciais ou diferenças não são apropriadas para representar este tipo de sistema. Os formalismos mais comuns utilizados para descrever e manipular SEDs são autômatos e redes de Petri (LAWSON, 2004; DAVID; ALLA, 2005; CASSAN-DRAS; LAFORTUNE, 2008). Neste trabalho, apenas sistemas modelados por autômatos são considerados. Autômatos são grafos direcionados, em que os vértices representam os estados, e os arcos são rotulados com eventos que provocam a mudança de um estado para outro (LAWSON, 2004). Quando modelado por autômatos, é possível construir um modelo de planta global complexo de um SED a partir de modelos mais simples de seus subsistemas.

Os sistemas ciber-físicos, como qualquer sistema de engenharia, são suscetíveis à ocorrência de falhas que podem afetar o comportamento esperado, podendo colocar em risco a segurança dos operadores ou agravar problemas nos equipamentos. Portanto, uma técnica de diagnóstico de falha que pode detectar com precisão a ocorrência de um evento de falha em SCFs mais complexos é uma tarefa fundamental que deve ser realizada.

Vários trabalhos na literatura abordam o problema de diagnóstico de falhas de SEDs modelados por autômatos (SAMPATH et al., 1995, 1996; DEBOUK et al., 2000; QIU; KUMAR, 2006; DAIGLE et al., 2007; LEFEBVRE; DELHERM, 2007; CARVALHO et al., 2012; CABASINO et al., 2012; BASILE, 2014; CABRAL et al., 2015; WHITE et al., 2019; CABRAL; MOREIRA, 2020; VERAS et al., 2021). Recentemente, uma nova arquitetura para diagnóstico, denominada Diagnóstico Síncrono Descentralizado, foi proposta. No

Diagnóstico Síncrono Descentralizado, diagnosticadores locais são calculados com base no comportamento livre de falha dos componentes do sistema, com o objetivo de reduzir o tamanho dos diagnosticadores para implementação. Embora esse método tenha sido implementado com sucesso, sua principal desvantagem é o crescimento da linguagem livre de falha do sistema para o diagnóstico, reduzindo a sua eficiência.

A fim de contornar este problema, um método de diagnóstico síncrono descentralizado com coordenador (DSDC) é proposto, refinando o status do diagnóstico por meio de *cluster* autômatos dos componentes locais. Para tanto, um protocolo de comunicação entre os estimadores locais e o coordenador é proposto. Este método impede o crescimento da linguagem livre de falha para o diagnóstico, garantindo o mesmo desempenho de diagnóstico do método centralizado tradicional. Além disso, uma implementação prática do método em um sistema didático de manufatura é apresentada.

# Objetivos

O objetivo principal deste trabalho é desenvolver um método descentralizado de diagnóstico síncrono em que a linguagem livre de falha observada e aceita para o diagnóstico síncrono seja igual à linguagem do sistema livre de falha observada.

Primeiramente, exploram-se métodos de diagnóstico de falhas em SEDs modelados como autômatos. Em seguida, uma técnica que refina o status do diagnóstico em um módulo de sincronização é proposta. Finalmente, é apresentado que este método impede o crescimento da linguagem livre de falhas para o diagnóstico.

## Objetivos específicos

- 1. Investigar métodos de diagnóstico de falhas em SEDs modelados como autômatos, especialmente o método de diagnóstico síncrono;
- 2. Desenvolver um método para eliminar o crescimento da linguagem livre de falhas aceita pelo diagnóstico síncrono;
- 3. Implementar o método em um sistema didático de manufatura;
- 4. Analisar o custo computacional do método;

# Metodologia

Primeiramente, este trabalho consiste em realizar um levantamento bibliográfico sobre os métodos de diagnóstico de falhas abordados na literatura, com ênfase no diagnóstico síncrono. A partir disso, um estudo sobre o crescimento da linguagem livre de falha aceita pelo diagnóstico síncrono é apresentado para corroborar com a proposta desta dissertação.

Propõe-se o desenvolvimento de um método para eliminar esse crescimento da linguagem com o objetivo de obter a mesma linguagem observada pelo sistema. Para tanto, os componentes livre de falha do sistema e suas observações locais para o diagnóstico são utilizados. Considerando esses componentes, um protocolo de comunicação utilizando

a estimativa local dos componentes e um diagnosticador para apresentar o *status* do diagnóstico são propostos. O protocolo de comunicação envia *clusters* correspondentes à observação de um evento para o diagnosticador, o qual realiza operações com esses *clusters* com o intuito de informar a ocorrência da falha. Além disso, o procedimento de diagnóstico considera as possíveis sincronizações de eventos não-observáveis em comum. Os algoritmos desenvolvidos foram implementados em um sistema didático de manufatura, considerando três estações que processam uma peça de trabalho.

## Resultados e discussões

O método proposto considera características relevantes de componentes de um determinado sistema, tais como um evento ser observável para um componente e não-observável para outro ou possuir eventos não-observáveis em um comum entre os componentes. O protocolo de comunicação envia os *clusters* de acordo com o evento observado pelos estimadores locais, entretanto, também considera os eventos não-observáveis em sua composição. No procedimento de diagnóstico, obtém-se tanto o resultado sobre ocorrência da falha, quanto a real sincronização dos eventos não-observáveis. Por conta dessa sincronização, é possível obter a mesma linguagem observada do sistema sem o crescimento da linguagem que pode ocorrer no diagnóstico síncrono. Portanto, sistemas que eram não-sincronamente diagnosticáveis e monoliticamente diagnosticáveis podem ser diagnosticados com o método desenvolvido. Além disso, a implementação em um sistema real não-diagnosticável de forma síncrona, representa a viabilidade do método com um custo computacional menor do que a implementação do modelo com a planta completa na maioria dos casos.

## Considerações finais

Neste trabalho, um método para o diagnóstico síncrono descentralizado com coordenador com o objetivo de eliminar o crescimento da linguagem aceita pelo diagnóstico é proposto. Uma revisão de literatura sobre arquiteturas de diagnóstico de falhas é apresentada a fim de contextualizar a problemática do crescimento da linguagem no diagnóstico síncrono. Um estudo de caso é apresentado para indicar que determinados sistemas podem ser monoliticamente diagnosticáveis e não-sincronamente diagnosticáveis. O método proposto visa contornar este problema com o desenvolvimento de dois algoritmos: um protocolo de comunicação e um procedimento de diagnóstico. Os algoritmos operam em conjunto e, com isso, conseguem identificar a ocorrência da falha mesmo em sistemas que ocorrem a não sincronização de eventos não-observáveis considerando o diagnóstico síncrono. A implementação do método em um sistema real com eventos não-observáveis em comum e não-sincronamente diagnosticável, uma planta didática de manufatura, mostra a eficácia do método. Além disso, em sistemas com muitas operações em paralelo, e que possuem em sua maior parte eventos observáveis, o método desenvolvido traz um custo computacional menor do que a implementação do modelo da planta completa com o mesmo poder de diagnóstico. Como trabalhos futuros, almeja-se implementar o método em CLPs e estudar possíveis atrasos de comunicação.

**Palavras-chave**: Sistemas a eventos discretos. Autômatos. Diagnóstico de falhas. Diagnóstico síncrono.

### ABSTRACT

Fault diagnosis is a fundamental task that must be performed in engineering systems in order to avoid undesired behaviors that can affect equipment or human safety. In this work, we consider fault diagnosis of Discrete Event Systems (DESs) modeled as automata. Recently, a new architecture for diagnosis called Decentralized Synchronous Diagnosis (DSD) has been proposed. In the DSD, local diagnosers are computed based on the fault-free behavior of the system components with the view to reduce the size of the local diagnosers for implementation. Although this method has been successfully implemented, its main drawback is the growth of the fault-free language of the system for diagnosis, which reduces the diagnosis efficiency. In order to circumvent this problem, in this work, we propose a decentralized synchronous diagnosis method with coordination (DSDC) that refines the diagnosis status using cluster automata of the local components. To do so, we also propose a communication protocol between local state estimators and the coordinator. We show that this method prevents the growth of the fault-free language for diagnosis, which guarantees the same diagnosis performance as the traditional centralized diagnosis method. Furthermore, a practical implementation of the method to a didactic manufacturing system is also presented.

Keywords: Discrete-Event Systems. Automata. Fault diagnosis. Synchronous Diagnosis.

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# LIST OF SYMBOLS

| Σ                | Set of events   |
|------------------|---|
| ε                | Empty trace   |
| $\ s\ $          | Length of a trace   |
| *                | Kleene-closure operation  |
| $\overline{L}$   | Prefix-closure operation on language $L$  |
| $P_s^l$          | Projection operation defined as $P_s^l: \Sigma_l^\star \to \Sigma_s^\star$                        |
| $\setminus$      | Set difference  |
| $P_s^{l^{-1}}$   | Inverse projection operation defined as $P_s^{l^{-1}}: \Sigma_s^{\star} \to 2^{\Sigma_l^{\star}}$ |
| G                | Automaton   |
| Q                | Set of states   |
| f                | Transition function   |
| $q_0$            | Initial state   |
| $\Gamma_G$       | Feasible event function of automaton $G$  |
| $\mathcal{L}(G)$ | Generated language of automaton $G$   |
| L                | Generated language of automaton $G$   |
| Ac(G)            | Accessible part of $G$  |
|                  | Parallel composition  |
| $G_{prod}$       | Product composition automaton   |
| $G_{par}$        | Parallel composition automaton  |
| $\Sigma_o$       | Set of observable events  |
| $\Sigma_{uo}$    | Set of unobservable events  |
| $P_o$            | Projection operation defined as $P_o: \Sigma^{\star} \to \Sigma_o^{\star}$                        |
| UR(q)            | Unobservable reach of state $q$   |
| Obs              | Observer automaton of $G$   |
| $\Sigma_f$       | Set of fault events   |
| $\sigma_f$       | Fault event   |
| $L_N$            | Fault-free language of $L$  |
| $\overline{L_F}$ | Faulty language of $L$  |
| $G_N$            | Automaton that models the fault-free behavior of the system                                       |
| $G_F$            | Automaton that models the faulty behavior of the system   |
| $G_d$            | Diagnoser automaton   |
| $G_l$            | Labeling automaton  |
| $D_i$            | Local diagnosers  |
| $L_{N_a}$        | Fault-free language for synchronous diagnosis   |
| $G_N^a$          | Automaton resultant from the parallel composition of observer automata                            |
| $\mathcal{C}$    | Coordinator of DSDC scheme  |
| $LM_i$           | Local measurement site  |
|                  |   |

| C           | Cluster automaton   |
|-------------|---|
| $E_i$       | Current state estimate of $D_i$                           |
| $C_i^{com}$ | Communicated local cluster automaton                      |
| R           | Set of initial states                                     |
| S           | Automaton computed by the cluster synchronous composition |
| Ι           | Set of indexes of local state estimators                  |

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### **1 INTRODUCTION**

The development of Industry 4.0 increases the implementation of automated systems in countless applications such as Industrial Internet of Things, Autonomous Robots, System Integration, and Cyber-security. In this context, automated systems are becoming more interconnected with more computation power between different systems and their components. Also, the system-human interaction is constantly changing to provide safer cooperation between them. In addition, these systems can be physically distributed or even built in a decentralized way. These features lead to integrated and more complex systems known as Cyber-Physical Systems (CPSs), which include virtual and physical environments.

A CPS is usually modeled as a Discrete Event System (DES). A DES is a dynamic system that has a discrete state space, and its evolution is driven by the occurrence of events that represent instantaneous changes in the system that can modify its current state. Therefore, since a large number of CPSs' processes do not depend directly on time passage but this event abstraction, a DES can be used to model these systems. The arrival or departure of loads on a warehouse, a sensor state change, the completion of a task, or a mechanical failure are examples of events. They can be classified as observable when their occurrence can be identified by a sensor or unobservable if their occurrence is not associated with a sensor.

Since the state evolution of a DES is driven by the occurrence of events, differential or difference equations are not appropriate to represent this type of system. The most common formalisms used to describe and analyze a DES behavior are automata and Petri nets (LAWSON, 2004; DAVID; ALLA, 2005; CASSANDRAS; LAFORTUNE, 2008). In this work, we only consider systems modeled as automata. Automata are directed graphs, where the vertices represent the states, and the arcs are labeled with events that provoke a change from a state to another (LAWSON, 2004). Usually, the automaton complete behavior model of a system can be obtained from the automata models of its subsystems.

Cyber-physical systems, like any engineering system, are subject to the occurrence of faults that can affect their expected behavior which can endanger the safety of operators or aggravate equipment problems. Therefore, a fault diagnosis technique that can accurately detect a fault event occurrence in more complex CPSs is a fundamental task that must be performed. Since the systems are more interconnected and interdependent, a fault event occurrence in one component can spread to its connected ones impacting the entire system. Thus, the fault diagnosis is even more relevant in Industry 4.0 applications analysis. In this work, we address the fault diagnosis problem for Discrete Event Systems modeled as automata to be applied in CPSs.

Several works in the literature address the problem of fault diagnosis of DESs modeled by automata (SAMPATH et al., 1995, 1996; DEBOUK et al., 2000; QIU; KUMAR,

2006; DAIGLE et al., 2007; LEFEBVRE; DELHERM, 2007; CARVALHO et al., 2012; CABASINO et al., 2012; BASILE, 2014; CABRAL et al., 2015; WHITE et al., 2019) and synchronous diagnosis (CABRAL; MOREIRA, 2020; VERAS et al., 2021). In the seminal work of Sampath et al. (1995, 1996), the authors proposed the notion of diagnosability of DESs and the monolithic diagnosis scheme. In order to diagnose the fault event occurrence, it is necessary to compare both fault-free and the post-fault behaviors of the global plant model. The system is diagnosable if the fault event can always be detected and isolated after a bounded number of events observations. The fault event is usually modeled as an unobservable event since its occurrence does not immediately cause a change in the sensors' readings (CASSANDRAS; LAFORTUNE, 2008).

In Sampath et al. (1995), a diagnoser that can verify the system diagnosability and provide a diagnosis status is proposed. In order to compute this diagnoser, the global system model is modified to build a twin-plant that corresponds to the global system model with labeled states. In this case, each state receives the label F if it is reached by a sequence of events that contains the fault, and N, otherwise. The diagnoser is then obtained by computing the observer automaton of the twin-plant. Although the diagnoser presented in Sampath et al. (1995) guarantees an accurate fault diagnosis status, its computation is, in general, avoided since, in the worst-case, the state-space of the diagnoser grows exponentially with the cardinality of the state-space of the plant model (SAMPATH et al., 1995, 1996; HASHTRUDI ZAD et al., 2003; QIU; KUMAR, 2006).

In Cabral et al. (2015), a Petri net diagnoser (PND) based on the fault-free model of the system is proposed. The PND provides the state estimate of the fault-free behavior model of the system after the observation of a sequence. If the state estimate is empty after an observation, the fault event is detected. Different from Sampath et al. (1995), the diagnoser grows polynomially according to the plant size. Furthermore, methods for implementation of the PND on Programmable Logic Controllers (PLC) are presented. Other works also address the fault diagnosis problem in a monolithic way for a robust diagnosis (CARVALHO et al., 2012), for computation of minimal diagnosis bases (SANTORO et al., 2017) and for systems modeled as Petri nets (CABASINO et al., 2012).

Although the monolithic diagnosis approach can be applied to several DESs, there are many applications where the diagnosis information is only available locally. For those systems, decentralized (DEBOUK et al., 2000; QIU; KUMAR, 2006; WANG et al., 2007) and distributed (QIU; KUMAR, 2005; KEROGLOU; HADJICOSTIS, 2014, 2018) architectures are more appropriated. In the following, these architectures will be presented.

In Debouk et al. (2000), the fault diagnosis approach called decentralized diagnosis is presented. In Protocol 3 of Debouk et al. (2000), the authors extend the work presented in Sampath et al. (1995) for a decentralized architecture. In this context, local diagnosers are computed based on the global system model, considering local observation sites which lead to local observable event sets. The local diagnosers do not communicate with each other.

The fault event occurrence is diagnosed when at least one local diagnoser identifies its occurrence. When it does, the local diagnoser sends the diagnostic status to a coordinator that informs the system operator that the fault has occurred. This Protocol is explored in several works in the literature, such as Qiu and Kumar (2006) and Wang et al. (2007). In Debouk et al. (2000), the notion of decentralized diagnosability, known as codiagnosability, is also presented. The definition of diagnosability can be seen as a particular case of the notion of codiagnosability. Methods to verify the codiagnosability are presented in Qiu and Kumar (2005) and Moreira et al. (2011).

A different architecture for DES, called distributed diagnosis, is proposed in Qiu and Kumar (2008) and Keroglou and Hadjicostis (2018). In this context, the local diagnosers can communicate between them and exchange information regarding event observations and the state estimate of the system. This exchanged information is used to refine the diagnosis, *i.e.*, to perform a more accurate diagnosis. The main drawback of the decentralized and distributed approaches is that the local diagnosers are obtained from the global plant model, which can grow exponentially with the number of system components. Thus, the local diagnosers can also grow exponentially, leading to a high computational cost for diagnosis.

In order to avoid the use of the global plant model for fault diagnosis, the modular architecture is proposed in Debouk et al. (2002) and Contant et al. (2006). In these works, it is assumed that the fault event is modeled in a single component of the system, and notions of modular diagnosability are presented. Moreover, the cited works also consider two assumptions: (i) there are no common unobservable events between the components, and (ii) the faulty component model has *persistent excitation*, *i.e.*, the faulty component model always generates a new event. In practice, the same fault event can occur in more than one component model and in Contant et al. (2006) a method for the verification of the persistent excitation property is not presented. Thus, assumptions (i) and (ii) limit the application of this diagnosis approach.

A comparison between the main diagnosis architectures proposed in the literature is shown in Figure 1. Notice that, in Figure 1, G represents the global plant model and  $G_1, G_2, \ldots, G_r$  are the component models of the global plant behavior G, where G is obtained composing the local system models. In Figure 1 (a),  $P_o$  represents the observation of observable events which are communicated to a single diagnoser  $G_d$ . In Figure 1 (b) and 1 (c), we present the decentralized and distributed scheme, respectively. In these schemes, local diagnosers represented by  $G_1, G_2$ , and  $G_3$  computed based on the global system models with local observations represented by  $P_{o_1}, P_{o_2}$ , and  $P_{o_3}$ . Notice that, in Figure 1 (c), the local diagnosers can communicate between each other through channels  $c_{1,2}, c_{2,3}$ , and  $c_{1,3}$ . Finally, in Figure 1 (d), the modular scheme is presented where it is supposed that the faulty component model is  $G_1$ . Thus, a single diagnoser  $G_{d_1}$ is computed in  $G_1$  with local event communication. In addition, the model behavior for



Figure 1 – Comparison between the main diagnosis architectures proposed in the literature:the monolithic scheme (a); the decentralized scheme (b); the distributed scheme (c); and the modular scheme (d).

the diagnosis is based on the faulty component.

More recently, a new diagnosis strategy, called synchronous diagnosis, has been proposed in Cabral and Moreira (2020). In this approach, local state estimators computed from the fault-free behavior automata of the system components are implemented. Notions of synchronous diagnosability and codiagnosability and a method to verify these properties are presented in Cabral and Moreira (2020). Since in the synchronous diagnosis strategy, the local diagnosers are based on the local system models instead of the global system model, the local diagnosers do not grow exponentially with the number of system components. Moreover, the assumptions needed for modular diagnosis are not considered in the synchronous approach.

Although the method has been successfully used in real systems, the drawback of the synchronous diagnosis strategy is that the fault-free observable language for synchronous diagnosis can be a larger set than the fault-free observable language of the system. This shows that monolithically diagnosable systems cannot be synchronously codiagnosable. In order to decrease the exceeding language for synchronous diagnosis, the distributed synchronous diagnosis scheme is proposed in Cabral and Moreira (2020). In this scheme, the local diagnosers can exchange information regarding state estimate and event observations. The distributed scheme can reduce the exceeding language but not



Figure 2 – Synchronous diagnosis schemes

eliminate it, *i.e.*, the distributed synchronous approach still can present an exceeding language. The architectures proposed for the synchronous diagnosis are depicted in Figure 2, where  $D_i[G_{N_i}]$  represents the local diagnosers computed from the local fault-free behavior models,  $\Sigma_o$  is the set of observed events,  $\Sigma_{i,o}$  is the set of local observations and  $ch_{i,j}$ represents the communication channel between the local diagnosers.

In this master thesis, we propose a decentralized synchronous diagnosis with coordination (DSDC) method, which eliminates the exceeding fault-free language accepted for the synchronous diagnosis approach to the fault-free language generated by the system. In the synchronous diagnosis scheme, the language grows due to the loss of unobservable events synchronization. In order to avoid this loss of synchronization, we propose in this work a coordinator that, based on the local state estimate, verifies the correct synchronization of unobservable events after the observation of an event. To do so, local state estimators send cluster automata of the fault-free behavior component models to the coordinator. We show that by using the DSDC method one can reconstruct the observed fault-free language of the system without using the global system model.

We show that the computational complexity is smaller than implementing the global system behavior model for diagnosis. Since we compute the unobservable reach of the fault-free behavior after each observable event occurrence, this state estimate can only grow if the local component models have a large number of transitions labeled with unobservable events. In practice, this feature has been modified with Industry 4.0 development, where more sensors are being used to communicate their signals reducing the number of unobservable events.

### 1.1 OBJECTIVES

The main goal of this work is to develop a decentralized synchronous diagnosis with coordination method where the observed fault-free language accepted for synchronous diagnosis is equal to the observed fault-free system language.

Firstly, we explore fault diagnosis schemes in the DES framework modeled as automata. Then, we propose a technique that refines the diagnosis status with a communication protocol and a coordinator. Finally, we show that this method prevents the growth of the fault-free language for diagnosis.

#### **1.1.1 Specific objectives**

- 1. Investigating fault diagnosis methods of DESs modeled as automata, especially the synchronous diagnosis method;
- 2. Develop a method to eliminate the exceeding fault-free language accepted by the synchronous diagnosis scheme;
- 3. Implement the method in a didactic manufacturing system;
- 4. Analyze the computational cost of the method.

## 1.2 WORK ORGANIZATION

This work is organized as follows. In Chapter 2, the fundamentals concepts about DESs modeled as automata are presented. In Chapter 3, the notion of diagnosability of DESs considering the classical approach and the definition of synchronous codiagnosability are introduced. Also, the growth of the language accepted for synchronous diagnosis is illustrated. The decentralized synchronous diagnosis with coordination method is proposed in Chapter 4. In Chapter 5, the decentralized synchronous diagnosis with coordination method applied to a real system is presented. Finally, in Chapter 6 the conclusions of this work along with its contributions and future works are presented.

### 2 FUNDAMENTALS OF DISCRETE EVENT SYSTEMS

A Discrete Event System (DES) is a dynamic system with a discrete state space, and its evolution depends on the occurrence of, usually, asynchronous events. Events can be seen as instantaneous actions that change the state reached by the system. In this work, only automata are considered as modeling formalism to describe DES.

In order to introduce the concepts of automata, we first consider the notion of languages and its characteristics. The formal definitions used in this work can also be found in Cassandras and Lafortune (2008).

#### 2.1 LANGUAGES

In this work,  $\Sigma$  is denoted as the event set of a Discrete Event System (DES) and  $\sigma$  as a generic event. A sequence of events forms a trace. If a trace does not contain any event, it is called the empty trace and the symbol  $\varepsilon$  is used to represent it. ||s|| represents the length of trace s. The empty trace  $\varepsilon$  has length equal to zero. Definitions of language and live language are stated in the sequel.

**Definition 2.1** (Language). A language L defined over an event set  $\Sigma$  is a set of finitelength traces formed from events in  $\Sigma$ .

**Definition 2.2** (Live language). A language L is said to be live if for all  $t \in L$ , exists  $\sigma$  such that  $t\sigma \in L$ .

For example, the language  $L = \{\varepsilon, a, aa, ab, bc, abc\}$  is defined over the event set  $\Sigma = \{a, b, c\}$  and it consists of six traces, including the empty trace  $\varepsilon$ . In the following, operations used to manipulate languages are presented.

#### 2.1.1 Language operations

Since languages are sets, all set operations can be applied to languages. The concatenation is the main operation involved in the construction of traces, and consequently languages, from an event set  $\Sigma$ . Consider the example aforementioned, where the trace  $aa \in L$  is an element of the language L. This trace is formed by the concatenation of the event a with another event a. The empty string  $\varepsilon$  is the identity element of the concatenation, *i.e.*,  $t\varepsilon = \varepsilon t = t$ , for any trace t.

**Definition 2.3** (Concatenation). Let  $L_1, L_2 \subseteq \Sigma^*$ , then:

 $L_1L_2 = \{t \in \Sigma^* : (t = t_1t_2) \text{ where } (t_1 \in L_1) \text{ and } (t_2 \in L_2)\}$ 

A trace s is in  $L_1L_2$  if it can be obtained by the concatenation of a trace  $t_1 \in L_1$ with a trace  $t_2 \in L_2$  The set of all finite traces that can be formed with the elements of  $\Sigma$  is denoted by  $\Sigma^*$  and it includes the empty trace  $\varepsilon$ . The notation  $\star$  represents the Kleene-closure operation. Languages defined over  $\Sigma$  are subsets of  $\Sigma^*$ . Sets  $\emptyset$ ,  $\Sigma$  and  $\Sigma^*$  are also languages.

**Definition 2.4** (Kleene-Closure). Let  $L \subseteq \Sigma^*$ , then:

$$L^{\star} = \{\varepsilon\} \cup L \cup LL \cup LLL \cup \dots$$

The concatenation of a finite number of elements of L is an element of  $L^*$ . The empty string  $\varepsilon$  represents the concatenation of "zero" elements. Moreover, the operation  $\star$  is idempotent, *i.e.*,  $(L^*)^* = L^*$ .

Some important concepts regarding traces are the prefix, subtrace and suffix. Let s = tuv, where s is a trace and  $t, u, v \in \Sigma^*$ , then it can be stated that: t is the *prefix* of s, u is the subtrace of s, and v is the suffix of s. Notice that,  $\varepsilon$  and s are also prefixes, subtraces, and suffixes of s.

The prefix-closure operation of a language L is defined in the sequel.

**Definition 2.5** (Prefix-closure). Let  $L \subseteq \Sigma^*$ , then:

$$\overline{L} = \{ t \in \Sigma^{\star} : (\exists u \in \Sigma^{\star}) [tu \in L] \}$$

The prefix-closure of L is the language represented by  $\overline{L}$  which contains all the prefixes of the traces of L. By definition, the language L is a subset of  $\overline{L}$ , *i.e.*  $L \subseteq \overline{L}$ . A language L is said to be *prefix-closed* if  $L = \overline{L}$ .

For a language  $L = \emptyset$ ,  $\overline{L} = \emptyset$  but if  $L \neq \emptyset$ , then  $\varepsilon \in \overline{L}$ . Furthermore,  $\emptyset^* = \{\varepsilon\}$  and  $\{\varepsilon\}^* = \{\varepsilon\}$ . Also, the concatenation between the empty set and a language is equal to the empty set, *i.e.*,  $\emptyset L = L\emptyset = \emptyset$ .

Another operation that can be applied to traces is the *projection*, from a larger set of events,  $\Sigma_l$ , to a smaller set of events,  $\Sigma_s$ , where  $\Sigma_s \subset \Sigma_l$ . Cassandras and Lafortune (2008) present the formal definition as follows.

**Definition 2.6** (Projection). The projection  $P_s^l: \Sigma_l^{\star} \to \Sigma_s^{\star}$  is defined as:

$$P_s^l(\varepsilon) = \varepsilon,$$

$$P_s^l(\sigma) = \begin{cases} \sigma, & \text{if } \sigma \in \Sigma_s, \\ \varepsilon, & \text{if } \sigma \in \Sigma_l \setminus \Sigma_s, \end{cases}$$

$$P_s^l(t\sigma) = P_s^l(t)P_s^l(\sigma), \text{ for all } t \in \Sigma_l^{\star}, \sigma \in \Sigma_l,$$

where  $\setminus$  denotes set difference.

According to definition 2.6, applying the projection to a trace s erases events from  $\Sigma_l$  that do not belong to  $\Sigma_s$ . The inverse projection can also be defined as follows.

**Definition 2.7** (Inverse projection). The inverse projection  $P_s^{l^{-1}}: \Sigma_s^{\star} \to 2^{\Sigma_l^{\star}}$  is defined as:

$$P_s^{l^{-1}}(u) = \{ t \in \Sigma_l^* : P_s^l(t) = u \}.$$

The inverse projection operation applied to a trace t with events from  $\Sigma_s$  results in a set constituted by all traces that can be formed with the events of  $\Sigma_l$  which projection is equal to trace u.

The projection and inverse projection operations can also be applied to languages. In this case, these operations are applied to all traces of the language.

**Example 1.** Let  $\Sigma_l = \{a, b, c\}$  and consider the subsets  $\Sigma_1 = \{a, b\}$  and  $\Sigma_2 = \{b, c\}$ . The language  $L = \{c, cca, cab, cbcab\} \subset \Sigma_l^*$ . Consider the two projections  $P_i : \Sigma_l^* \to \Sigma_i^*, i = 1, 2$ . We have that:

$$P_{1}(L) = \{\varepsilon, a, ab, bab\}$$
$$P_{2}(L) = \{c, cc, cb, cbcb\}$$
$$P_{1}^{-1}(\{\varepsilon\}) = \{c\}^{\star}$$
$$P_{2}^{-1}(\{b\}) = \{a\}^{\star}\{b\}\{a\}^{\star}$$

In the next section, the automata formalism that is used to represent languages is presented.

#### 2.2 AUTOMATA

An automaton is a device that is capable of representing a language giving welldefined rules and it is defined in the following (CASSANDRAS; LAFORTUNE, 2008).

**Definition 2.8.** An automaton, denoted by G, is a four-tuple

$$G = (Q, \Sigma, f, q_0)$$

where Q is the set of states,  $\Sigma$  is the finite set of events,  $f: Q \times \Sigma \to Q$  is the partial transition function, and  $q_0$  is the initial state.

In this work, the marked states symbolism is not used and it is omitted from the tuple definition.

The transition function  $f(q_1, \sigma) = q_2$  represents that there is a transition from state  $q_1$  to state  $q_2$  labeled with the event  $\sigma$ , which can be extended to any trace of the generated language of the automaton. The feasible event function is defined as  $\Gamma_G: Q \to 2^{\Sigma}$ , where it is the set of all events  $\sigma$  for which  $f(q, \sigma)!$ , and "!" denotes that the function is defined.

An automaton is represented by a directed graph called state transition diagram. The vertices represent the states, and the arcs are labeled with events from one vertex to another (LAWSON, 2004). In order to represent the initial state of the automaton, an



Figure 3 – State transition diagram of automaton G of Example 2.

arc that does not have a previous state is included. In the following, an example of an automaton and its state transition diagram is presented.

**Example 2.** Let G be an automaton which state transition diagram is illustrated in Figure 3. The state and event sets of G are  $Q = \{0, 1\}$  and  $\Sigma = \{a, b\}$ , respectively. The initial state of G is  $q_0 = 0$ . The feasible event function is defined as:  $\Gamma_G(0) = \{a, b\}$ and  $\Gamma_G(1) = \{b\}$ . The transition function is defined as: f(0, a) = 0, f(0, b) = 1 and f(1, b) = 1.

The notion of the language generated by an automaton is presented in the following.

**Definition 2.9** (Generated language). The generated language of an automaton  $G = (Q, \Sigma, f, q_0), \mathcal{L}(G)$ , is

$$\mathcal{L}(G) = \{ t \in \Sigma^{\star} : f(q_0, t)! \},\$$

For the sake of simplicity, in this work, the generated language of G is represented as L. The language L represents all the traces that can be built by following the transitions of the state transition diagram starting at the initial state, *i.e.*,  $f(q_0, t)!$ . Thus, a trace  $t \in L$  if, and only if, it corresponds to an admissible path in the state transition of G. It is important to notice that L is prefix-closed by definition since a trace in L is only possible if all its prefixes are also possible to be generated by G. Furthermore, if f is a total function over its domain, then  $L = \Sigma^*$ . If  $Q = \emptyset$ , the language generated by G is said to be live.

For two automata  $S = (Q_S, \Sigma, f_S, q_{0,S})$  and  $G = (Q, \Sigma, f, q_0)$ , S is said to be a subautomaton of G if  $f_S(q_{0,S}, t) = f(q_0, t)$  for all  $t \in \mathcal{L}(S)$ . Notice that this condition implies that  $Q_S \subseteq Q$ ,  $q_{0,S} = q_0$ , and  $\mathcal{L}(S) \subseteq L$ . This definition also implies that the state transition diagram of S is a subgraph of that of G (CASSANDRAS; LAFORTUNE, 2008).

In the next section, automata operations are defined.

#### 2.2.1 Operations on automata

There are some operations that can be used to modify the language in order to modify an automaton. In this context, there are unary operations for a single automaton and operations for more than one automaton that are usually used for compositions (CASSANDRAS; LAFORTUNE, 2008).



Figure 4 – Automaton G (a) and Ac(G) (b).

The unary operation transforms the state transition diagram of an automaton while the event set  $\Sigma$  remains unchanged. In the following, the definition of accessible part of an automaton is presented.

**Definition 2.10** (Accessible part). The accessible part of an automaton G, Ac(G), is defined as:

$$Ac(G) = (Q_{ac}, \Sigma, f_{ac}, q_0),$$

where  $Q_{ac} = \{q \in Q : (\exists s \in \Sigma^{\star}) [f(q_0, s) = q]\}$ , and  $f_{ac} = f|_{Q_{ac} \times \Sigma \to Q_{ac}}$ .

The notation  $f|_{Q_{ac} \times \Sigma \to Q_{ac}}$  means that function f is restricted to a smaller domain of the accessible states  $Q_{ac}$ .

The accessible part of an automaton G produces an automaton Ac(G), where all the states and its related transitions that are not reachable from the initial state  $q_0$  are deleted. It is important to notice that this operation does not affect the generated language of G,  $\mathcal{L}(G)$ .

In the following, an example to show the accessible part of an automaton operation is presented.

**Example 3.** Let G be the automaton depicted in Figure 4(a) with the set of states  $Q = \{0, 1, 2, 3\}$ . The accessible part of G is represented in Figure 4(b), where  $Q_{ac} = \{0, 1, 2\}$ .

The composition operations are used to obtain a single automaton from two or more automata. In this work, only one composition operation on automata is defined: the parallel compositions. This operation is used to compute an automaton model of a system from its subsystems models (CASSANDRAS; LAFORTUNE, 2008).

In general, systems are composed by smaller components or subsystems that interact between themselves. The component behavior can be internal or coupling, and are modeled by *private* and *common* events, respectively. The parallel composition, also named *synchronous composition*, is the operation which is capable of integrating individual systems components while considering their private behavior. This operation is formally defined as follows. **Definition 2.11** (Parallel composition). Let  $G_1 = (Q_1, \Sigma_1, f_1, q_{0,1})$  and  $G_2 = (Q_2, \Sigma_2, f_2, q_{0,2})$  be two automata. The parallel composition of  $G_1$  and  $G_2$  is the automaton:

$$G_1 \| G_2 = Ac(Q_1 \times Q_2, \Sigma_1 \cup \Sigma_2, f_1 \|_2, (q_{0,1}, q_{0,2})),$$

where

$$f_{1||2}((q_1, q_2), \sigma) = \begin{cases} (f_1(q_1, \sigma), f_2(q_2, \sigma)) & \text{if } \sigma \in \Gamma_{G_1}(q_1) \cap \Gamma_{G_2}(q_2); \\ (f_1(q_1, \sigma), q_2) & \text{if } \sigma \in \Gamma_{G_1}(q_1) \setminus \Sigma_2; \\ (q_1, f_2(q_2, \sigma)) & \text{if } \sigma \in \Gamma_{G_2}(q_2) \setminus \Sigma_1; \\ undefined, & otherwise. \end{cases}$$

According to definition 2.11, a common event, *i.e.*, an event in  $\Sigma_1 \cap \Sigma_2$ , can only be executed if  $G_1$  and  $G_2$  executes it simultaneously. The private events, *i.e.*, those in  $(\Sigma_1 \setminus \Sigma_2) \cup (\Sigma_2 \setminus \Sigma_1)$  can be executed whenever they are possible in  $G_1$  or in  $G_2$ . Therefore, the parallel composition allows each component to execute their private behavior and only synchronizes the common behavior of the components.

The parallel composition is equivalent to the product composition if  $\Sigma_1 = \Sigma_2$  since all transitions will be synchronized. If  $\Sigma_1 \cap \Sigma_2 = \emptyset$ , then  $G_1 || G_2$  is the concurrent behavior of  $G_1$  and  $G_2$  because there are no synchronized transitions. Let  $P_i = (\Sigma_1 \cup \Sigma_2)^* \to \Sigma_i^*$  be two projections for i = 1, 2. The language generated by  $G_1 || G_2$  is equal to  $L(G_1 || G_2) =$  $P_1^{-1}(L(G_1)) \cap P_2^{-1}(L(G_2))$ . Moreover, this operation has the associative property, *i.e.*,  $(G_1 || G_2) || G_3 = G_1 || (G_2 || G_3)$ .

In the following, an example of product and parallel composition operations is presented.

**Example 4.** Let  $G_1 = (Q_1, \Sigma_1, f_1, q_{0,1})$  and  $G_2 = (Q_2, \Sigma_2, f_2, q_{0,2})$  be two automata, where  $\Sigma_1 = \{a, b, c\}$  and  $\Sigma_2 = \{a, b, d\}$ . The state transition diagrams of  $G_1$  and  $G_2$  are shown in Figure 5 (a) and 5 (b), respectively. The automata  $G_{prod} = G_1 \times G_2$  and  $G_{par} = G_1 || G_2$  that correspond to the product and parallel composition are presented in Figures 6 (a) and 6 (b), respectively. Notice that in automaton  $G_{prod}$  all transitions are labeled with events from  $\Sigma_1 \cap \Sigma_2 = \{a, b\}$ , whereas  $G_{par}$  models the synchronization of  $G_1$  and  $G_2$  and their concurrent behavior through events  $\Sigma_1 \cup \Sigma_2 = \{a, b, c, d\}$ .

#### 2.2.2 Partially-observed automata

The event set of an automaton G can be partitioned as  $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$ , where  $\Sigma_o$ and  $\Sigma_{uo}$  denote the set of observable and unobservable events, respectively. An event is observable when its occurrence can be detected by a sensor, for example. Fault events are usually modeled as unobservable events since their occurrence does not immediately provoke a change in sensor readings (CASSANDRAS; LAFORTUNE, 2008). In order



Figure 5 – Automata  $G_1$  and  $G_2$  of Example 4.



Figure 6 – Automata  $G_{prod}$  and  $G_{par}$  of Example 4.

to improve the readability of this work, the unobservable events are represented inside brackets in the automata transition state diagrams.

The observed language of a system G can be obtained from its generated language L by applying the projection operation  $P_o(L)$ , where  $P_o : \Sigma^* \to \Sigma_o^*$ . Given a system with observable and unobservable events, it is necessary to know the set of possible states reachable from a state  $q \in Q$  after the occurrence of an observable event. The unobservable reach represents this set of states and its definition is presented in the following.

**Definition 2.12** (Unobservable reach). The unobservable reach of a state  $q \in Q$ , denoted as UR(q), is defined as:

$$UR(q) = \{ y \in Q : (\exists t \in \Sigma_{uo}^{\star}) [(f(q, t) = y)] \}.$$

The unobservable reach is extended to sets of states  $A \subseteq Q$  as:

$$UR(A) = \bigcup_{q \in A} UR(q).$$

Definition 2.12 shows that the unobservable reach of a state  $q \in Q$  is a set formed by all states reached from q by sequences of transitions labeled with unobservable events. The unobservable reach can be used to build an observer automaton from G,  $Obs(G, \Sigma_o)$ , that generates the observed language of G,  $P_o(L)$ . This automaton is defined as follows.

**Definition 2.13** (Observer automaton). The observer automaton of G with respect to a set of observable events  $\Sigma_o$ , denoted as  $Obs(G, \Sigma_o)$ , is given by:

$$Obs(G, \Sigma_o) = (Q_{obs}, \Sigma_o, f_{obs}, q_{0,obs}),$$



Figure 7 – State transition diagram of automaton G of Example 5 (a), and observer automaton of G,  $Obs(G, \Sigma_o)$  (b).

where  $Q_{obs} \subseteq 2^Q$ .  $f_{obs}$  and  $q_{0,obs}$  are obtained from the Algorithm 1 (CASSANDRAS; LAFORTUNE, 2008; BASILIO et al., 2010).

### Algorithm 1 Observer automaton

**Input:**  $G = (Q, \Sigma, f, q_0)$ , and the observable event set  $\Sigma_o$ , where  $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$ . **Output:** Observer automaton  $Obs(G, \Sigma_o) = (Q_{obs}, \Sigma_o, f_{obs}, q_{0,obs}).$ 1: Define  $q_{obs} \leftarrow UR(q_0), Q_{obs} \leftarrow \{q_{0,obs}\}$  and  $Q_{obs} \leftarrow Q_{obs}$ 2:  $Q_{obs} \leftarrow Q_{obs}$  and  $Q_{obs} \leftarrow \emptyset$ 3: for each  $B \in Q_{obs}$  do  $\Gamma_{obs}(B) \leftarrow (\bigcup_{q \in B} UR(q)) \cap \Sigma_o$ 4: for each  $\sigma \in \hat{\Gamma}_{obs}(B)$  do 5: $f_{obs}(B, \sigma) \leftarrow UR(\{q \in Q : (\exists y \in B)[q = f(y, \sigma)]\})$ 6: end for 7:  $\tilde{Q}_{obs} \leftarrow \tilde{Q}_{obs} \cup f_{obs}(B, \sigma)$ 8: 9: end for 10:  $Q_{obs} \leftarrow Q_{obs} \cup Q_{obs}$ 11: Repeat lines 2 to 10 until all accessible part of  $Obs(G, \Sigma_o)$  is constructed

In the sequel, an example of an observer automaton of a system G is presented.

**Example 5.** Let G be an automaton which state transition diagram is shown in Figure 7 (a). The state set of G is  $Q = \{0, 1, 2, 3\}$  and the event set of G is  $\Sigma = \Sigma_o \bigcup \Sigma_{uo} = \{a, b, \sigma_{uo}\}$ , where  $\Sigma_o = \{a, b\}$  and  $\Sigma_{uo} = \{\sigma_{uo}\}$ . The observer automaton of G,  $Obs(G, \Sigma_o)$ , is illustrated in Figure 7 (b). If we consider that the system has executed the trace  $t = a\sigma_{uo}b$ , the observed trace is  $P_o(t) = ab$ , where  $P_o : \Sigma^* \to \Sigma_o^*$ . It is important to notice that the state reached after the observation of the trace  $P_o(t) = ab$  in  $Obs(G, \Sigma_o)$  is  $q = \{2\}$ . The states of  $Obs(G, \Sigma_o)$  correspond to state estimates of G after the observation of a trace.

#### 2.3 FINAL REMARKS

In this chapter, the formal definition of a DES language and the automata formalism used to represent DESs behavior are presented. In this work, only DESs modeled as automata are considered and it is important to analyze their behavior. The unary and composition operations have also been presented in this chapter.

In the next chapter, the problem of fault diagnosis of DES for the monolithic (SAMPATH et al., 1995) and decentralized synchronous schemes (CABRAL; MOREIRA, 2020) are presented.

### **3 DIAGNOSIS OF DES**

In this chapter, the notion of diagnosability of Discrete Events Systems considering the monolithic approach (SAMPATH et al., 1995) is presented. This method is explored in this chapter because it is the first one presented in the literature and it can be used to explain the diagnosis problem in the context of DESs. The monolithic approach uses the global system model in order to diagnose the fault event occurrence. However, the global system model can grow exponentially with the number of system components in the worst-case scenario.

Thus, in Cabral and Moreira (2020) the synchronous diagnosis approach (CABRAL; MOREIRA, 2020) is presented in order to reduce this computational complexity by using only the fault-free behavior component models for diagnosis. The synchronous diagnosis has been successfully implemented in manufacturing systems and it has a smaller computational complexity than the monolithic approach. However, the fault-free accepted language for synchronous diagnosis can be a larger set than the observed fault-free language of the global system.

The idea of this Master thesis is to refine the synchronous diagnosis strategy in order to restore the diagnosis power of the monolithic approach, i.e, diagnose systems that are diagnosable and may be not synchronously diagnosable, without the exponential computational cost with respect to the number of system components. Therefore, in this chapter, both methods are presented.

#### 3.1 MONOLITHIC DIAGNOSIS OF DES

The notion of diagnosability of a language L is to identify an occurrence of determined unobservable event from the observation of the language generated by the system. Since fault events are modeled as unobservable events, it is said that the system is diagnosable with respect to the projection  $P_o: \Sigma^* \to \Sigma_o^*$  and the fault event, if the fault event occurrence can be identified. Let G be the automaton that models a system and L be the language generated by G. The fault event set is denoted as  $\Sigma_f$ , where  $\Sigma_f \subseteq \Sigma_{uo}$ . For the sake of simplicity, it is assumed that there is only one fault event  $\sigma_f$ , *i.e.*,  $\Sigma_f = \{\sigma_f\}$ . If the system has more than one fault event type, each fault type can be considered separately (WANG et al., 2007). In the following, the definition of faulty and fault-free traces of a system is presented (CASSANDRAS; LAFORTUNE, 2008).

**Definition 3.1** (Faulty and fault-free traces). A faulty trace is a sequence s which contains the fault event  $\sigma_f$ . On the other hand, a fault-free trace does not contain it.

The fault-free language  $L_N \subset L$  denotes the set of all fault-free traces of L. Notice that,  $L_N = \overline{L_N}$ . In addition, the set of all fault traces of L is given by  $L_F = L \setminus L_N$ . The

subautomaton of G that generates the language  $L_N$  and  $\overline{L_F}$  are represented as  $G_N$  and  $G_F$ , respectively.

In Sampath et al. (1995), the definition of language diagnosability is presented considering two assumptions:

A1. The language generated by the system is live;

A2. There is no cycle of unobservable events in the system.

Then, the following definition of language diagnosability can be stated (SAMPATH et al., 1995).

**Definition 3.2** (Language diagnosability). The prefix-closed and live language L is diagnosable with respect to the projection  $P_o: \Sigma^* \to \Sigma_o^*$  and  $\Sigma_f$  if

$$(\exists z \in \mathbb{N}) (\forall s \in L_F) (\forall st \in L_F, ||t|| \ge z \Rightarrow P_o(st) \notin P_o(L_N))$$

where  $\|.\|$  denotes the length of a trace.

It is also important to remark that there are works for fault diagnosis that do not consider assumption A2. In this work, assumption A2 is not necessary as well.

The definition 3.2 indicates that L is diagnosable, if and only if, all fault traces with arbitrarily long length do not have the same projection as any fault-free trace of  $L_N$ . Thus, if L is diagnosable, it is always possible to detect and isolate the occurrence of fault events within a bounded number of event occurrences.

In Sampath et al. (1995), a diagnoser automaton, denoted as  $G_d$ , that can be used to verify the diagnosability of L and to diagnose a system, is presented. This diagnoser is built based on a labeling automaton, denoted as  $G_l$ , computed from the plant model G. The automaton  $G_l$  is obtained by labeling the states of G according to the traces that reach them. If a trace that contains the fault event  $\sigma_f$  reaches a state of G, it is labeled with F, otherwise, it is labeled with N. The diagnoser automaton  $G_d$  is the observer of  $G_l$  with respect to its observable events, *i.e.*,  $G_d = Obs(G_l, \Sigma_o)$ .

**Definition 3.3** (Diagnoser automaton). The diagnoser automaton  $G_d$  with respect to the faulty set  $\Sigma_f$  and the observable events set  $\Sigma_o$  is given by:

$$G_d = (Q_d, \Sigma_o, f_d, q_{0,d}),$$

where  $Q_d \subseteq 2^{Q \times \{N,F\}}$ . The transition function  $f_d$ , and the initial state  $q_{0,d}$  are defined according to Algorithm 2.

The state transition diagram of automaton  $A_l$  is illustrated in Figure 8. It is important to notice that the language generated by  $G_l$  and G are the same. Furthermore, there are two types of states  $q \in Q$  according to the trace that reaches q,  $q_l = (q, N)$  if

### Algorithm 2 Diagnoser automaton $G_d$

Input:  $G = (Q, \Sigma, f, q_0)$ . Output:  $G_d = (Q_d, \Sigma_o, f_d, q_{0,d})$ 1: Define automaton  $A_l = (Q_l, \Sigma_f, f_l, q_{0,l})$ , where  $Q_l = N, F, q_{0,l} = N, f_l(N, \sigma_f) = F$ , and  $f_l(F, \sigma_f) = F$ 

- 2: Compute the labeling automaton  $G_l = G ||A_l|$
- 3: Compute the diagnoser automaton  $G_d = Obs(G_l, \Sigma_o)$



Figure 8 – State transition diagram of automaton  $A_l$ .

q is reached by a fault-free trace and  $q_l = (q, F)$  if q is reached by a faulty trace. The generated language of  $G_d$  is the natural projection of the generated language of G, L, *i.e.*,  $L(G_d) = P_o(L)$ .

If the automaton  $G_d$  reaches a state labeled only with F, it indicates that the fault event has certainly occurred and it is diagnosed. On the other hand, if a state is labeled only with N, it represents that the fault event has not been executed by the system. States that have both labels are called uncertain states, indicating that the occurrence of the fault event is not certain. This occurs when an observed generated trace can be mapped both in a fault-free trace and in a faulty trace. In order to verify the diagnosability of a language L using  $G_d$ , it is necessary to search for indeterminate cycles in  $G_d$ . Indeterminate cycles are cycles of uncertain states that are associated to both faulty and fault-free cycles in the plant G. If there is an indeterminate cycle in  $G_d$ , the language L generated by G is not diagnosable.

In the sequel, an example that illustrates the construction of the diagnoser automaton  $G_d$  for a given plant G is presented.

**Example 6.** Let G be the system depicted in Figure 9(a), such that  $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo} = \{a, b, c, \sigma_u, \sigma_f\}$ , where  $\Sigma_o = \{a, b, c\}$  and  $\Sigma_{uo} = \{\sigma_u, \sigma_f\}$ . Automaton  $G_l = G ||A_l|$  is illustrated in Figure 9(b). Finally, in the Figure 9(c) the diagnoser automaton  $G_d$  computed from the observer of  $G_l$  with respect to its observable event set  $\Sigma_o$ ,  $G_d = Obs(G_l, \Sigma_o)$  is presented.

Notice that the initial state of  $G_d$  is  $\{0N\}$ , which corresponds to the unobservable reach of the initial state of  $G_l$ . After the occurrence of event a,  $G_d$  reaches state  $\{1N; 2F\}$ . The observations of traces ac and acb leads to states  $\{3N; 4N; 5F\}$  and  $\{0N; 2F\}$ , respectively. It is important to remark that all these states are labeled with N and F, corresponding to uncertain states, which means that the occurrence of the fault event is uncertain. There is an uncertain cycle formed by the states  $\{1N; 2F\}$ ,  $\{3N; 4N; 5F\}$ ,  $\{0N; 2F\}$  and it is necessary to verify if this cycle is also indeterminate. In this case, all the states of the



Figure 9 – Automaton G (a),  $G_l$  (b),  $G_d$  (c), and  $G'_d$  (d). Adapted from (CABRAL, 2017).

unique cycle of  $G_l$  associated to this uncertain cycle have the label N, i.e., only with states reached by fault-free traces. Thus, this cycle is not indeterminate. If the system executes the fault trace  $a\sigma_f(cb)^n$ , the fault event is diagnosed when  $G_d$  reaches state {5F}. Since there are no indeterminate cycles in  $G_d$ , the language of G is diagnosable with respect to the projection  $P_o: \Sigma^* \to \Sigma_o^*$  and  $\Sigma_f$ .

Let us now consider that the observable event set of G is  $\Sigma'_o = \{b, c\}$  and the unobservable event set is  $\Sigma_{uo} = \{a, \sigma_u, \sigma_f\}$ . The diagnoser  $G'_d$  considering  $\Sigma'_o$  as the set of observable events is shown in Figure 9 (d). Notice that the states  $\{0N; 1N; 2F\}$  and  $\{3N; 4N; 5F\}$  forms an uncertain cycle. Differently from the uncertain cycle of  $G_d$ , this is an indeterminate cycle since it is associated with cycles in  $G_l$  labeled with N and F, for example, cycle formed by the states  $\{0N\}$ ,  $\{1N\}$ ,  $\{3N\}$ , and  $\{4N\}$ , and  $\{2F\}$  and  $\{5F\}$  in  $G_l$ . Thus, the language generated by G, L, is not diagnosable with respect to the projection  $P'_o: \Sigma^* \to \Sigma^*_o$  and  $\Sigma_f$ .

**Remark 3.1.** The diagnoser presented in this work is used to illustrate the diagnosability definition (Definition 3.2). However, this work does not compute the observers for a diagnoser computation (SAMPATH et al., 1995).

It is important noticing that the diagnoser automaton  $G_d$  is computed based on an observer, and its construction is in general avoided since, in the worst-case, the state-space of the diagnoser grows exponentially with the cardinality of the state-space of the plant model.
### 3.2 SYNCHRONOUS DIAGNOSIS

In the decentralized diagnosis scheme proposed in (CABRAL; MOREIRA, 2020) the system G is supposed to be formed by r local components, such that  $G = ||_{i=1}^{r} G_i$ . Local diagnosers  $D_i$  are implemented for each local component  $G_i$  and they are computed from the fault-free behavior models of  $G_i = (Q_i, \Sigma_i, f_i, q_{0,i}), G_{N_i} = (Q_{N_i}, \Sigma_i \setminus \Sigma_f, f_{N_i}, q_{0,i}),$  $i = 1, \ldots, r$ . In this setting, the set of local events  $\Sigma_i$  is partitioned into observable,  $\Sigma_{i,o}$ , and unobservable,  $\Sigma_{i,uo}$ , event sets, such that  $\Sigma_i = \Sigma_{i,o} \cup \Sigma_{i,uo}$ . It is important to remark that in the decentralized setting, an event can be observable to local diagnoser  $D_i$  and unobservable to local diagnoser  $D_j$ , *i.e.*,  $\Sigma_{i,o} \cap \Sigma_{j,uo}$  is not necessarily equal to the empty set for  $i, j \in \{1, \ldots, r\}$  and  $i \neq j$ .

Since the decentralized synchronous diagnosis (DSD) scheme proposed in Cabral and Moreira (2020) is performed based on the fault-free local component models  $G_{N_i}$ and with local observations, the fault-free language for synchronous diagnosis,  $L_{N_a}$ , can be larger than the fault-free system language  $L_N$  due to the loss of synchronizations (CABRAL; MOREIRA, 2020). Language  $L_{N_a}$  can be written as:

$$L_{N_a} = \bigcap_{i=1}^r P_{i,o}^{o^{-1}}(P_{i,o}(L_{N_i})),$$

where  $P_{i,o}^o: \Sigma_o^{\star} \to \Sigma_{i,o}^{\star}$  and  $P_{i,o}: \Sigma^{\star} \to \Sigma_{i,o}^{\star}$  are projections. Then, the following definition of synchronous codiagnosability can be stated (CABRAL; MOREIRA, 2020).

**Definition 3.4** (Synchronous codiagnosability). Let  $G_N = ||_{i=1}^r G_{N_i}$ , where  $G_{N_i}$  is the automaton that models the fault-free behavior of  $G_i$ . Assume that  $L_{N_i}$  denotes the language generated by  $G_{N_i}$ , for i = 1, ..., r. Let  $P_o : \Sigma^* \to \Sigma_o^*$ , with  $\Sigma_o = \bigcup_{i=1}^r \Sigma_{i,o}$ . Then, L is said to be synchronously codiagnosable with respect to  $P_o$ ,  $L_{N_i}$ , i = 1, ..., r, and  $\Sigma_f$  if

$$(\exists z \in \mathbb{N}) (\forall s \in L_F) (\forall st \in L_F, ||t|| \ge z \Rightarrow P_o(st) \notin L_{N_a}).$$

Definition 3.4 of synchronous codiagnosability of a language L is equivalent to definition 3.2 of diagnosability for a system where the fault-free language is  $L_{N_a}$  and the faulty language is  $L_F$ .

Since  $P_o(L_N) \subseteq L_{N_a} = ||P_o(L_{N_i})|$  (CABRAL; MOREIRA, 2020), then a system can be diagnosable with respect to  $P_o$ ,  $L_N$  and  $\Sigma_f$  according to Definition 3.2 and not synchronously codiagnosable according to Definition 3.4. The growth of the fault-free observable language  $L_{N_a}$  for DSD scheme is associated with the lost of synchronization of common unobservable events between two or more components of the system.

In Cabral and Moreira (2020) two architectures for synchronous diagnosis are presented: (i) the centralized and (ii) the decentralized. In the centralized scheme, all local diagnosers  $D_i$  are implemented in one unique site and they receive all information regarding event observations trough a unique communication channel. Thus, the centralized



Figure 10 – Automata  $G_1$  and  $G_2$  of case study.

synchronous diagnosis is a particular case of the decentralized synchronous diagnosis, since in the centralized setting, an observable event is observable to all components where it is defined.

**Remark 3.2.** In Cabral and Moreira (2020), a method for the verification of synchronous codiagnosability is presented. Although the verification is exponential according to the components number, the synchronous diagnosis method considers only the fault-free behavior model of the system components. Therefore, this growth is avoided for the diagnosis.

In the sequel, a case study is presented to show the growth of the fault-free language  $L_{N_a}$  for the synchronous diagnosis.

#### 3.3 CASE STUDY

Consider a system G composed of two modules  $G_1$  and  $G_2$  depicted in Figure 10. The event sets of the two modules are  $\Sigma_1 = \{a, c, e, g, \sigma_1\}, \Sigma_2 = \{e, h, \sigma_1, \sigma_2, \sigma_f\}$ . The observable event sets of  $G_1$  and  $G_2$  are  $\Sigma_{1,o} = \{a, c, e, g\}, \Sigma_{2,o} = \{e, h\}$ , respectively. The sets of unobservable events of  $G_1$  and  $G_2$  are  $\Sigma_{1,uo} = \{\sigma_1\}, \Sigma_{2,uo} = \{\sigma_1, \sigma_2, \sigma_f\}$ , respectively. The fault event set is  $\Sigma_f = \{\sigma_f\}$ . Automaton  $G = G_1 || G_2$ , and the composed fault-free behavior automaton  $G_N$  are shown in Figures 11 and 12, respectively. The fault event at  $G_1$  and  $G_2$ , denoted by  $G_{N_1}$  and  $G_{N_2}$ , respectively, are shown in Figure 13. All automata considered in this case study are taken from Cabral (2017).

Since all synchronous diagnosis schemes are based on the observation of the faultfree behavior of the system components, we can compare the accepted fault-free languages for the different diagnosis methods using observers<sup>1</sup>. According to definition 2.13, the language generated by an observer is the projection of the generated language of the original automaton (CASSANDRAS; LAFORTUNE, 2008). The computation of an observer is presented in Algorithm 1. In the sequence, we present the observer automata of  $G_{N_1}$ ,  $G_{N_2}$ ,

<sup>&</sup>lt;sup>1</sup> It is importante to remark that the use of observers in this document is only to simplify the case study. In Cabral et al. (2017) and Cabral and Moreira (2020) observers are also avoided in order to escape from their computational exponential growth in the worst case scenario.



Figure 11 – Automaton G of case study.



Figure 12 – Automaton $\,G_N$  of case study.



Figure 13 – Automata  $G_{N_1}$  and  $G_{N_2}$  of case study.



Figure 14 – Automata  $Obs(G_{N_1}, \Sigma_{1,o})$  and  $Obs(G_{N_2}, \Sigma_{2,o})$  of case study.



Figure 15 – Automaton  $Obs(G_N, \Sigma_o)$  of case study.



Figure 16 – Automaton  $G_N^a$  of case study,  $G_N^a = Obs(G_{N_1}, \Sigma_{1,o}) \| Obs(G_{N_2}, \Sigma_{2,o}).$ 



Figure 17 – Cardinality of the exceeding language generated by the decentralized synchronous diagnosis scheme  $L_{Exc,N_a}^{\leq n}$  ( $\circ$ ) for different values of n for case study.

and  $G_N$ ,  $Obs(G_{N_1}, \Sigma_{1,o})$ ,  $Obs(G_{N_2}, \Sigma_{2,o})$ , and  $Obs(G_N, \Sigma_o)$ , respectively, in Figures 14 and 15.

Since, in the decentralized synchronous diagnosis scheme, state estimators of the fault-free system component models are implemented in parallel, we can model the fault-free language  $L_{N_a}$  by making the parallel composition of the observers  $G_N^a = Obs(G_{N_1}, \Sigma_{1,o}) || Obs(G_{N_2}, \Sigma_{2,o})$ , depicted in Figure 16 (CABRAL; MOREIRA, 2020). In the sequel, we compare the generated language by  $G_N^a$  with the generated language by  $Obs(G_N, \Sigma_o)$ . In order to do so, we define languages  $L_{N_o}^{\leq n}$  and  $L_{N_a}^{\leq n}$  formed of all possible observed fault-free traces of length less than or equal to a given number  $n \in \mathbb{N}$  of  $Obs(G_N, \Sigma_o)$  and  $G_N^a$ , respectively as follows.

$$L_{N_o}^{\leq n} := \{ s \in \mathcal{L}(Obs(G_N, \Sigma_o)) : \|s\| \le n \},$$

$$(1)$$

$$L_{N_{a}}^{\leq n} := \{ s \in \mathcal{L}(G_{N}^{a}) ) : \| s \| \leq n \},$$
(2)

where ||s|| denotes the length of trace s.

Thus, we can define the exceeding language  $L_{Exc,N_a}^{\leq n} = L_{N_a}^{\leq n} \setminus L_{N_o}^{\leq n}$ . Language  $L_{Exc,N_a}^{\leq n}$  corresponds to the traces with length less than or equal to  $n \in \mathbb{N}$  that are accepted as fault-free in the decentralized synchronous diagnosis method and do not belong to the fault-free language of the system model. The cardinality of the exceeding language  $L_{Exc,N_a}^{\leq n}$  for the decentralized synchronous diagnosis scheme is presented in Figure 17. Note that the cardinality of the exceeding language of this scheme significantly increases as n grows. For example, for n = 10, there are 586 more traces in  $L_{Exc,N_a}^{\leq n}$  than in  $L_{Exc,N_o}^{\leq n}$ .

Since each exceeding trace in the decentralized synchronous diagnosis scheme can lead to an increase in the delay bound for diagnosis, the reduction of the accepted fault-free language is of great importance. Moreover, when we consider arbitrary long values of n, this accepted language growth can lead a diagnosable system to be not synchronously codiagnosable. Particularly, in this case study, the language generated by G is diagnosable according to Definition 3.2 and it is not synchronously diagnosable according to Definition 3.4.

### 3.4 FINAL REMARKS

In this chapter, the definitions of language diagnosability and synchronous codiagnosability are presented. In order to diagnose a fault using the monolithic scheme it is necessary to construct a diagnoser automaton based on an observer and search for indeterminate cycles in the diagnoser. If there is at least one indeterminate cycle, the language generated by the system is not diagnosable. Methods that avoid the use of observers for verification of diagnosability and online diagnosis have also been proposed in the literature.

Furthermore, a case study to verify the fault-free language growth in the decentralized synchronous diagnosis is presented. The concept of observers presented in Sampath et al. (1995) are used to illustrate it. Since the local component models are implemented in parallel, the fault-free language accepted can be seen through the parallel composition of the local observers. The case study aforementioned shows that a diagnosable system can be not synchronously codiagnosable because of this language growth.

In the next chapter, we propose the decentralized synchronous diagnosis with coordination scheme in order to reduce the fault-free language accepted for synchronous diagnosis,  $L_{N_a}$ , into the observed system language,  $P_o(L_N)$ . Thus, the synchronous codiagnosability verification is not needed for the diagnosis method proposed in this work. Since the fault-free observed language of the system according to the DSDC method is equal to  $P_o(L_N)$ .

# 4 DECENTRALIZED SYNCHRONOUS DIAGNOSIS WITH COORDINA-TION

In this chapter, the decentralized synchronous diagnosis with coordination scheme is presented. The goal of the method is to provide a diagnosis scheme with the same diagnosis power as the monolithic approach proposed in Sampath et al. (1995) without using the global plant model for diagnosis. The use of the global automaton model of the system is avoided in order to provide a diagnosis method that has better computational complexity than other approaches that need to implement the whole system model. Since, in this work, only an online diagnosis scheme is proposed, unless stated otherwise, it is assumed that all systems are monolithic diagnosable according to Definition 3.2.

As presented in the previous chapter, the fault-free observed language for the synchronous diagnosis scheme,  $L_{N_a}$ , can be a larger set than the observed fault-free system language,  $P_o(L_N)$ , when there are unobservable events in common between two or more components of the system. This is due to the loss of synchronization of unobservable events, and in order to maintain the same diagnosis power, one needs to reduce  $L_{N_a}$  into  $P_o(L_N)$ . Thus, in this work, a decentralized synchronous diagnosis with coordination scheme is proposed with the view to preserve the synchronization of unobservable events without the use of the fault-free global system model. To do so, the method is based on local state estimators of the fault-free component models of the system that can send cluster automata to a coordinator after the observation of an event. These clusters are composed of the unobservable reach of the local fault-free models after an observed event. The coordinator uses these clusters to build the correct state estimate of the fault-free global model of the system and verifies if an observed event is feasible in the current state estimate. If the event is not feasible, the fault event occurrence is detected.

In the sequel, the diagnosis scheme proposed in this work is presented in details.

#### 4.1 DIAGNOSIS SCHEME

In this work, we introduce a decentralized synchronous diagnosis with coordination (DSDC) scheme based on local state estimators of the fault-free component models of the system  $D_i$  that are implemented in parallel and a coordinator C that receives state estimate information from  $D_i$ , i = 1, ..., r. Coordinator C uses the local state estimation of  $D_i$  to refine the diagnosis and provide the fault event occurrence status. In this regard, suppose that the plant model consists of r components, *i.e.*,  $G = ||_{i=1}^r G_i$  and, associated with each component  $G_i$ , for  $i \in \{1, ..., r\}$  there is a  $D_i$  based on the fault-free component model,  $G_{N_i}$ . Each module  $G_i$  has a local measurement site  $LM_i$  that provides observation of events directly to its state estimator  $D_i$ . The set of events of  $G_i$  is defined as  $\Sigma_i = \sum_{i,o} \bigcup \Sigma_{i,uo}$ , where  $\Sigma_{i,o}$  and  $\Sigma_{i,uo}$  denote the sets of observable and unobservable events of  $G_i$ , respectively. The set of events of G is denoted as  $\Sigma = \bigcup_{i=1}^r \Sigma_i$ . The observable and



Figure 18 – Decentralized Synchronous Diagnosis scheme.

unobservable events of G are denoted as  $\Sigma_o = \bigcup_{i=1}^r \Sigma_{i,o}$  and  $\Sigma_{uo} = \Sigma \setminus \Sigma_o$ , respectively.  $\Sigma_f \subseteq \Sigma_{uo}$  denotes the fault event set.

In the DSDC framework, when an event  $\sigma_o \in \Sigma_{i,o}$  is observed by the local measurement site  $LM_i$ , this information is sent to the local state estimator  $D_i$ , which updates the current state estimate of  $G_{N_i}$ . Then,  $D_i$  sends to the coordinator a subautomaton of  $G_{N_i}$ , referred in this work as a cluster automaton  $C_i^{com}$ , composed of the states of the last state estimate for which  $\sigma_o$  is feasible, the states of the state estimate reached after the occurrence of  $\sigma_o$ , and all of its unobservable transitions.

After a cluster  $C_i^{com}$  is communicated to the coordinator, a composition between all local clusters is built by the coordinator in order to keep tracking the possible synchronizations of unobservable events. This is necessary to prevent the diagnosis scheme to consider fault-free traces that cannot be generated by the global plant model. These exceeding fault-free traces, caused by the lost of synchronization of unobservable events, cause the growth of the fault-free language for the DSD scheme,  $L_{N_a}$ , presented in (CABRAL; MOREIRA, 2020).

In this work, the communication between the local measurement sites  $LM_i$  and local state estimators  $D_i$ , and between local state estimators  $D_i$  and the coordinator are assumed to be ideal, *i.e.*, there are no communication delays and/or package losses. The DSDC scheme is depicted in Figure 18.

### 4.2 PROBLEM FORMULATION

In order to correctly synchronize common unobservable events, we propose the DSDC scheme, where the local state estimators  $D_i$  send clusters  $C_i^{com}$  of  $G_{N_i}$  to the coordinator  $\mathcal{C}$  that can select the correct traces of  $G_{N_i}$  according to the observed trace generated by the system. To do so, a communication protocol between the local state estimators  $D_i$  and coordinator  $\mathcal{C}$ , and a fault diagnosis procedure that indicates if a fault has occurred after the observation of a trace are proposed. Before these algorithms are

presented, it is first necessary to introduce the concepts of cluster automaton and natural cluster of an automaton.

**Definition 4.1** (Cluster automaton). A cluster  $C = (Q_C, \Sigma_C, f_C, \emptyset)$  is an automaton that has no initial states.

Given a subset of states of an automaton  $G = (Q, \Sigma, f, q_0), E \subseteq Q$ , and an observable event  $\sigma_o \in \Sigma_o$ , we define set  $B \subseteq E$  composed of the states of E such that  $\sigma_o$  is feasible:

$$B = \{q \in E : f(q, \sigma_o)!\}.$$
(3)

Then, the following definition of natural cluster of an automaton can be stated.

**Definition 4.2** (Natural cluster automaton). Let  $G = (Q, \Sigma, f, q_0)$ . Let  $E \subseteq Q$  be a subset of states of G and consider an observable event  $\sigma_o \in \Sigma_o$ . The natural cluster automaton  $NC(E, \sigma_o) = (Q_C, \Sigma_C, f_C, \emptyset)$  is a subautomaton of  $G = (Q, \Sigma, f, q_0)$  and is defined as:

•  $Q_C = B \cup (\cup_{q \in E} UR(f(q, \sigma_o)));$ 

• 
$$\Sigma_C = \{\sigma_o\} \cup \Sigma_u;$$

• 
$$f_C: Q_C \times \Sigma_C \to Q_C$$
, where

$$f_C(q, \sigma) = \begin{cases} f(q, \sigma_o) & \text{if } (q \in B) \land (\sigma = \sigma_o); \\ f(q, \sigma_u) & \text{if } (q \in Q_C \setminus B) \land (\sigma_u \in \Sigma_u); \\ undefined, \text{ otherwise.} \end{cases}$$

The natural cluster  $NC(E, \sigma_o)$  of G defined in Definition 4.2 corresponds to the states  $q \in E$  such that  $\sigma_o$  is feasible, the transitions  $(q, \sigma_o, q')$ , for  $q \in E$ , the unobservable reach of states q', and the unobservable transitions between the elements of UR(q'). The following example illustrates the natural cluster introduced in Definition 4.2.

**Example 7.** Consider automaton  $G = (Q, \Sigma, f, q_0)$  of Figure 19, where its set of observable and unobservable events are  $\Sigma_o = \{a, b\}$  and  $\Sigma_{uo} = \{\sigma_1\}$ , respectively. Let  $E = \{1, 3\}$ , where  $E \subseteq Q$ . The natural cluster  $NC(\{1, 3\}, b)$  is shown in Figure 20. Notice that since event b is feasible in state 1, i.e.,  $b \in \Gamma_G(1)$ , state 1 belong to  $NC(\{1, 3\}, b)$ .

The communication protocol of the DSDC method is given by Algorithm 3. Algorithm 3 is initialized in lines 1-5, where each local state estimator  $D_i$ , for i = 1, ..., r, sends a subautomaton of  $G_{N_i}$ ,  $S_{0,i}$ , corresponding to the states in the unobservable reach of its initial state and their related unobservable transitions. Then, in lines 6-16 the communication procedure is detailed when an event is observed. If an observed event  $\sigma_o \in \Sigma_{i,o}$ 



Figure 19 – Automaton G of Example 7.

$$1 \xrightarrow{b} 2 \xrightarrow{\sigma_1} 3 \xrightarrow{\sigma_1} 4$$

Figure 20 – Cluster  $C(\{1,3\}, b)$  of Example 7.

Algorithm 3 DSDC communication protocol **Input**:  $G_{N_i} = (Q_{N_i}, \Sigma_i \setminus \Sigma_f, f_{N_i}, q_{0,i})$ , for  $i \in \{1, ..., r\}$ . 1: for i = 1, ..., r do Compute  $E_i \leftarrow UR(q_{0,i})$ 2: Compute the subautomaton  $S_{0,i} = (E_i, \Sigma_{i,u}, f_{S_i}, q_{0,i})$  of  $G_{N_i}$ , where  $f_{S_i}(q, \sigma) =$ 3:  $f_{N_i}(q, \sigma)$  for  $q \in E_i$  and  $\sigma \in \Sigma_{i,uo}$ Send  $S_{0,i}$  to the Coordinator 4: 5: end for 6: Wait for the observation of an event  $\sigma_o \in \Sigma_{i,o}$  generated by the system 7: for each  $G_{N_i}$  such that  $\sigma_o \in \Sigma_{i,o}$  do if  $\sigma_o \in \Gamma_{G_{N_i}}(E_i)$  then 8: Compute cluster  $C_i^{com} \leftarrow NC_i(E_i, \sigma_o)$  and send it to the Coordinator 9: Compute  $E'_i \leftarrow \bigcup_{q_i \in E_i} UR(f(q_i, \sigma_o))$ 10: Update  $E_i \leftarrow E'_i$ 11: else 12:Send  $C_i^{com} \leftarrow \emptyset$  to the Coordinator 13:end if 14: 15: **end for** 16: Return to line 6

is feasible in the current state estimate of  $D_i$ ,  $E_i$ , the natural cluster  $NC_i(E_i, \sigma_o)$  is computed and it is communicated to the coordinator C. Otherwise,  $C_i^{com} = \emptyset$  is communicated to the coordinator C, which uses this information to directly diagnose the fault event occurrence.

It is important to remark that the computation of  $NC_i(E_i, \sigma_o)$  that is required in Line 9 of Algorithm 3 according to Definition 4.2 can be done in linear time by using any graph search algorithm (CORMEN et al., 2009).

#### 4.3 DIAGNOSIS PROCEDURE

In the DSDC scheme, a coordinator C needs to compute a composition between r clusters each time a new event  $\sigma_o$  is observed in order to verify if  $\sigma_o$  is feasible according to

the fault-free behavior of the system. According to Algorithm 3, after an initialization, the coordinator C receives a new cluster  $C_i^{com} = NC_i(E_i, \sigma_o)$  from all local state estimators  $D_i$ , such that  $\sigma_o \in \Sigma_{i,o}$ , each time an event  $\sigma_o$  is observed by  $LM_i$ . Each new cluster is incorporated in a previous stored one using a cluster union operation, defined in the following<sup>1</sup>. The definition of the cluster synchronous composition is also presented in the sequel.

**Definition 4.3** (Cluster union). Let  $C_1 = (Q_1, \Sigma_1, f_1, \emptyset)$  and  $C_2 = (Q_2, \Sigma_2, f_2, \emptyset)$  be two clusters. The union operation of  $C_1$  and  $C_2$ , is defined as  $C = C_1 \sqcup C_2 = (Q, \Sigma, f, \emptyset)$ , where

- $Q = Q_1 \cup Q_2;$
- $\Sigma = \Sigma_1 \cup \Sigma_2;$
- $f: Q \times \Sigma \rightarrow Q$ , where

$$f(q, \sigma) = \begin{cases} f_1(q_1, \sigma_1) = q'_1 & \text{if } q_1, q'_1 \in Q_1 \text{ and } \sigma_1 \in \Sigma_1; \\ f_2(q_2, \sigma_2) = q'_2 & \text{if } q_2, q'_2 \in Q_2 \text{ and } \sigma_2 \in \Sigma_2; \\ undefined, & otherwise. \end{cases}$$

**Definition 4.4** (Cluster synchronous composition). Let  $C_1 = (Q_1, \Sigma_1, f_1, \emptyset)$  and  $C_2 = (Q_2, \Sigma_2, f_2, \emptyset)$  be two cluster automata. The synchronous composition between  $C_1$  and  $C_2$  is defined as  $sync((C_1, C_2), R) = Ac(Q_1 \times Q_2, \Sigma_1 \cup \Sigma_2, f, R)$ , where  $R \subseteq Q_1 \times Q_2$  is a set of initial states and

$$f((q_1, q_2), \sigma) = \begin{cases} (f_1(q_1, \sigma), f_2(q_2, \sigma)) & \text{if } \sigma \in \Gamma_{C_1}(q_1) \cap \Gamma_{C_2}(q_2); \\ (f_1(q_1, \sigma), q_2) & \text{if } \sigma \in \Gamma_{C_1}(q_1) \setminus \Sigma_2; \\ (q_1, f_2(q_2, \sigma)) & \text{if } \sigma \in \Gamma_{C_2}(q_2) \setminus \Sigma_1; \\ undefined, & otherwise. \end{cases}$$

Note that Definition 4.4 is equivalent to the parallel composition between automata using R as the set of the initial states. The coordinator C uses the unobservable part of the communicated clusters to correctly follow the fault-free global behavior of the system by syncing the clusters of all system components after the observation of an event. In the following, the coordinator C, that computes the fault diagnostic status, for the DSDC scheme proposed in this work is formally presented in Algorithm 4. Algorithm 4 runs together with Algorithm 3.

The idea of Algorithm 4 is to verify if the observation of an event is indeed feasible in the fault-free behavior of the system, considering the synchronization of unobservable events. If the occurrence of an observable event is not possible in the fault-free system behavior, the algorithm informs the occurrence of the fault event and stops. To do so,

<sup>&</sup>lt;sup>1</sup> The cluster union operation is inspired in the graph union operation presented in Harary (1969).

### Algorithm 4 Diagnosis procedure

**Input**:  $\Sigma_i = \Sigma_{i,o} \cup \Sigma_{i,uo}$  for  $i = 1, ..., r, \Sigma_o = \bigcup_{i=1}^r \Sigma_{i,o}$ , subautomata  $S_{0,i}$ , and communicated clusters  $C_i^{com}$ , for  $i = 1, \ldots, r$ . **Output**: Diagnosis decision. 1: Define  $R \leftarrow \emptyset$  and  $I \leftarrow \emptyset$ 2: Define  $C'_i \leftarrow \emptyset$ , for  $i = 1, \ldots, r$ 3: After receiving subautomata  $S_{0,i}$ ,  $i = 1, \ldots, r$ , compute  $S = \prod_{i=1}^{r} S_{0,i}$ 4: Assign to R the initial state of S5: For  $S_{0,i} = (E_i, \Sigma_{i,u}, f_{S_i}, q_{0,i})$ , define  $C_i \leftarrow (E_i, \Sigma_{i,u}, f_{S_i}, \emptyset), i = 1, ..., r$ 6: Wait for the observation of an event  $\sigma_o \in \Sigma_{i,o}$ 7: for each  $C_i^{com}$  communicated from  $D_i$  do if  $C_i^{com} = \emptyset$  then 8: Inform the occurrence of the fault event and Stop 9: 10: else Define  $C_i \leftarrow C_i \sqcup C_i^{com} = (Q_{C_i}, \Sigma_i, f_{C_i}, \emptyset)$ 11:Define  $I \leftarrow I \cup \{i\}$ 12:end if 13:14: **end for** 15: Compute  $S \leftarrow sync((C_1, C_2, \ldots, C_r), R) = (Q_S, \Sigma_N, f_S, R)$ 16: if  $f_S(q, \sigma_o)$  is undefined for all  $q \in Q_S$  then Inform the occurrence of the fault event and Stop 17:18: **else** Set  $R \leftarrow \{q' \in Q_S : f_S(q, \sigma_o) = q'\}$ , for all  $q, q' \in Q_S$ 19:for each  $i \in I$  do 20:Compute  $C'_i = (Q'_i, \Sigma_i, f'_i, \emptyset)$  by removing from  $C_i$  all states and transitions that 21: are not reachable after  $\sigma_o$ Define  $C_i \leftarrow C'_i$ 22:23: end for 24: end if 25: Define  $I \leftarrow \emptyset$ 26: Return to line 6

Algorithm 4 is initialized in lines 1-5 by assigning set  $\emptyset$  to R, I, and  $C'_i$ . In line 3,  $S_{0,i}$ , communicated in the initialization of Algorithm 3, are used to compute automaton S. The initial state of S is assigned to variable R in line 4. Clusters  $C_i$  are then initialized using automata  $S_{0,i}$  in line 5.

From lines 6 to 26, the main cycle of Algorithm 4 is carried out when a new event is observed by at least one local state estimator  $D_i$  that communicates a new cluster to the coordinator. If an observable event  $\sigma_o$  is common to more than one system component and it occurs, each corresponding local state estimator  $D_i$  will communicate a new natural cluster  $C_i^{com}$ . For each  $C_i^{com}$  communicated, clusters  $C_i$  are updated in lines 7-14. If the empty automaton is communicated, then the fault is diagnosed, and the algorithm stops, as indicated in lines 8 and 9 of Algorithm 4. Otherwise,  $C_i$  is updated in line 11 to consider the behavior of the communicated cluster from  $D_i$ ,  $C_i^{com}$ , using the cluster union operation. Set I is then updated to record the indexes of local state estimators that have communicated in this run due to the observation of  $\sigma_o$ .

In line 15 of Algorithm 4, automaton S is computed by the cluster synchronous composition considering R as the set of initial states. If there is no observable transitions labeled with  $\sigma_o$  in S, then the occurrence of  $\sigma_o$  is not feasible in the fault-free system behavior. Therefore, the fault is diagnosed and the coordinator informs the occurrence of the fault event and stops. Otherwise, in line 19, a new set of initial states R is computed, formed by all states of S that are immediately reached by an observable transition labeled with  $\sigma_o$ , in order to be considered after a new event observation. In lines 21 and 22, the clusters  $C_i$  are updated to consider only the possible unobservable behavior after the occurrence of  $\sigma_o$ . Finally, the set of indexes I is defined as the empty set in line 25, and Algorithm 4 waits for a new event observation by  $D_i$ .

The following theorem guarantees that the fault-free language considered in the method presented in this work is equal to the observable fault-free language  $P_o(L_N)$  of the system.

**Theorem 4.1.** Consider a system G whose generated language is L and the fault-free language is  $L_N$ . The observable fault-free language considered in the DSDC scheme is equal to the observable fault-free language of the system  $P_o(L_N)$ ,  $P_o: \Sigma^* \to \Sigma_o^*$ .

**Proof.** The proof is done by induction with respect to the size of an observed trace s for the state estimate provided by the DSDC scheme and the state estimate of the fault-free automaton model of the system  $G_N$ .

• ||s|| = 0:

For ||s|| = 0, algorithms 3 and 4 are initialized. This process is done by the communication of the subautomata  $S_{0,i}$ ,  $i = 1, \ldots, r$ , computed in Algorithm 3 to the coordinator C, which is carried out in lines 1-4 of Algorithm 3. Note that the state set of  $S_{0,i}$  is  $E_i$  which is equal to  $UR(q_{0,i})$ , where  $q_{0,i}$  is the initial state of  $G_{N_i}$ . In addition, the transitions of  $S_{0,i}$  correspond to the unobservable transitions of  $G_{N_i}$  related to the states belonging to  $E_i$ . After Algorithm 4 receives subautomata  $S_{0,i}$ , automaton  $S = ||_{i=1}^r S_{0,i}$  is computed in line 3. Since  $S = ||_{i=1}^r S_{0,i}$ , the states of S correspond to the initial state estimate of automaton  $G_N = ||_{i=1}^r G_{N_i}$  for the observed sequence  $\varepsilon$ .

•  $||s|| = n, n \in \mathbb{N}$ :

Suppose now that a trace  $s = s'\sigma_o$ , such that ||s|| = n, has been observed by the DSDC scheme. Consider automaton  $S = (Q_S, \Sigma_N, f_S, R)$ , computed in line 15 of Algorithm 4. For all  $q \in Q_S$ , such that  $q' = f_S(q, \sigma_o)$ , UR(q') correspond to the state estimate of automaton  $G_N$  after the observation of trace s.

•  $||s|| = n + 1, n \in \mathbb{N}$ : Let us suppose that  $s = s' \sigma_o \sigma'_o$ , such that ||s|| = n + 1. Automaton S computed in the previous step, *i.e.*, after the observation of trace  $s'\sigma_o$ , contains the correct state estimate of  $G_N$  after the observation of  $\sigma_o$ . Notice that, after obtaining automaton S, set R is computed in line 19 of Algorithm 4 and it stores all states of S reached after event  $\sigma_o$ . In the sequence, for each local component i such that  $\sigma_o \in \Sigma_{i,o}$ , clusters  $C_i$  are updated in lines 21 and 22 in order to consider only the unobservable behavior after  $\sigma_o$ . It is important noticing that, according to the previous step, if the operation  $sync((C_1, C_2, \ldots, C_r), R)$  is performed, the result is equal to an automaton that corresponds to the state estimate of  $G_N$  after the observed trace  $s'\sigma_o$ .

After the observation of  $\sigma'_o$ , clusters  $C_i^{com}$  are communicated to the coordinator  $\mathcal{C}$ , where  $\sigma'_o \in \Sigma_{i,o}$ . These clusters correspond to all possible occurrences of  $\sigma'_o$  at the current state estimate of  $G_{N_i}$  and all their unobservable continuations. In line 11 of Algorithm 4, the cluster union operation is performed, which guarantees that clusters  $C_i$  are updated to also consider the information available in  $C_i^{com}$ . Then, in line 15, a new automaton S is built using clusters  $C_i$  and the set of initial states R, computed after the observation of  $s'\sigma_o$ . Since the *sync* operation is equivalent to the parallel composition considering all elements of R as the initial state of the resulting automaton, the unobservable reach of the states of automaton S reached after  $\sigma'_o$  correspond to the state estimate of  $G_N$  after the observed trace s. This result is achieved since the communicated clusters correspond to the unobservable reach of automata  $G_{N_i}$  after the observation of s, which concludes the proof.

In the following, an example is presented to illustrate the use of Algorithm 4 for the Decentralized Synchronous Diagnosis with coordination process.

**Example 8.** Consider the system  $G = \|_{i=1}^{3} G_{i}$ , where  $G_{1}$ ,  $G_{2}$  and  $G_{3}$  are depicted in Figure 21, automata G is shown in Figure 22 and the fault-free model  $G_{N}$  is illustrated in Figure 23. The sets of events of  $G_{1}$ ,  $G_{2}$  and  $G_{3}$  are, respectively,  $\Sigma_{1} = \Sigma_{1,o} \cup \Sigma_{1,uo} = \{a, b, d, \sigma_{1}, \sigma_{2}\}, \Sigma_{2} = \Sigma_{2,o} \cup \Sigma_{2,uo} = \{a, c, \sigma_{1}, \sigma_{f}\}$  and  $\Sigma_{3} = \Sigma_{3,o} \cup \Sigma_{3,uo} = \{a, e, g, \sigma_{1}\},$  where  $\Sigma_{1,o} = \{b, d\}, \Sigma_{2,o} = \{a, c\}$  and  $\Sigma_{3,o} = \{a, e, g\}$  are the sets of observable events of  $G_{1}$ ,  $G_{2}$  and  $G_{3}$ , and  $\Sigma_{1,uo} = \{a, \sigma_{1}, \sigma_{2}\}, \Sigma_{2,uo} = \{\sigma_{1}, \sigma_{f}\}$  and  $\Sigma_{3,uo} = \{\sigma_{1}\}$  are the sets of unobservable events of  $G_{1}$ ,  $G_{2}$  and  $G_{3}$ . The set of observable events of G is  $\Sigma_{o} = \Sigma_{1,o} \cup \Sigma_{2,o} \cup \Sigma_{3,o} = \{a, b, c, d, e, g\}$  and the set of fault events is  $\Sigma_{f} = \{\sigma_{f}\}$ .



Figure 21 – Automata  $G_1$ ,  $G_2$  and  $G_3$  of Example 8.



Figure 22 – Automaton G of Example 8.



Figure 23 – Automaton  $G_N$  of Example 8.

Suppose that the system G has generated the fault trace  $s_f = bc\sigma_f a^n$ ,  $n \in \mathbb{N}$ , which its local projections are  $P_{1,o}(s_f) = b$ ,  $P_{2,o}(s_f) = ca^n$  and  $P_{3,o}(s_f) = a^n$ , where  $P_{i,o}: \Sigma^{\star} \to \Sigma_{i,o}^{\star}, i \in \{1, 2, 3\}, are projections.$  It is important to notice that this system is not synchronously codiagnosable since the local projections of the fault trace can be observed in the fault-free component models  $G_{N_1}, G_{N_2}, and G_{N_3}$ , presented in Figure 24. The DSDC scheme is illustrated by using Algorithms 3 and 4 after each observed event.



Figure 24 – Automata  $G_{N_1}$ ,  $G_{N_2}$  and  $G_{N_3}$  of Example 8.

### • Observed trace $\varepsilon$

Before any event is observed, Algorithm 3 sends  $S_{0,1}$ ,  $S_{0,2}$ , and  $S_{0,3}$ , shown in Figure 25, to the Coordinator. Then  $S = S_{0,1} ||S_{0,2}||S_{0,3}$ , shown in Figure 26, is computed by Algorithm 4 that stores the initial state of S in  $R = \{(0,0,0)\}$ .

| $\rightarrow 0$ | $\rightarrow 0$ | $\rightarrow 0$ |
|-----------------|-----------------|-----------------|
| (a) $S_{0,1}$   | (b) $S_{0,2}$   | (c) $S_{0,3}$   |

Figure 25 – Subautomata  $S_{0,1}$  (a),  $S_{0,2}$  (b), and  $S_{0,3}$  (c) of Example 8.

Figure 26 – Initial composition  $S = S_{0,1} ||S_{0,2}|| S_{0,3}$  of Example 8.

At this stage, Algorithms 3 and 4 wait for a new observed event by  $D_1$ ,  $D_2$  or  $D_3$ .

#### • Observed trace b

When event b occurs, it is observed by  $D_1$ , and  $C_1^{com}$ , depicted in Figure 27, is communicated.

$$(0) \xrightarrow{b} (1) \xrightarrow{[a]} (4)$$

Figure 27 – Cluster automaton  $C_1^{com}$  after observation of event b.

In Algorithm 4, the union operation of  $C_1$  and  $C_1^{com}$  is computed in line 11, and the result is assigned to  $C_1$ , depicted in Figure 28 (a). Notice that, in this case,  $C_1$  is

equal to  $C_1^{com}$ . Since only local state estimator  $D_1$  observes event b, set I is updated to  $I = \{1\}$ . Clusters  $C_2$  and  $C_3$  are not modified as shown in Figure 28 (b) and 28 (c), respectively.



Figure 28 – Cluster automata  $C_1$ ,  $C_2$ , and  $C_3$  after observation of event b.

A new S is computed, according to line 15 of Algorithm 4 and it is depicted in Figure 29. Since a transition labeled with b, namely ((0,0,0), b, (1,0,0)), exists in S of Figure 29, the fault event is not detected and set R is updated to the states of S reached by transitions labeled with b, i.e.,  $R = \{(1,0,0)\}$  in line 19 of Algorithm 4.



Figure 29 - S after observation of event b.

After that,  $C'_1$ , shown in Figure 30, is computed and it is used to update  $C_1$ , according to lines 21 and 22 of Algorithm 4 by removing the states and transitions that are not reachable after event b. Algorithm 4 then updates set  $I = \emptyset$  and waits for a new observed event.

$$1 \xrightarrow{[a]} 4$$

Figure 30 – Cluster automaton  $C_1 = C'_1$  after observation of event b.

#### • Observed trace bc

The next generated event is c, observed by state estimator  $D_2$  that sends  $C_2^{com}$ , depicted in Figure 31. Then, the cluster automaton  $C_2$  shown in Figure 32 (b) is updated and, in this case, it is equal to  $C_2^{com}$ . Notice that cluster automata  $C_1$  and  $C_3$ , illustrated in Figure 32 (a) and 32 (c), respectively, do not change.



Figure 31 – Cluster automaton  $C_2^{com}$  after observation of trace bc.



Figure 32 – Cluster automata  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace bc.

Set I is updated to  $I = \{2\}$  and a new S is computed using  $R = \{(1,0,0)\}$  as the initial state, as shown in Figure 33.

$$\rightarrow$$
 1,0,0  $\xrightarrow{c}$  1,1,0

Figure 33 - S after observation of trace *bc*.

After that, set R is updated to  $R = \{(1, 1, 0)\}$  and a new  $C_2$  is computed, as presented in Figure 34.



Figure 34 – Cluster automaton  $C_2 = C'_2$  after observation of trace bc.

#### • Observed trace bca

When event a occurs, state estimators  $D_2$  and  $D_3$  send clusters  $C_2^{com}$  and  $C_3^{com}$ , depicted in Figures 35 and 36, respectively.



Figure 35 – Cluster automaton  $C_2^{com}$  after observation of trace *bca*.

$$\underbrace{0}_{a} \underbrace{2}_{[\sigma_1]} \underbrace{3}$$

Figure 36 – Cluster automaton  $C_3^{com}$  after observation of trace *bca*.

Clusters  $C_2$  and  $C_3$ , illustrated in Figure 37(b) and 37(c), are updated, and  $C_3$ , in this case, is equal to  $C_3^{com}$ , shown in Figure 36. Notice that cluster  $C_1$ , shown in Figure 37(a), does not change.



Figure 37 – Cluster automata  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace *bca*.

Set I is then computed, and it is equal to  $I = \{2, 3\}$ . Notice that since  $R = \{(1, 1, 0)\}$ , which corresponds to the initial state of S, and events a and  $\sigma_1$  are common to all three components but are not feasible in all three states, (1,1,0), no transitions leave state (1,1,0) of S. Thus, S is equal to a graph composed of only state (1,1,0), shown in Figure 38, and since there are no transitions in S labeled with event a, the fault event occurrence is detected in line 17 of Algorithm 4.

Figure 38 - S after observation of trace *bca*.

**Remark 4.1.** It is important to remark that the method presented in this work can be implemented even in the case where there are events that are observable to one component and unobservable to other components, as it is the case in Example 8.

# 4.4 COMPUTATIONAL COMPLEXITY ANALYSIS

Since mostly operations of Algorithms 3 and 4 have linear computational complexity and are performed on automata  $G_{N_i}$ , i = 1, ..., r, the computational complexity of the DSDC scheme can be analyzed according to the construction of S in line 15 of Algorithm 4. To do so, let  $Q_i$  denote the set of states of  $G_i$ , i = 1, ..., r, and suppose that all  $G_i$  have the same number of states  $|Q_i|$ . Due to the cluster synchronous composition performed in line 15 of Algorithm 4, automaton S can have, in the worst case scenario,  $|Q_i|^r$  states and  $|Q_i|^r \times (|\Sigma_{uo}| + 1)$  transitions. Thus, automaton S can grow exponentially with the number of system components.

However, this complexity order is achieved only if the majority of transitions of the component models are labeled with unobservable events since the communicated clusters  $C_i^{com}$  are composed of states and unobservable transitions reached after an observed event. Thus, the DSDC method performs better than fault diagnosis techniques based on the global system model, as it is the case in Example 8, where  $G_N$  has 31 states, while the sum of states of  $G_{N_1}$ ,  $G_{N_2}$  and  $G_{N_3}$  is equal to 15 and automaton S has, in the worst case, two states and one transition.

It is important to remark that one of the tendencies of Industry 4.0 is the increasing number of sensors in order to achieve more information of the system. Therefore, in practice, the number of unobservable events tend to be much smaller than the number of observable events, which also contributes to a better performance of the method presented in this work. This result can be seen in the next chapter, where a practical application of the DSDC method is presented.

# 4.5 FINAL REMARKS

In this chapter, the decentralized synchronous diagnosis with coordination (DSDC) scheme has been presented. The DSDC method is based on the implementation of state estimators computed from the fault-free component models of the system components. As in Cabral and Moreira (2020), if an event is observed and it is not feasible in the current state estimate of all state estimators, the fault event occurrence is diagnosed. Otherwise, the local state estimators that observed the event send clusters composed of the state estimates and their related unobservable transitions to a coordinator. This coordinator verifies if the observed event is indeed possible to occur in the synchronization of all local clusters. If the answer is negative, the fault event is diagnosed.

The method is formalized using two algorithms: (i) a communication protocol, that establishes when the cluster automata must be communicated from the state estimators to the coordinator; and (ii) the diagnosis procedure, that provides the diagnosis status after the observation of an event. Although, in the worst case scenario, the method is exponential with the number of the system components, it performs better than other methods based on the global system model, mainly for systems with a low number of unobservable events/transitions.

The DSDC scheme can be seen as a method to compute the state estimate of the global fault-free behavior model online, without the need to store the whole automaton model. Thus, it preserves the diagnosis power of the monolithic approach presented in Sampath et al. (1995) without the use of observers.

In the next chapter, the DSDC method is applied to a didactic manufacturing system.

### **5 DSDC METHOD APPLIED TO A MANUFACTURING SYSTEM**

In this chapter, the DSDC method is applied to a didactic manufacturing system. The system has unobservable events in common to two components and it is previously known to be diagnosable according to Definition 3.2 and not synchronously codiagnosable according to Definition 3.4. Therefore, one can use the monolithic diagnosis approach to diagnosis the fault event, however, it leads to a higher computation cost.

This chapter is organized as follows. First, the controlled plant and the system model are presented and then the fault-free behavior system components necessary to run the DSDC method are shown. A fault trace that cannot be diagnosed using the synchronous decentralized diagnosis scheme is used to illustrate the DSDC method. This fault trace is successfully diagnosed using the DSDC approach. Finally, final remarks about this implementation are drawn at the end of the chapter.

### 5.1 CASE STUDY SYSTEM

The controlled plant is a workpiece assembly manufacturing system of the manufacturer FESTO (FESTO, 2006) installed at the Industrial Informatics and Automation Laboratory of the Federal University of Santa Catarina. This didactic manufacturing system consists of six stations: (i) the Distributing station that removes workpieces from the magazine and transfer them to the next station through a robotic arm with a suction cup; (ii) the Testing station that either reject a workpiece or make it available to the subsequent station according to its height; (iii) the Separating station that separates the workpiece according to its positioning; (iv) the Pick and Place station which positions a lid over the workpiece; (v) the Fluidic Muscle Press station presses the lid to lock it in the workpiece; and (vi) the Sorting Station sorts the workpieces according to its material and color. In this work, only the first three stations functioning on their operation cycle are considered. The system schematics is presented in Figure 39 and a top view picture of the real system is shown in Figure 40.

The first three stations are designed to select workpieces with the appropriated height and in the right position. The detailed behavior is presented as follows: In the Distributing station, a pneumatic cylinder pushes a workpiece out of the magazine. A robotic arm with a suction cup turns on the vacuum and delivers it to the Testing station. Then, the workpiece is allocated in an elevator that moves it to be tested according to its height. If the workpiece is not higher than the standard model, a pneumatic cylinder pushes it to a conveyor belt with air pockets and the workpiece is delivered to the Separating station. Otherwise, it is discarded. Then, in the Separating station, the workpiece is allocated under a sensor to evaluate if it is rightly positioned. If the answer is yes, the workpiece is transferred to the following station. Otherwise, it is discarded.



(c)

Figure 39 – Stations considered for this case study: (a) Distributing station; (b) Testing station; and (c) Separating station.



Figure 40 – Real didactic manufacturing system of Industrial Computing and Automation Laboratory.



Figure 41 – Automata models: (a) Distributing station -  $G_1$ , (b) Testing station -  $G_2$  and (c) Separating station -  $G_3$ .



Figure 42 – Fault-free behavior of the stations: (a) Distributing station -  $G_{N_1}$ , (b) Testing station -  $G_{N_2}$  and (c) Separating station -  $G_{N_3}$ .

### 5.2 SYSTEM MODEL

In order to implement the DSDC method, it is first necessary to model the plant according to its controlled behavior. It is considered that the global system is composed of three stations: (i) the Distributing station; (ii) the Testing station; and (iii) the Separating station. The automata models of the stations and their fault-free behaviors are illustrated in Figures 41 and 42, respectively. The global plant model, G, has 159 states and 369 transitions and its fault-free behavior model,  $G_N$ , has 96 states and 218 transitions, and are omitted due to their large number of states and transitions.

The initial state of automaton  $G_1$  represents that the workpiece is ready to be transported. The robotic arm leaves the neutral position in order to grab the workpiece, modeled as event  $u_l$ , and  $G_1$  reaches state 1. This means that the robotic arm is ready to turn on the suction cup. When the workpiece is grabbed by the suction cup, modeled as event  $s_c$ , the system evolves to state 2. If the Testing station is ready to receive the workpiece, the robotic arm moves towards the right position to deliver the workpiece, and event  $s_{2r}$  occurs and the system evolves to state 3. The occurrence of event  $u_r$  indicates the rising edge of the right position sensor of the robotic arm. This leads the system to reach state 4, where the robotic arm can deliver the workpiece to the Testing station. After that, the suction cup is turned off, event  $d_2$  occurs, and the system evolves to state 5. In this state, the workpiece is successfully delivered and the robotic arm can return to its neutral position when event  $d_r$  occurs, leading  $G_1$  back to this initial state.

In this system, the fault event, represented by  $\sigma_f$ , models the suction cup malfunctioning. Therefore, the workpieces cannot be grabbed by the robotic arm and transported to the Testing station. It is important to notice that the robotic arm trajectory is not affected by the fault event and the system cannot recognize if the workpiece was successfully delivered. The fault behavior is modeled in the cycle formed by  $(6, s_{2r}, 7)$ ,  $(7, u_r, 8)$ ,  $(8, d_r, 9)$ , and  $(9, u_l, 6)$  transitions in  $G_1$ .

The initial state of automaton  $G_2$  represents that the station has no workpiece in the elevator and it is waiting to receive a new workpiece. When event  $s_{2r}$  occurs, the station can receive a new workpiece and reaches state 1 meaning that it is ready to test it. The occurrence of event  $d_2$  illustrates that the workpiece was delivered to the elevator and the system reaches state 2. When event  $d_s$  occurs, it means that the robotic arm has returned to a safe position and the elevator can rise to test the workpiece height. Then, the system evolves to state 3, which represents that the workpiece has been tested and can be either accepted or discarded. If the workpiece has an acceptable height, then the pneumatic cylinder pushes the workpiece to a conveyor belt with air pockets, modeled as event  $g_p$ . After that, the workpiece is in the conveyor belt and the elevator is in an upper position, illustrated by state 4. The successful deliver to the separating station is represented by the occurrence of event  $d_3$ . Then, the system reaches state 6, where there are no workpieces in the workstation and event  $r_s$  can occur returning the system to its initial state. If the workpiece has greater height than the accepted, it is discarded, which is modeled as event  $b_p$ . The system now evolves from state 3 to state 5, where the pneumatic cylinder pushes the workpiece to a discard ramp. When event  $d_p$  occurs, the workpiece was discarded and the system can return to its initial state. The cycle  $(0,s_{2r},1),(1,d_s,7),(7,g_p,8)$ , and  $(8,r_s,0)$  represents the behavior of the station when the fault has occurred and the robotic arm of the previous station only attempts to deliver a workpiece. Since the system only tests if a piece has a greater height, when the first station fails to deliver a real workpiece, the Testing station considers that a good height workpiece was tested.

The automaton  $G_3$  represents the Separating station and its essential functioning. The initial state illustrates that the station is ready to receive a workpiece. When event  $d_3$  occurs, the system evolves to state 1, where the workpiece has been placed. If the station detects a workpiece, modeled as event  $d_e$ , the process starts leading the system to state 2. The occurrence of event  $e_p$  means that the workpiece was successfully verified and the system is available to receive another workpiece. The behavior model of the Separating station was simplified, since it does not affect the diagnosability status of the whole system.

The sets of events of  $G_1$ ,  $G_2$ , and  $G_3$  are  $\Sigma_1 = \{u_l, s_c, s_{2r}, u_r, d_2, d_r, \sigma_f\}, \Sigma_2 =$ 

 $\{s_{2r}, d_2, d_s, g_p, r_s, b_p, d_3, d_p\}$ , and  $\Sigma_3 = \{d_3, d_e, e_p\}$ , respectively, where the sets of observable events of each component are  $\Sigma_{1,o} = \{u_l, s_{2r}, u_r, d_r\}$ ,  $\Sigma_{2,o} = \{s_{2r}, d_s, g_p, b_p\}$ , and  $\Sigma_{3,o} = \{d_e, e_p\}$ . Thus, the set of events of  $G = G_1 || G_2 || G_3$  is  $\Sigma = \{u_l, s_c, s_{2r}, u_r, d_2, d_r, \sigma_f, d_s, g_p, r_s, b_p, d_3, d_p, d_e, e_p\}$ , where the set of observable events of G is  $\Sigma_o = \Sigma_{1,o} \cup \Sigma_{2,o} \cup \Sigma_{3,o} = \{u_l, s_{2r}, u_r, d_r, d_s, g_p, b_p, d_e, e_p\}$  and the set of unobservable events of G is  $\Sigma_{uo} = \Sigma \setminus \Sigma_o$ .

In the sequel, we present the list of states and events of automata  $G_1$ ,  $G_2$ , and  $G_3$ .

| State | Meaning  |
|-------|--|
| 0     | Workpiece is in the right position to be transported                       |
| 1     | Robotic arm is in the right position to turn on the suction cup            |
| 2     | Workpiece has been grabbed by the robotic arm with the suction cup on      |
| 3     | Robotic arm is moving towards the position to deliver the workpiece        |
| 4     | Robotic arm is ready to deliver the workpiece to the next station          |
| 5     | Workpiece was successfully delivered                                       |
| 6     | Workpiece has not been grabbed by the robotic arm with the suction cup off |
| 7     | Robotic arm moving towards the delivering position without a workpiece     |
| 8     | Robotic arm is at the delivering position without a workpiece              |
| 9     | Robotic arm is returning to its neutral position                           |

| Table $I = $ States of $G_1$ | Table | 1 - 5 | states | OI | $G_1$ . |
|------------------------------|-------|-------|--------|----|---------|
|------------------------------|-------|-------|--------|----|---------|

| State | Meaning   |
|-------|---|
| 0     | The station is waiting for a new workpiece                            |
| 1     | Station is ready to test another workpiece                            |
| 2     | Workpiece delivered to the elevator                                   |
| 3     | Workpiece height tested by the sensor                                 |
| 4     | Workpiece in the conveyor belt and elevator in upper position         |
| 5     | Pneumatic cylinder pushed the workpiece off                           |
| 6     | Station with no workpiece waiting to return to its initial state      |
| 7     | Phantom workpiece height tested by the sensor                         |
| 8     | Phantom workpiece in the conveyor belt and elevator in upper position |

Table 2 – States of  $G_2$ .

| State | Meaning   |
|-------|---|
| 0     | The station is waiting for a new workpiece        |
| 1     | Workpiece is in the right position to be verified |
| 2     | Workpiece is detected by the sensor               |

Table 3 – States of  $G_3$ .

# 5.3 DSDC SCHEME APPLIED TO THE CASE STUDY

In this section, Algorithms 3 and 4 are applied to this manufacturing system in order to diagnose the fault event occurrence. Suppose that the system has generated

| Event       | Meaning  |
|-------------|--|
| $u_l$       | Robotic arm leaves the neutral position                    |
| $s_c$       | Suction cup is turned on                                   |
| $s_{2r}$    | Testing station is ready to receive the workpiece          |
| $d_2$       | Workpiece is delivered to the elevator                     |
| $u_r$       | Rising edge of the right position of the robotic arm       |
| $d_r$       | Falling edge of the right position of the robotic arm      |
| $\sigma_f$  | The suction cup fails                                      |
| $d_s$       | Robotic arm returns to a safe position                     |
| $g_p$       | Pneumatic cylinder pushes the workpiece to a conveyor belt |
| $\dot{d_3}$ | Workpiece is delivered to separating station               |
| $r_s$       | Returns the system to its initial state                    |
| $d_p$       | Workpiece is discarded                                     |
| $\dot{b_p}$ | Pneumatic cylinder pushes the workpiece to a ramp          |
| $d_e$       | Workpiece is detected                                      |
| $e_p$       | Workpiece is verified                                      |

Table 4 – Events of  $G_1$ ,  $G_2$ , and  $G_3$ .

the fault trace  $s_f = u_l \sigma_f (s_{2r} u_r d_r d_s g_p u_l r_s)^n$ ,  $n \in \mathbb{N}$ , where its observation is  $P_o(s_f) = u_l (s_{2r} u_r d_r d_s g_p u_l)^n$ , for  $P_o : \Sigma^* \to \Sigma_o^*$ . It is important to notice that this system is not synchronously codiagnosable since the local projections of the fault trace can be observed in the fault-free component models presented in Figure 42.

#### • Observed trace $\varepsilon$

At first, Algorithm 3 sends subautomata  $S_{0,1}$ ,  $S_{0,2}$ , and  $S_{0,3}$  depicted in Figure 43 to the Coordinator. After that, Algorithm 4 computes  $S = S_{0,1} ||S_{0,2}||S_{0,3}$  shown in Figure 44 and stores the initial state of S in  $R = \{(0,0,0)\}$ .



Figure 43 – Subautomata  $S_{0,1}$  (a),  $S_{0,2}$  (b), and  $S_{0,3}$  (c).



Figure 44 – Composition  $S = S_{0,1} || S_{0,2} || S_{0,3}$ .

#### • Observed trace $u_l$

Algorithms 3 and 4 wait for a new event observation by the local state estimators. When event  $u_l$  occurs, it is observed by  $D_1$  that communicates cluster  $C_1^{com}$  depicted in Figure 45. Since  $D_2$  and  $D_3$  do not observe any event, their clusters are equal to the ones sent before as Figure 46 illustrates.



Figure 46 –  $C_2$  and  $C_3$  after observation of event  $u_l$ .

In Algorithm 4, the union operation of  $C_1$  and  $C_1^{com}$  is computed in line 11, and the result is assigned to  $C_1$  shown in Figure 47. Notice that, in this case,  $C_1$  and  $C_1^{com}$  are the same clusters. Since only local state estimator  $D_1$  observes event  $u_l$ , set I is updated to  $I = \{1\}$ .



Figure 47 –  $C_1$  after observation of event  $u_l$ .

Then, a new S is computed, according to line 15 of Algorithm 4 and it is depicted in Figure 48. Since a transition labeled with  $u_l$  exists in S of Figure 48, the fault is not detected and set R is updated to  $R = \{(1,0,0)\}.$ 



Figure 48 – S after observation of event  $u_l$ .

After that, only  $C_1$  is updated from  $C'_1$  as shown in Figure 49, according to lines 21 and 22 of Algorithm 4. The states and transitions that are not reachable after event  $u_l$  are removed and the set I is updated to  $I = \emptyset$ . Then, the Algorithms 3 and 4 wait for a new event occurrence.



Figure 49 –  $C'_1$  after observation of event  $u_l$ .

#### • Observed trace $u_l s_{2r}$

The next event,  $s_{2r}$ , is observed for local state estimators  $D_1$  and is  $D_2$ . The cluster automata  $C_1^{com}$  and  $C_2^{com}$  illustrated in Figure 50 are sent to the diagnoser. Then,



Figure 45 –  $C_1^{com}$  after observation of event  $u_l$ .

 $C_1$  and  $C_2$  depicted in Figure 51 (a) and 51 (b), respectively, are updated after the union operation. Notice that  $C_1$  and  $C_2$  are equal to  $C_1^{com}$  and  $C_2^{com}$ , respectively. It is important to notice that the cluster automaton  $C_3$  - shown in Figure 51 (c) - has not changed.



Figure 50 –  $C_1^{com}$  and  $C_2^{com}$  after observation of trace  $u_l s_{2r}$ .



Figure 51 – Cluster automata  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r}$ .

According to line 12 of Algorithm 4, set I is updated to  $I = \{1, 2\}$  and a new S is computed and it is shown in Figure 52.



Figure 52 – S after observation of trace  $u_l s_{2r}$ .

After that, set R is updated to  $R = \{(3, 1, 0)\}$  and new  $C_1$  and  $C_2$  are computed from  $C'_1$  and  $C'_2$ , as presented in Figure 53 (a) and 53 (b). Then, Algorithm 4 updates set  $I = \emptyset$ .



Figure 53 –  $C'_1$  and  $C'_2$  after observation of trace  $u_l s_{2r}$ .

# • Observed trace $u_l s_{2r} u_r$

The next generated event is  $u_r$ , and it is observed by  $D_1$  which sends  $C_1^{com}$  to the diagnoser as illustrated in Figure 54. Then,  $C_1$  is updated as shown in Figure 55 (a) and cluster automata  $C_2$  and  $C_3$  remain the same as depicted in Figure 55 (b) and 55 (c). In this case,  $C_1$  is equal to  $C_1^{com}$ .



Figure 54 –  $C_1^{com}$  after observation of trace  $u_l s_{2r} u_r$ .



Figure 55 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r$ .

After that, set I is equal to  $I = \{1\}$  and a new S is computed, as presented in Figure 56. Then, set *R* is updated to  $R = \{(4, 1, 0)\}.$ 

$$\rightarrow 3, 1, 0 \xrightarrow{u_r} 4, 1, 0 \xrightarrow{[d_2]} 5, 2, 0$$

Figure 56 – S after observation of trace  $u_l s_{2r} u_r$ .

Cluster  $C_1$  is updated as shown in Figure 57 (a) and cluster automata  $C_2$  and  $C_3$ are not modified as depicted in Figure 57 (b) and 57 (c). Then, set I is updated to  $I = \emptyset.$ 



Figure 57 –  $C'_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r$ .

# • Observed trace $u_l s_{2r} u_r d_r$

The occurrence of event  $d_r$  is observed by  $D_1$  and  $C_1^{com}$  is sent to the Diagnoser as illustrated in Figure 58. After that,  $C_1$  is updated according to line 11 of Algorithm 4 as presented in Figure 59. Clusters  $C_2$  and  $C_3$  are the same as shown in Figure 57 (b) and 57 (c).

$$5 \xrightarrow{d_r} 0$$

 $\underbrace{5}_{r} \underbrace{0}_{r}$ Figure 58 –  $C_1^{com}$  after observation of trace  $u_l s_{2r} u_r d_r$ .

$$(4) \xrightarrow{[d_2]} (5) \xrightarrow{d_r} (0)$$

Figure 59 –  $C_1$  after observation of trace  $u_l s_{2r} u_r d_r$ .

Then, set I is updated to  $I = \{1\}$  and a new S is computed, according to line 15 of Algorithm 4 and it is presented in Figure 60. Since there is a transition labeled as  $d_r$  in S, set R is updated to  $R = \{(0, 2, 0)\}.$ 



Figure 60 – S after observation of trace  $u_1 s_{2r} u_r d_r$ .

After that, a new cluster automaton  $C_1$  is computed from  $C'_1$  as shown in Figure 61 (a).  $C_2$  and  $C_3$  illustrated in Figure 61 (b) and 61 (c) do not change since there was not an event observation in  $D_2$  or  $D_3$ . Also, Algorithm 4 updates set  $I = \emptyset$ .



Figure 61 –  $C'_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r$ .

### • Observed trace $u_l s_{2r} u_r d_r d_s$

When event  $d_s$  occurs, local state estimator  $D_2$  communicates two clusters  $C_2^{com}$  to the diagnoser according to line 8 of Algorithm 3 and it is presented in Figure 62 (a) and 62 (b).



Then,  $C_2$  is updated after the union operation and it is depicted in Figure 63 (a).  $C_1$  and  $C_3$  are not modified as illustrated in Figure 63 (b) and 63 (c).



Figure 63 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s$ .

Set I is updated to  $I = \{2\}$  and a new S is computed from these clusters as shown in Figure 64.



Figure 64 – S after observation of trace  $u_l s_{2r} u_r d_r d_s$ .

After that, set R is updated to  $R = \{(0,3,0)\}$  and the cluster  $C_2$  is computed from  $C'_2$  presented in Figure 65 (b). In this case,  $C_1$  and  $C_3$  - depicted in Figure 65 (a) and 65 (c) - are the same as before the event observation. At last, set I is updated to  $I = \emptyset$ .



Figure 65 –  $C_1$ ,  $C'_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s$ .

## • Observed trace $u_l s_{2r} u_r d_r d_s g_p$

The next generated event is  $g_p$  and it is observed by  $D_2$ . Then, the local state estimator communicates  $C_2^{com}$  as illustrated in Figure 66.



Figure 66 –  $C_2^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p$ .

After that,  $C_2$  is updated as shown in Figure 67. In this case,  $C_2$  is equal to  $C_2^{com}$ . Since event  $g_p$  was not observed by  $D_1$  or  $D_3$ , their clusters were not modified and they are depicted in Figure 65 (a) and 65 (c).



Figure 67 –  $C_2$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p$ .

Set I is updated to  $I = \{2\}$  and a new S - depicted in Figure 68 - is computed.

$$\rightarrow \underbrace{0,3,0}^{g_p} \underbrace{0,4,0}^{[d_3]} \underbrace{0,6,1}^{[r_s]} \underbrace{0,0,1}$$

Figure 68 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p$ .

After that, set R is updated to  $R = \{(0, 4, 0)\}$ , a new  $C'_2$  is computed and  $C_2$  is updated as shown in Figure 69 (b).  $C_1$  and  $C_3$  are not modified and they are illustrated in Figure 69 (a) and 69 (c), respectively. Then, set I is updated to  $I = \emptyset$ .



Figure 69 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p$ .

# • Observed trace $u_l s_{2r} u_r d_r d_s g_p u_l$

After the second occurrence of event  $u_l$ , Algorithm 3 sends  $C_1^{com}$  to the diagnoser and it is depicted in Figure 70.



Figure 70 –  $C_1^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l$ .

Then,  $C_1$  is updated and equal to  $C_1^{com}$  as shown in Figure 71 (a).  $C_2$  and  $C_3$  are the same as before the event observation as depicted in Figure 71 (b) and 71 (c), respectively. A new S is computed from  $C_1$ ,  $C_2$ , and  $C_3$  as presented in Figure 72. At this point, set I is updated to  $I = \{1\}$ . Notice that events  $d_3$  and  $r_s$  are now possible in S.



Figure 71 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l$ .



Figure 72 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l$ .

After that, Algorithm 4 updates  $R = \{(1, 4, 0), (1, 6, 1), (1, 0, 1)\}$ . Cluster automaton  $C_1$ , illustrated in Figure 73 (a) is computed according to lines 21 and 22 of Algorithm 4. Also, set I is updated to  $I = \emptyset$ .  $C_2$  and  $C_3$  are not modified and can be seen in Figure 73 (b) and 73 (c).



Figure 73 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l$ .

# • Observed trace $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$

When event  $s_{2r}$  occurs for the second time, it is observed by  $D_1$  and  $D_2$ . These local state estimators communicate  $C_1^{com}$  and  $C_2^{com}$  to the diagnoser as shown in Figure 74 (a) and 74 (b), respectively.



Figure 74 –  $C_1^{com}$  and  $C_2^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

Then,  $C_1$  and  $C_2$  are computed according to the operation union of each cluster as depicted in Figure 75 (a) and 75 (b), respectively.  $C_3$  remains the same cluster automaton as seen in Figure 75 (c). Algorithm 4 updates  $I = \{1, 2\}$  according to line 12.



Figure 75 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

In this point, a new S is computed and it is depicted in Figure 76. Notice that S has three initial states since  $R = \{(1, 4, 0), (1, 6, 1), (1, 0, 1)\}.$ 



Figure 76 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

Then, set R is updated to  $R = \{(3, 1, 1)\}$ . Cluster automata  $C_1$  and  $C_2$  are updated, as presented in Figure 77 (a) and 77 (b), respectively.  $C_3$  does not have any modification as can be seen in Figure 77 (c). Algorithm 4 then updates set  $I = \emptyset$  and waits a new event occurrence.



Figure 77 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

# • Observed trace $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r$

The occurrence of event  $u_r$  is observed by  $D_1$  which sends  $C_1^{com}$  to the diagnoser as illustrated in Figure 78.

$$(3 \xrightarrow{u_r} (4) \xrightarrow{[d_2]} (5)$$

Figure 78 –  $C_1^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r$ .

After that,  $C_1$  is computed as shown in Figure 79 and set I is updated to  $I = \{1\}$ . In this case,  $C_1$  is equal to  $C_1^{com}$ .



Figure 79 –  $C_1$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r$ .

A new S is computed by Algorithm 4 and it is shown in Figure 80. Set R is now updated to  $R = \{(4, 1, 1)\}.$ 



Figure 80 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r$ .

A new  $C'_1$  - presented in Figure 81 (a) - is computed and  $C_1$  is updated. Notice that  $C_2$  and  $C_3$  do not change as depicted in Figure 81 (b) and 81 (c), respectively. Set I is updated to  $I = \emptyset$ .



Figure 81 –  $C'_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r$ .

• Observed trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r$ 

The next generated event is  $d_r$  and it is observed by  $D_1$  which communicates the cluster automaton  $C_1^{com}$  as shown in Figure 82. Then,  $C_1$  is updated as presented in Figure 83 (a). Clusters  $C_2$  and  $C_3$  are not modified since  $D_2$  or  $D_3$  observe the event  $d_r$  and can be seen in Figure 83 (b) and 83 (c), respectively. Algorithm 4 then updates  $I = \{1\}$ .

$$(5 \xrightarrow{d_r} 0)$$

Figure 82 –  $C_1^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r$ .



Figure 83 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r$ .

After that, a new S is computed from  $C_1$ ,  $C_2$ , and  $C_3$  as illustrated in Figure 84. Algorithm 4 now updates set R to  $R = \{(0, 2, 1)\}.$ 



Figure 84 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r$ .

Then,  $C_1$  is updated according to lines 21 and 22 from Algorithm 4 as depicted in Figure 85 (a).  $C_2$  and  $C_3$  are the same as before the observed event as shown in Figure 85 (b) and 85 (c), respectively. Set I is now updated to  $I = \emptyset$ .



Figure 85 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r$ .

#### • Observed trace $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s$

The second occurrence of event  $d_s$  is observed by  $D_2$  which communicates two clusters  $C_2^{com}$  - depicted in Figure 86 (a) and 86 (b) - to the diagnoser.



Figure 86 –  $C_2^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s$ .

Then,  $C_2$  is updated after the union operation according to line 11 of Algorithm 4 and it is depicted in Figure 87 (a).  $C_1$  and  $C_3$  are not modified as shown in Figure 87 (b) and 87 (c). Algorithm 4 updates set  $I = \{2\}$ .



Figure 87 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s$ .

A new S is computed as presented in Figure 88. Then R is updated to  $R = \{(0,3,1)\}$ .

| →(0,2,1)- | $\xrightarrow{d_s}$ (0,3,1) |
|-----------|-----------------------------|
| $\sim$    | $\sim$                      |

Figure 88 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s$ .

After that,  $C_2$  is updated, presented in Figure 89 (b), according to lines 21 and 22 of Algorithm 4 by removing the states and transitions that are not reachable after event  $d_r$ . C1 and  $C_3$  are not modified as illustrated in Figure 89 (a) and 89 (c), respectively. Then, set I is updated to  $I = \emptyset$ .



Figure 89 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s$ .

### • Observed trace $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p$

When event  $g_p$  occurs for the second time, it is observed by  $D_2$  which communicates  $C_2^{com}$  as shown in Figure 90.

$$3 \xrightarrow{g_p} 4 \xrightarrow{[d_3]} 6 \xrightarrow{[r_s]} 0$$

Figure 90 –  $C_2^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p$ .

After that,  $C_2$  is updated as depicted in Figure 91. In this case,  $C_2$  is equal to  $C_2^{com}$ . Clusters  $C_1$  and  $C_3$  - depicted in Figure 89 (a) and 89 (c) - are not modified at this point since local state estimators  $D_1$  and  $D_3$  do not observe event  $g_p$ . Then, set I is update to  $I = \{2\}$ .


Figure 91 –  $C_2$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p$ .

A new S is now computed by Algorithm 4 and it is shown in Figure 92. Then, set R is updated to  $R = \{(0, 4, 1)\}.$ 



Figure 92 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p$ .

After that, cluster  $C_2$  is updated, illustrated in Figure 93 (b).  $C_1$  and  $C_3$  are not updated as presented in Figure 93 (a) and 93 (c), respectively. Then, Algorithm 4 updates set  $I = \emptyset$ .



Figure 93 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p$ .

• Observed trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l$ 

After the third occurrence of event  $u_l$ ,  $D_1$  sends  $C_1^{com}$  to diagnoser as depicted in Figure 94.



Figure 94 –  $C_1^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l$ .

Then,  $C_1$  is updated and, in this case, it is equal to  $C_1^{com}$  as shown in Figure 95. After that, set I is updated to  $I = \{1\}$ .



Figure 95 –  $C_1$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l$ .

A new S is computed from  $C_1$ ,  $C_2$ , and  $C_3$  as presented in Figure 96. Then, set R is updated to  $R = \{(1, 4, 1)\}.$ 



Figure 96 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l$ .

After that,  $C_1$  is updated as shown in Figure 97 (a). Clusters  $C_2$  and  $C_3$  are not modified and can be seen in Figure 97 (b) and 97 (c), respectively. Then, set I is updated to  $I = \emptyset$ .

$$\underbrace{1}_{(a) C_1}^{[s_c]} \underbrace{2}_{(b) C_2} \underbrace{4}_{(b) C_2}^{[d_3]} \underbrace{0}_{(c) C_3}^{[d_3]} \underbrace{0}_{(c) C_3}^{[d_3]} \underbrace{1}_{(c) C_3}$$

Figure 97 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l$ .

• Observed trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ 

Finally, the third occurrence of event  $s_{2r}$  is observed by  $D_1$  and  $D_2$  which send  $C_1^{com}$  and  $C_2^{com}$  as presented in Figure 98 (a) and 98 (b), respectively.



Figure 98 –  $C_1^{com}$  and  $C_2^{com}$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

Then, clusters  $C_1$  and  $C_2$  are computed according to the operation union of each cluster as illustrated in Figure 99 (a) and 99 (b), respectively.  $C_3$  is not modified at this stage as shown in Figure 99 (c). After that, set I is updated to  $I = \{1, 2\}$ .



Figure 99 –  $C_1$ ,  $C_2$ , and  $C_3$  after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

Then, a new S is computed as shown in Figure 100. Notice that, since  $R = \{(1, 4, 1)\}$ , event  $s_{2r}$  is common to two components but not feasible in both states, no transition leave state (2, 4, 1) of S. Therefore, S is equal to a graph composed of two states (1, 4, 1) and (2, 4, 1), presented in Figure 100. Since there are no transitions in S labeled with event  $s_{2r}$ , the fault occurrence is detected in line 17 of Algorithm 4.



Figure 100 – S after observation of trace  $u_l s_{2r} u_r d_r d_s g_p u_l s_{2r} u_r d_r d_s g_p u_l s_{2r}$ .

## 5.3.1 Concluding remarks of the case study

It is important to notice that the fault occurrence is detected after the third observation of event  $s_{2r}$ . Moreover, in the worst-case scenario for this example, S has 9 states and the sum of states of  $G_{N_1}$ ,  $G_{N_2}$ , and  $G_{N_3}$  is equal to 26. On the other hand, the fault-free behavior automaton,  $G_N$ , has 96 states.

Notice also the growth of the cardinality of the exceeding language  $L_{Exc,N_a}^{\leq n}$  for the DSD scheme presented in section 3.3 applied to this practical implementation in Figure 101. For example, for n = 9, there are 9,093 more traces in  $L_{N_a}^{\leq n}$ . Notice that the cardinality of the exceeding language considerably increases as n grows, which is avoided in the DSDC scheme proposed in this work.



Figure 101 – Cardinality of the exceeding language generated by the decentralized synchronous diagnosis scheme  $L_{Exc,N_a}^{\leq n}$  ( $\circ$ ) for different values of n.

## 5.4 FINAL REMARKS

In this chapter, the DSDC method is applied to a real system. A didactic manufacturing system installed at Industrial Computing and Automation Laboratory of Federal University of Santa Catarina was considered. In order to implement the DSDC method, the controlled plant is first modeled from three component models where a brief explanation of their operations is presented. The fault-free component models are used as input to the communication protocol algorithm, which runs together to the diagnosis procedure.

In this case, the system is not synchronously codiagnosable since the local projections of the fault trace can be observed in the fault-free models of the local components. In order to diagnose the fault event occurrence, a fault trace generated by the system was considered and the DSDC method is applied to each event observation of the fault trace. The exceeding language accepted for the decentralized synchronous diagnosis for this implementation is also presented.

## 6 CONCLUSION

Recently, the synchronous diagnosis strategy has been presented in the literature. In this scheme, state estimators of the fault-free system component models are implemented in parallel, and the fault event is diagnosed when an observable event that is not feasible in the current state estimate of at least one fault-free component model occurs. Although, by using this method, an exponential complexity order, with the number of the system components, is avoided for diagnosis, the fault-free language accepted by the synchronous diagnosis can be larger than the observable fault-free language generated by the system. Thus, a system may be diagnosable and not synchronously codiagnosable due to this property, which reduces the diagnosis efficiency. Thus, in this work, a decentralized synchronous diagnosis with coordination (DSDC) for discrete event systems modeled as automata is proposed with the view to eliminate this fault-free language growth.

In the DSDC method, two algorithms run together: the communication protocol and the diagnosis procedure, also called coordinator. Local state estimators send the information regarding event observations, and the coordinator provides the diagnosis status. The method uses cluster automata of the fault-free component models to online computed the current fault-free state estimate of the global system model in order to verify if an observed event is feasible. If an observed event is not feasible in the current fault-free state estimate, the fault event occurrence is diagnosed. The main advantage of the proposed method is that the fault-free language accepted by the DSDC is equal to the observable fault-free language of the global system. In general, the DSDC method has a smaller computational complexity than traditional diagnosis approaches. Moreover, the same diagnosis power is achieved as the monolithic technique using only the fault-free component models for diagnosis.

The method was implemented to a real system that is not synchronously codiagnosable. It is shown that the DSDC method successfully diagnose the fault event occurrence after a bounded number of event observations. The implementation of the method illustrates that if the system is monolithically diagnosable, the DSDC method can be applied.

The main contributions of this work are highlighted in the following:

- An algorithm for the communication protocol between local state estimators and a coordinator is proposed;
- A cluster automata synchronous composition is proposed;
- An algorithm for the coordinator that detects if the fault event has been occurred and synchronizes the common unobservable events is proposed;
- The DSDC method is guaranteed to have the same diagnosis power as the monolithic diagnosis approach;

• An application of the DSDC method in a real system is presented.

In the sequel, future research themes that can be carried out from this work are presented.

- A full implementation of the DSDC method using PLCs and a Supervisory Control and Data Acquisition (SCADA) approach is currently being investigated.
- In this work, the communication between local state estimators and the coordinator is supposed to be ideal, which cannot always be guaranteed. A DSDC method that is robust to communication delays and/or package losses is an open research theme.

The results presented in this thesis have been submitted for presentation in the next International Conference on Automatic Control and Soft Computing (CONTROLO 2022) as Mayer et al. (2022).

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