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# VISUALITY AND MATHEMATICAL VISUALIZATION: SEEKING NEW FRONTIERS*

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*This paper aims to establish a new theoretical perspective which can provide reflections for the research of visualization in mathematics education. The history of how visual technologies have affected ways of seeing things leads to the understanding of the construction of mathematical looking. The concept of visuality, taken from the North American study on visual culture, is proposed here as a tool for analysing visual practices and mathematics education, explaining that visual thinking is shaped by means of visual regimes. Finally, it presents a discussion focused on benefits of visual culture and a commentary describes, by connecting theories, its potential for future research.*
*Keywords: Visuality, Mathematical Looking, Mathematical Visualization, Theoretical Perspective, Mathematics Education.*

## INTRODUCTION

The issue of visualization in the teaching and learning of geometry has been a main research focus for several groups, because they have recognized its importance for student comprehension of mathematical and geometrical knowledge (Presmeg, 1986; Zimmerman & Cunningham, 1991; Guttiérrez, 1996; Duval, 1999; Biza et al, 2009).

According to Presmeg (2006), research on visualization in teaching and learning mathematics began in the 70's and early 80's and was first based on theoretical psychology. In the 90's, when the importance of this research field was then recognized in mathematics education, it became focused on different issues as diverse as: curriculum development; effectiveness of visualization for learning mathematics; student´s seeming reluctance to engage (and difficulty to do) with visualization; gender differences; etc. Therefore, a trend stands out linked to the embodied cognition and affective aspects. Since 2000 we have observed an increase in investigations on semiotic aspects for a more deeply understanding of the concept of image and representation.

Although we have seen an increasing interest in this research area as well as the broadening of research questions and the considering of theoretical diversity that encompasses psychology, semiotics, anthropology, we are still very far from exhausting the issue that there is a clear need to continue the search for new theoretical perspectives.

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Lerman (2010) argues that the multiplicity and divergence of theories is not necessarily a problem, but is indispensable to enlarge the range of theories for discussing many complex

issues that interact with the teaching and learning of mathematics. However, the issue is not to get a bunch of theories, but rather to find ways to connect them. By proposing the term "networking of theories", Bikner-Ahsbahs & Prediger (2010) have discussed strategies and methods for connecting theoretical approaches.

In the study of connecting theories, Wedege (2009) addressed the use of the terms theory, theoretical approach and theoretical perspective, and considered the synonymy of the two former ones, as follows:

> A theoretical approach is based on a system of basis of basic theoretical principles combined with a methodology, […], hence, guiding and directing thinking and action. A theoretical perspective is a filter for looking at the world based on theoretical principles, thus with consequences for the construction of the subject and problem field in research; that is the field to be investigated (Wedege, 2009, p. 1667).

In accordance to Wedege´s terminological understanding, in this text I adopt the use of cultural perspective in the sense of a theoretical perspective in mathematics education research.

According to Bikner-Ahsbahs et al (2010), the beginning of the connection of theories is marked by distinguishing among theories and their understanding. Thus, as a purpose of this paper I am seeking a cultural approach to recognize a theoretical perspective suitable for the use of visualization in mathematics education.

Since about 5 years, I have been proposing to discuss the visual thinking under the principles of cultural studies. In this context, I am trying to understand the construction of the mathematical looking as well as thinking about methodologies for mathematics education, by exploring the history of visual technologies in seeing things (Flores, 2007; 2010).

Particularly in this paper, I emphasize the concept of visuality, which is the sum of discourses that inform how we see and provide the basis for an analysis of visual practices, exploring the role of mathematical concepts in visual regimes. For that, at first I briefly present some cultural visual principles, and then I address two examples of how visuality can work in mathematics education research.

## VISUAL CULTURE AND VISUALITY PRINCIPLES

Visual Culture is a new interdisciplinary field that combines arts, philosophy, anthropology, and cultural studies, by focusing on images (Brennan & Jay, 1996; Sturken & Cartwright, 2001; Dikovitskaya, 2005). According to Dikovitskaya (2005), it considers the visual image as a focal point in the process through which meaning is made in a cultural context. An important mark point in the development of this field was the book entitled "Vision and Visuality", organized by Foster in 1988. Combining the two terms, i.e. vision and visuality, the author argued that they could not be simply distinguished, but rather dialectically constructed. Although vision is considered a physical process and visuality a social fact, vision is both social and historical. On the other hand, visuality also involves the body and the psyche.

Visuality has become an important keyword for the field of visual culture (Mirzoeff, 2006). It involves both historically constructed techniques and discursive determinations. Thus, visuality is a more suitable term in visual culture than visualization, because the former means the visual in the overlap between representation and cultural power. Hence, practices of looking create ways of seeing in to historical and cultural specificities. Individual visuality conflicts with its own production of intersubjectivity, thus defining the dialectic of the gaze. "Our visual experiences do not take place in isolation; they are enriched by memories and images from many different aspects of our lives" (Sturken and Cartwright, 2001, p.2).

Visual methodologies have been offered to studies of vision, scopic regimes and cultural practices of visuality in different fields (e.g. history, art history, art, philosophy, etc.). Recently, such methodologies have been introduced for many educational purposes. In this scenario, I have proposed (Flores, 2010) that principles and methodologies from visual culture studies can be potentially applied to mathematical visualization research, as outlined below:

- To think about the visual through the cultural, social practices, and power relations in which images and practices of visuality - that is, ways of looking and producing looks - are involved.
- To study the history of various technologies of visual culture, such as the technique of perspective, photography, cinema, exploring the relationship between technology and construction of sight.
- To consider space, distance, perspective, light, volume, depth, as statements that are conceptualized in a discursive practice and embodied in techniques and effects through images.
- To examine different visual regimes emphasizing the role of mathematics in the maintenance of visual homogenization, ocularcentrism and panoptical regime.
- To use the theory of perspective to operate as a diagram, a working hypothesis to think about knowledge, looks, and representing images.

In this context, I have also suggested to use the term "visuality" instead of "visualization", because the former leads to a deconstruction of the founding principles of sense of vision and perception. In contrast, visualization is understood as a process of construction and transformation of mental images, whereas visuality is the sum of discourses that inform how we see. Thus, while the latter is concerned with learning geometry´s concepts and visual skills, visuality discusses visual practices in the context of history and culture.

## VISUALITY AS A TOOL IN MATHEMATICS EDUCATION RESEARCH

My proposal considers visuality as a tool to analyse visual regimes historically constructed and visual source as a place for research. It sets out to address both theoretical and methodological strategies for mathematics education research. In one sense, the visual sources can be the focus for an analysis of different visual practices. On the other hand, they could provide the basis of the practice of mathematical looking. This means that such perspective involves both understanding the construction of the mathematical looking as well as the design of research problems and methodologies in the field.

Thus, in this paper I highlight two works written by our research group in order to demonstrate how this theoretical perspective works, as follows:

In the first example, Zago and Flores (2010) used modern local artists' paintings exhibiting characteristics of a classical system of visuality to exercise the mathematics looking. By means of those paintings and involving concepts of harmony, symmetry, parallelism and perspective, it was concluded that everyone may do it differently. This is because our visual thinking is shaped within an already built field of techniques and discourses about visuality.

In the following picture (Fig. 1) called "phantasy woman" by Rodrigo de Haro, we can see the layout of lines suggesting proportionality in pictorial space. In fact, the visuality employed on classical visual regime becomes so dominant that one constitutes a *habitus*, formatting our gaze to be technique, geometric and rational. In this case, perspective is not simply a visual technique but a way of seeing. Thus, mathematical knowledge is not only typical for paintings, but it is also an element for organizing the pictorial space and our thoughts.

Figure 1. Rodrigo de Haro's Painting Phantasy Woman
("Mulher Fantasia" in Portuguese)

In this example, the concepts of perspective were suggested as a method for relating art and mathematics education. From one side we have the paintings that bring out the modernized thought of the artist, and on the other side we have the mathematical looking that elaborate technique and geometrics knowledge. Thus, mathematical knowledge and art can be connected because they both affect and support each other.

In the second example, Flores and Wagner (2012) studied basic concepts involved in the perspective technique from Alberti's Treatise on Painting published in 1435. In order to better understand the complexity of such technique, it was applied in Renaissance paintings, not only to show the design in perspective, i.e. the geometry of an art, but also to discuss practices of looking (Fig. 2).

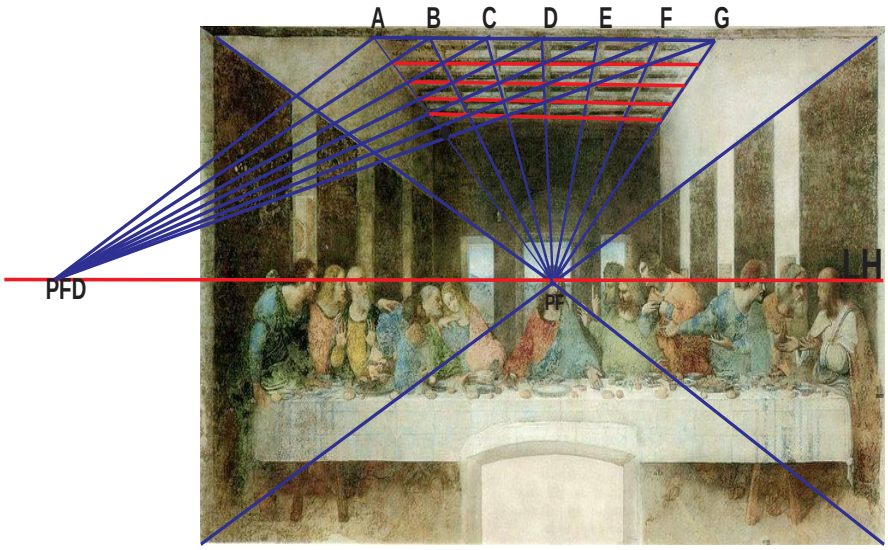


Figure 1. Leonardo da Vinci's famous painting, The Last Suppper.
("Il Cenacolo" in Italian)

The perspective emphasizes a scientific and mechanical view toward ordering and depicting of the pictorial space, and defines the central position of the spectator's view. So, the technique of perspective appears to us as a model of vision that produces the three-dimensional as well as rationalized space of perspectival vision.

In this latter example, the Cartesian visual regime was suggested in order to overcome the geometry teaching as usual. Our study has also examined the set of statements considered as truth, that is, proportionality, symmetric, perspective as notions for organizing the pictorial space and elaborating way of seeing things.

Although both examples presented above have not been applied in practice yet, they could be helpful in dealing with mathematical visualization activities for classroom and teacher development. By means of such activities, we could analyse how teachers or students put into practice ways of looking, reproducing discursive practices that have become embodied in techniques and effects.

## CONCLUDING REMARK

Considering the need for further studies on the components of overarching theories of visualization, this paper aimed to present a point of view that includes historical and cultural aspects of looking to deal with research on visualization in mathematics education. It also intended to promote a better understanding of how visual culture could be useful for research on mathematics visualization, contributing to build up theoretical principles involving historical studies of visuality and constitution of human subject; methodological strategies to connect art and mathematics education; tactics to understanding modes of responding to visuality, as social patterns of looking.

For future studies, this paper advocates in seeking for the studies of visual culture in order to formulate new research questions in mathematics education such as: different practices of looking and their role for determining mathematical looking; different ways of representing

space and their connection with technologies of power and invention of mathematical concepts; analyses of images and search of how one created discursive practices involving mathematical statements; visual methodologies and their request in mathematics education.

As a closing remark, further studies are necessary to establish such perspective in an empirical study, legitimising a meta-theoretical level in mathematics visualization research. In order to do that, discourse analyses and social practices of learning must be first linked. Anyway, applications within classroom at different levels of teaching and learning of mathematics and also with teacher's development remain still uncovered.

**References**


Bikner-Ahsbah, A., Prediger, S. (2010). Networking of Theories – An Approach for Exploiting the Diversity of Theoretical Approaches. In Sriraman, B &English, L. (Eds.). *Theories of Mathematics Education*: Seeking New Frontiers, pp. 483-506. London, New York: Springer.

Bikner-Ahsbah, A., Dreyfus, T., Kidron, I., Radford, L., Artigue, M., Sabena, M. (2010). In Pinto, M. M. F. & Kawasaki, T. F. (Eds.). *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 145-175. Belo Horizonte, Brazil: PME.

Biza, I., Nardi, E. & Zachariades, T. (2009). Teachers' views on the role of visualization and didactical intentions regarding proof. *Proceedings of CERME 6*, Vol. 1, pp.261-270. Lyon: France.

Brennan, T & Jay, M. (1996). *Vision in context*: historical and contemporary perspectives on sight. New Work: Routledge.

Dikovitskaya, M. (2005). *Visual Culture*. The study of the visual after the cultural turn. Cambridge, Massachusetts/London, England: The MIT Press.

Duval, R. (1999). Representation, vision and visualization: cognitive functions in mathematical thinking. Basic issues for learning. In F. Hitt & Santos, M (Eds). *Proc. 21st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. (pp.3-26). Cuernavaca, Morelos, Mexico.

Foster, H. (1988). *Vision and Visuality*. Seattle: Bay Press.

Flores, C. R. (2007). *Olhar, Saber, Representar*: sobre a representação em perspectiva. São Paulo: Musa Editora.

Flores, C. R. (2010). Cultura visual, visualidade, visualização matemática. *Zetetikè*, v. 18, pp. 271-293. Número Temático Linguagens e práticas culturais: perspectiva para a Educação Matemática.

Flores, C. R. & Wagner, D. R. (2012). Práticas do olhar na pintura do Renascimento: contribuições para a educação matemática. *Revista de Matemática, Ensino e Cultura*, in press.

Guttiérrez, A. (1996). Visualization in 3 – dimensional geometry: in search of a framework. In L. Puig e A. Gutiérrez (Eds.). *Proceedings of 20th PME conference* (Vol. 3, pp. 19-26). Valencia: Universitat de València, Dept. de Didàctica de la Matemática.

Lerman, S. (2010). Theories of Mathematics Education: Is Plurality a Problem? In Sriraman, B &English, L. (Eds.). *Theories of Mathematics Education*: Seeking New Frontiers, pp. 99-109. London, New York: Springer.

Mirzoeff, N. (2006) *On visuality*. Journal of Visual Culture, v. 5, p. 53-79.

Presmeg, N. (1986). Visualization in high school mathematics. *For the Learning of Mathematics*, 6(3), 42-46.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics: Emergence from psychology. In A.Gutierrez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education* (pp. 205–235). Dordrecht, NL: Sense Publishers.

Sturken, M. & Cartwright, L. (2001). *Practices of Looking*: an introduction to visual culture. Oxford/New York: Oxford University Press.

Zago, H. da S. & Flores, C. R. (2010). Uma proposta para relacinar arte e educação matemática. *Relime*, 13 (3), pp. 337-354.

Wedege, T. (2009). Combining and Coordinating Theoretical Perspectives in Mathematics Education Research. *Proceedings of CERME 6*, Vol. 1, pp.1665-1674. Lyon: France.

Zimmermann, W. & Cunningham, S. (Eds.). (1991). *Visualization in Teaching and Learning Mathematics.*Washington: MAA.