UNIVERSIDADE FEDERAL DE SANTA CATARINA
CENTRO SOCIOECONÔMICO
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA

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Heterogeneity in Unemployment Expectations and Evolutionary Dynamics

Florianópolis

2023

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Dissertação submetida ao Programa de Pós-Graduação em Economia (PPGEco) do Centro Socioeconômico (CSE) da Universidade Federal de Santa Catarina como requisito parcial para a obtenção do título de Mestre em Economia.

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Florianópolis
2023

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Pitta, João Luiz Toogood
Heterogeneity in Unemployment Expectations and Evolutionary Dynamics / João Luiz Toogood Pitta ; orientador, Jaylson Jair da Silveira, 2023.
63 p.

Dissertação (mestrado) - Universidade Federal de Santa Catarina, Centro Sócio-Econômico, Programa de Pós-Graduação em Economia, Florianópolis, 2023.

Inclui referências.

1. Economia. 2. Expectativas de Desemprego. 3. Salário Eficiência. 4. Heterogeneidade. 5. Teoria dos Jogos Evolucionários. I. Silveira, Jaylson Jair da . II. Universidade Federal de Santa Catarina. Programa de Pós Graduação em Economia. III. Título.

João Luiz Toogood Pitta

Heterogeneity in Unemployment Expectations and Evolutionary Dynamics

O presente trabalho em nível de Mestrado foi avaliado e aprovado em 28 de março de 2023, pela banca examinadora composta pelos seguintes membros:

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Florianópolis, 2023.

To my beloved wife,
Letícia Kern da Rosa,
who causes my life
to be full of joy.

ACKNOWLEDGEMENTS

As with any good academic achievement, the current work benefited greatly from the assistance of others. First, I would like to thank my supervisor, Professor Jaylson Jair da Silvera, for his guidance and help. His ability to see hidden paths where there were seemingly only dead ends was the reason I got so far. Working alongside him allowed me not only to profit from his precise comments, but also to enhance my skills as a researcher. Second, I would like to thank *Fundação de Amparo à Pesquisa e Inovação do Estado de Santa Catarina* - FAPESC (Foundation for the Support of Research and Innovation of the State of Santa Catarina) for providing me with a scholarship during my research phase, as well as *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior* - CAPES (Brazilian Coordination for the Improvement of Higher Education Personnel) for providing me with a scholarship during my course phase. This research could not have been done without their financial support. Third, I would like to thank the (i) qualifying examination committee, composed by Prof. Roberto Meurer, Prof. Guilherme de Oliveira, and Prof. Helberte João França Almeida, for their comments and insights into the early version of this work, (ii) the final examination committee, composed by Prof. Roberto Meurer and Prof. Paulo Victor da Fonseca, for their attentive read of this Thesis and suggestions for improvements and further research, and (iii) the Professors from the Graduate Program in Economics at the Federal University of Santa Catarina (UFSC) for their invaluable teachings. At last, I would like to thank my wife, Letícia Kern da Rosa, who not only helped me to go through all the difficulties but also provided me with a thorough review of the written text and English grammar. If any misspelling remains, the fault is entirely mine.

"Two things that remain eternally true and complement each other, in my view are: don't snuff out your inspiration and power of imagination, don't become a slave to the model; and, the other, take a model and study it, for otherwise your inspiration won't take on material form."

(Vincent van Gogh, letter #280 to Theo van Gogh.
The Hague, Sunday, November 5th, 1882.)

RESUMO

A persistência na heterogeneidade das expectativas de desemprego dos trabalhadores, assim como revelam dados obtidos a partir de pesquisas de opinião, é um bom indicador de que o custo da perda de trabalho é diferente entre trabalhadores. Como há evidências empíricas robustas de que maiores compensações salariais são eficientes em extrair mais esforço dos trabalhadores, é razoável supor que a provisão de esforço também será heterogênea entre os diferentes trabalhadores. Partindo destas considerações e assumindo que as expectativas são exógenas, Silveira and Lima (2021) propõe um modelo de salário eficiência aumentado com heterogeneidade de expectativas de desemprego e analisam como diferentes frequências de distribuição das expectativas na população de trabalhadores afeta a taxa de desemprego de equilíbrio. Há um suficiente consenso que nos permite argumentar que a heterogeneidade de expectativas é um fenômeno melhor explicado pelas teorias de racionalidade limitada, e não pela hipótese de expectativas racionais. Como consequência, pesquisas recentes têm investigado como as pessoas formam suas expectativas de desemprego e têm sugerido que elas deveriam ser tratadas de forma endógena num modelo de salário eficiência. Ao oferecer um raciocínio básico acerca de como os trabalhadores revisam suas expectativas, o presente trabalho foi capaz de propor um modelo dinâmico com flutuações endógenas nas expectativas de desemprego dos trabalhadores em que o equilíbrio de curto-prazo é dado pelo modelo proposto por Silveira and Lima (2021). O modelo conclui que a persistência na heterogeneidade de expectativas de desemprego dos trabalhadores é um equilíbrio evolucionariamente estável.

Palavras-chave: Expectativa de Desemprego. Salário Eficiência. Heterogeneidade. Sistemas Dinâmicos. Teoria dos Jogos Evolucionários.

RESUMO EXPANDIDO

Introdução

Há robusta evidência empírica de que as firmas que pagam uma compensação salarial maior são eficientes em extrair maiores níveis de esforço dos trabalhadores. Assumindo que a compensação salarial não é absoluta, mas relativa ao custo esperado por cada trabalhador caso perca o emprego, é de se imaginar que diferentes expectativas de desemprego por parte dos trabalhadores irão resultar em diferentes níveis de esforço, *coeteris paribus*. As pesquisas de expectativa dos consumidores, como a realizada pela Universidade do Michigan desde 1946, revelam não só uma heterogeneidade persistente nas expectativas de desemprego, mas também que o padrão da heterogeneidade varia substancialmente com o passar do tempo. Logo, é de se imaginar que a cada período do tempo haja trabalhadores realizando níveis distintos de esforço, bem como que estes níveis variam conforme o padrão de heterogeneidade varia. Silveira and Lima (2021) partem deste raciocínio para construir um modelo de salário eficiência aumentado com heterogeneidade de expectativas de desemprego. Como resultado, os autores concluem que o padrão da heterogeneidade afeta os níveis de emprego e salário de equilíbrio.

Uma importante hipótese adotada por Silveira and Lima (2021) é que o padrão da heterogeneidade de expectativas de desemprego é exógeno. No entanto, há diversas evidências empíricas que sugerem que tais expectativas deveriam ser tratadas de forma endógena num modelo de salário eficiência. Este é o caso, por exemplo, de Malgarini and Margani (2008), que sugerem que as pessoas formam suas expectativas com base nas suas próprias experiências, e Kuchler and Zafar (2019), que sugerem que estar desempregado afeta o grau de pessimismo sobre o desemprego futuro das pessoas. Estes resultado estão de acordo com as teorias de racionalidade limitada inauguradas por Simon (1955). Tais teorias enfatizam a incapacidade do ser humano em tomar decisões plenamente racionais, e concluem que a busca pela utilização eficiente dos recursos mentais leva as pessoas a agirem muitas vezes com base em heurísticas (isto é, adotando comportamentos que as afastam daquele esperado por um otimizador racional). Curtin (2019) argumenta que a maioria das expectativas econômicas são feitas sob medida para atender necessidades específicas do indivíduo em um dado contexto, e, além disso, que as pessoas não possuem nenhuma tendência natural a formar expectativas sobre eventos econômicos que não possuem impacto direto em suas vidas. Logo, as informações mais relevantes para se formular expectativas não vêm de estatísticas nacionais, mas das próprias experiências que as pessoas enfrentam no mercado.

Se considerarmos que as expectativas de desemprego dos trabalhadores são endógenas num modelo de salário eficiência, temos que o desemprego de equilíbrio gerado por um padrão de heterogeneidade inicial irá fazer com que as pessoas reformulem suas expectativas. Esta revisão pode levar a alterações no padrão de heterogeneidade que, por sua vez, levarão a um novo desemprego de equilíbrio. Este processo pode fazer com que o padrão de heterogeneidade e a taxa de desemprego convirjam para um equilíbrio de longo-prazo ou oscilem indefinidamente, a depender de como o processo for desenhado. Portanto, entender de que maneira as pessoas reformulam suas expectativas de desemprego e quais são as consequências no longo-prazo é de suma importância para se entender como que a heterogeneidade de expectativas afeta os resultados alcançados por um modelo de salário eficiência.

Objetivos

O objetivo geral da presente dissertação é avaliar quais são as implicações de longo-prazo ao se considerar que os trabalhadores revisam suas expectativas de desemprego com base na realidade que eles próprios enfrentam no mercado de trabalho, assumindo que o equilíbrio de curto-prazo é dado pelo modelo proposto por Silveira and Lima (2021). Para isso, é necessário primeiro entender de que forma a maneira como os trabalhadores revisam suas expectativas pode ser determinada de modo consistente com as teorias de racionalidade limitada. Feito isto, uma dinâmica é proposta para que se possa determinar como as frações de trabalhadores otimistas, pessimistas e neutros com relação ao desemprego futuro evoluem com o tempo. O comportamento destas frações e as implicações para os resultados de longo-prazo do modelo são então analisados.

Metodologia

A presente dissertação desenvolve um sistema dinâmico que, numa primeira definição, pode ser entendido como um modelo que descreve a evolução temporal de um conjunto de variáveis. De maneira precisa, um sistema dinâmico é composto por três elementos. O primeiro deles é o espaço de estados, que nada mais é do que o conjunto de valores que as variáveis que evoluem diretamente com o tempo (isto é, as frações de trabalhadores pessimistas, otimistas e neutros) podem assumir. O segundo é a definição dos valores que o tempo pode assumir no modelo (no presente caso, o tempo será considerado contínuo, iniciando-se em zero e podendo ir até infinito). O último elemento é a regra evolucionária que conecta o estado atual do sistema com a sua taxa de variação (ou seja, a regra que determina de que maneira os trabalhadores revisam as suas estratégias). Um campo do conhecimento que utiliza sistemas dinâmicos para descrever a evolução, tanto em contextos biológicos como sociais, é a Teoria dos Jogos Evolucionários. Dessa forma, o modelo proposto se assemelha sobremaneira com os chamados jogos populacionais, que fornecem um arcabouço para se estudar a interação estratégica entre um grande número de agentes, de tal forma que cada indivíduo possui um peso insignificante na população.

Resultados e Discussão

Busca-se cumprir o objetivo geral propondo-se uma regra evolucionária que conecta o padrão atual de heterogeneidade de expectativas de desemprego com a taxa de variação das frações de pessimistas, otimistas e neutros através da taxa de desemprego de equilíbrio temporária (ou de curto-prazo). Esta regra, que é consistente com as teorias de racionalidade limitada, leva a um novo padrão de heterogeneidade e uma consequente nova taxa de desemprego de equilíbrio temporário. A presente dissertação conclui que a regra evolucionária proposta leva o sistema a alcançar um equilíbrio de longo-prazo, que é tanto único quanto estável. Além disso, tal equilíbrio possui a propriedade de estar sempre localizado no interior do espaço de estados (isto é, o equilíbrio será polimórfico). Com isso, temos que a heterogeneidade de expectativas de desemprego por parte dos trabalhadores é um equilíbrio assintoticamente estável do modelo, replicando o importante fato estilizado de que há uma heterogeneidade persistente nas expectativas de desemprego dos trabalhadores.

Considerações Finais

A presente dissertação buscou propor uma regra comportamental consistente com as teorias de racionalidade limitada que descreve como, num modelo de salário eficiência, os trabalhadores revisam as suas expectativas de desemprego a partir da taxa de desemprego do equilíbrio temporário dado pelo modelo apresentado por Silveira and Lima (2021). Tal regra leva a uma dinâmica evolucionária que possui um único equilíbrio de longo-prazo, que é tanto assintoticamente estável quanto plenamente polimórfico. Ou seja, a dinâmica evolucionária gerada pela regra proposta é capaz de replicar o fato estilizado de que há uma heterogeneidade persistente nas expectativas de desemprego dos trabalhadores. Tal resultado contribui para a literatura sobre os efeitos gerados pela heterogeneidade de expectativas de desemprego, tanto a nível teórico quanto empírico.

Palavras-chave: Expectativa de desemprego. Salário Eficiência. Heterogeneidade. Sistemas Dinâmicos. Teoria dos Jogos Evolucionários.

ABSTRACT

The persistence of heterogeneity among workers' unemployment expectations, as revealed by survey data, is a good indicator that the cost of job loss is different across workers. As there is robust empirical evidence that a higher wage compensation is efficient in eliciting more effort from workers, it is only reasonable to assume that the provision of effort will also be heterogeneous across different workers. Drawing on this insight and assuming that expectations are exogenous, Silveira and Lima (2021) put forward an efficiency wage model augmented with heterogeneous unemployment expectations and analysed how different frequency distributions of expectations in the population of workers affect the equilibrium rate of unemployment. There is enough consensus to argue that heterogeneity in expectations is a phenomenon best explained by the theories of bounded rationality and not by the rational expectations hypothesis. As a consequence, recent research has investigated how people form their unemployment expectations, and they suggest that they should actually be considered endogenous in an efficiency wage model. Providing a basic reasoning for how workers revise their expectations, the present work was able to set forth a dynamic model of endogenous fluctuations in workers' unemployment expectations, where the short-run equilibrium is given by the model put forward by Silveira and Lima (2021). The model concludes that the persistence of heterogeneity among workers' unemployment expectations is an evolutionary stable equilibrium.

Keywords: Unemployment Expectations. Efficiency Wage Models. Heterogeneity. Dynamic Systems. Evolutionary Game Theory.

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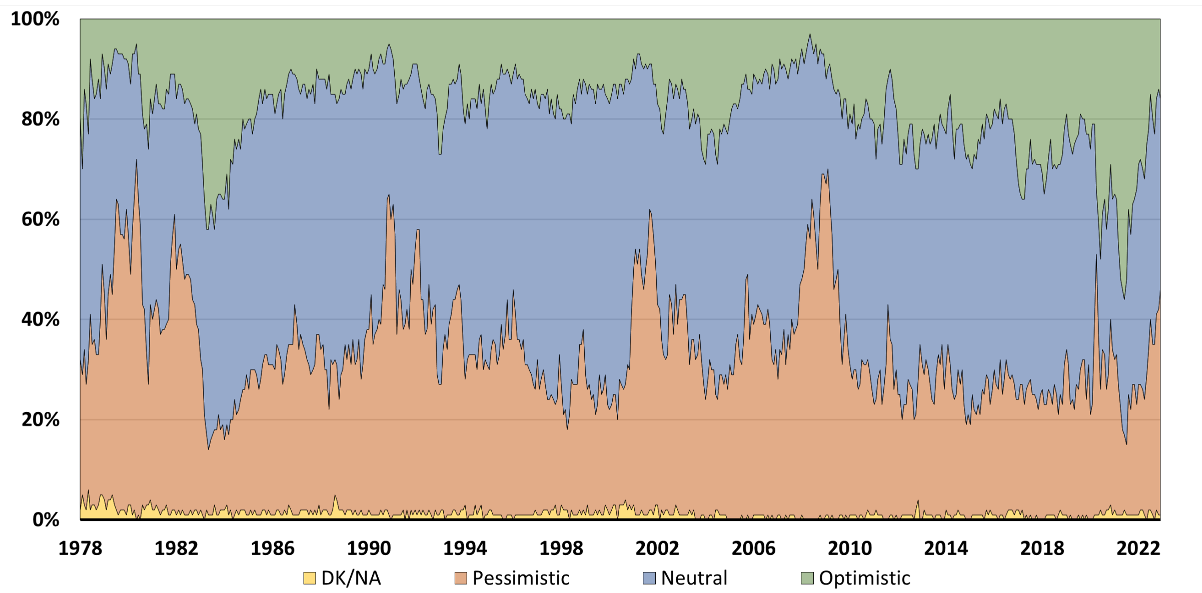
1 INTRODUCTION

The University of Michigan has been conducting a survey on consumers' expectations in the US since 1946. In regard to unemployment expectations, consumers are asked the following question: *"How about people out of work during the coming 12 months — do you think that there will be more unemployment than now, about the same, or less?"*. Figure 1.1 shows the monthly data ranging from January 1978 to December 2022. The consumers that answer *less unemployment* are labelled *optimistic* (green), those who answer *more unemployment* are labelled *pessimistic* (orange), and those answering *about the same* are labelled *neutral* (blue). In yellow are those who didn't answer or didn't know. Among other interesting features, Figure 1.1 shows that (i) there is a persistent heterogeneity among consumers' unemployment expectations and that (ii) the frequency distribution of expectations vary substantially across different periods of time.

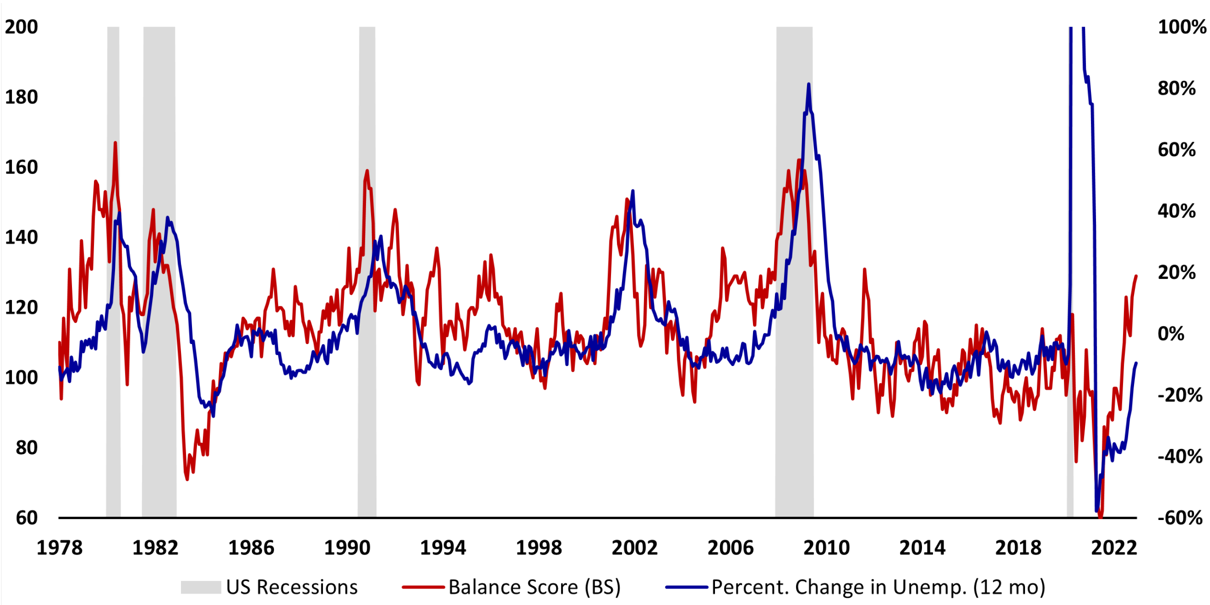
Figure 1.1 – Ratio of Optimistic, Pessimistic and Neutral Consumers



Source: University of Michigan Survey of Consumers.

According to Richard T. Curtin, who directed the University of Michigan Survey of Consumers from 1976 to 2022 and is now a Director Emeritus of the same institution, there is strong empirical evidence that consumers' unemployment expectations are a good predictor for future unemployment (Curtin, 2003; Curtin, 2019). Figure 1.2 shows the balance score (BS) of the unemployment expectations and the yearly percent change in the seasonally adjusted unemployment level[1] (shaded areas are periods of recession). The BS is equal to the ratio of pessimists minus the ratio of optimists plus 100. Hence, this qualitative measure varies from 0 (all consumers are optimists) to 200 (all consumers are pessimists), with a balance between optimism and pessimism being equal to 100. As can be seen from Figure 1.2, the BS foreshadows actual changes in the unemployment level to a high degree[2].

Figure 1.2 – Balance Score (left axis) and Yearly Percent Change in Seasonally Adjusted Unemployment Level (right axis)



Source: University of Michigan Survey of Consumers, US Bureau of Labor Statistics and National Bureau of Economic Research.

Drawing on the stylised facts about the persistence of heterogeneity among consumers' unemployment expectations, Silveira and Lima (2021) set forth an efficiency wage model with heterogeneous provision of effort across workers caused by differences in expected cost of job loss (that, in turn, are caused by differences in unemployment expectations). As a consequence, the short-run equilibrium rate of unemployment will depend on the ratio of workers holding pessimistic and optimistic expectations. The authors conclude that whether a higher proportion of workers having pessimistic unemployment expectations leads to a lower or higher unemployment rate depends on the prevailing configuration of heterogeneity in unemployment expectations across workers. Depending

[1] Notice that the BS is compared to the changes in unemployment level, not its rate. It is due to the nature of the question asked by the survey.

[2] During the COVID-19 pandemics, the unemployment level inordinately surged from 7.2 million people in March 2020 to 23 million people in the following month. Hence, the percent change from April 2019 (6 million people) to April 2020 was a record high for the sample: 290%. For that reason, the right axis was limited to a value of 100% in order to prevent distortions.

on this configuration, the model yields the positive relation between the BS and changes in unemployment, as suggested by recent empirical evidence.

A key feature of the model presented in Silveira and Lima (2021) is that the expectations' profile (i.e., the ratio of pessimistic, optimistic, and neutral workers) is determined exogenously. However, recent literature on the formation of expectations, as will be presented in Chapter 2, suggests that they are determined endogenously. If we incorporate this idea into the model, a given initial profile of expectations would yield a short-run equilibrium rate of unemployment, but this rate would change as the expectations are updated period after period. This dynamic process could either lead to a permanent motion in the unemployment rate or converge to a long-run equilibrium rate of unemployment.

The present work aims to present a dynamic version of Silveira and Lima (2021), considering that workers' formulate their expectations endogenously and following myopic rules. The proposed model concludes that the expectation's profile will converge to an equilibrium that is both unique, stable, and heterogeneous. Hence, the model replicates the stylized fact that there is a permanent heterogeneity among workers' unemployment expectations. As a result, the unemployment rate will also converge to a long-run equilibrium. To achieve this goal, Chapter 2 presents a literature review on the role of heterogeneity in economics, with emphasis on heterogeneity in unemployment expectations, as well as the theories of bounded rationality. Next, Chapter 3 puts forward a model of endogenous fluctuation in workers' unemployment expectations, where the short-run equilibrium is given by the model set forth by Silveira and Lima (2021). Then, Chapter 4 analyses the long-run equilibrium of the model. Finally, Chapter 5 makes final remarks. All mathematical proofs are presented in Appendix A.

2 LITERATURE REVIEW

The diversity of economic agents is one of the most fundamental human characteristic that leads us to trade and to the consequent emergence of markets. This is a well established paradigm since at least the marginal revolution in the late 19^{th} century. However, the economic profession only began to fully embrace the importance of heterogeneity, both in theoretical and empirical models, in the late 20^{th} century. Since then, many fields of economics have completely changed, and new ones have been born. Among the many features that vary across different people are their expectations, whose origins are largely explained by the theories of bounded rationality. As we saw in Chapter 1, there is a persistent heterogeneity in consumers' unemployment expectations, and the frequency distributions of optimists, pessimists and neutrals change considerably across different periods of time. This stylised fact is what motivates the heterogeneous-expectations-augmented efficiency wage model put forward by Silveira and Lima (2021), which serves as a description of the short-run equilibrium of the dynamic model to be proposed in Chapter 4. Since the fundamental hypothesis of the dynamic model is that expectations are endogenous, a thorough understanding of how expectations are formed and the reasons for the empirically observed heterogeneity is of utmost importance. That having been said, Section 2.1 puts forward a brief discussion on the important role that heterogeneity plays in modern economic theory and practice. Then, Section 2.2 explores the theories of bounded rationality that try to explain the existence of heterogeneity in expectations (a phenomenon usually disregarded by the rational expectations hypothesis). At last, Section 2.3 presents some key facts and evidence on the expectations of future unemployment.

2.1 HETEROGENEITY IN ECONOMICS

According to Giri (2017), the analysis of heterogeneity is nowadays one of the cornerstones of modern economics. However, it has not always been like this. Until the 1970s, the economic profession, led by the *Cowles Commission*, was mostly concerned with estimating *ad hoc* aggregate relationships that, in accordance with Heathcote, Storeslette and Violante (2009), largely abstracted from individual behaviour and differences across economic agents. However, the new stylised facts that emerged from the oil shocks of the 1970s undermined the use of those models, and critiques such as Lucas (1976) gave birth to the creation of new methods. In consequence, Kydland and Prescott (1982) engendered the first generation of dynamic general equilibrium models, whose main characteristic is that the aggregate behaviour was derived from the micro-behaviour of single agents. However, those first models were also characterised by the use of representative (i.e. homogeneous) agents. According to Heathcote, Storeslette and Violante (2009), there are two main reasons for this choice. First, economists lacked the tools to solve dynamic models with heterogeneous agents. Second, it was not obvious that incorporating heterogeneity would lead to sufficient improvement in the comprehension of topics such as business cycles and economic growth.

However, Heathcote, Storeslette and Violante (2009) highlight that microeconometric work, such as in labour economics, has revealed enormous cross-sectional dispersion and individual volatility for workers and firms. As Heckman (2001) points out in his Nobel Lecture, "the most important discovery from microeconometric investigations was the evidence on the pervasiveness of heterogeneity and diversity in economic life". In dealing with such diversity in micro data, the traditional econometric methodologies were challenged, and problems that appeared unimportant when examining aggregate averages became central (Heckman, 2001). As a result, the search for solutions to these new problems in order to address the policy concerns that motivated most of the collection of micro data in the first place gave way to the field of microeconometrics. Then, as new evidence showing the validity of incorporating heterogeneity was emerging, along with the development of new tools to solve models with heterogeneous agents, the representative-agent abstraction was challenged.

Pursuant to Heathcote, Storeslette and Violante (2009), the introduction of heterogeneity into economic models brings three important improvements. First, heterogeneity affects both the levels and dynamics of aggregate equilibrium quantities and prices. For example, Heathcote (2005) finds that changes in the timing of taxes that would be neutral in a representative agent model turn out to have large real effects in a model in which heterogeneous households face a borrowing constraint[1]. Second, introducing heterogeneity can change the answer to welfare questions. Lucas (1987) suggested a way of calculating the welfare gain associated with the elimination of business cycles and concluded that the gains are very small. This result poses the following question: why should we employ so much effort on stabilisation policies? Krusell and Smith (1999) argue that one possible explanation is that the welfare effects of business cycles are asymmetric across different groups of consumers. They find that even though business cycles do not affect the average household (i.e., representative agent) so much, as in Lucas (1987), some consumers may suffer significantly, especially the poor and the unemployed. However, in spite of the fact that the welfare gain was estimated to be larger in a model with heterogeneous households, Krusell and Smith (1999) suggest that it is still small. A different conclusion,

[1] Another contribution that is worth noting is the Heterogeneous Agent New Keynesian (HANK) model put forward by Kaplan, Moll and Violante (2018) and its implications for monetary policy. They find that, in stark contrast to Representative Agent New Keynesian (RANK) models, the direct effects of interest rate shocks are small, while the indirect effects (i.e., the general equilibrium response in household disposable income) are large.

though, was found by Storesletten, Telmer and Yaron (2001) and Krusell *et al.* (2009), whose models, when calibrated for the US economy, yield large welfare gains derived from stabilisation policies. Third, Heathcote, Storeslette and Violante (2009) highlight that many macro questions of great relevance simply cannot be addressed without allowing for at least some heterogeneity. It is the special case of studies regarding social security policies and the impacts caused by demographic transition. In summary, heterogeneity matters not only in the sense that it is a more realistic depiction of reality, but also because it changes important results and policy implications when compared to models based on representative agents.

2.2 BOUNDED RATIONALITY AND HETEROGENEOUS EXPECTATIONS

The central role of expectations in economic theory goes back at least as far as the works of John Muth, Robert Lucas and Thomas Sargent in the 1960's and 1970's[2] (Muth, 1961; Sargent, 1971; Lucas, 1972). According to Curtin (2019), the view on expectations held by those and many authors that followed required that economic agents had knowledge of past economic developments, correct interpretations of ongoing trends, and the ability and willingness to make detailed calculations. Under this paradigm, often referred to as the rational expectations hypothesis, there is not much space for heterogeneity. If people have different expectations at first but are rational and fully informed, then in time they will learn to hold the same expectations. However, as Kenneth Arrow puts it, "one of the things that microeconomics teaches you is that individuals are not alike. There is heterogeneity, and probably the most important heterogeneity here is heterogeneity of expectations"(Colander; Holt; Rosser, 2004, p. 301). In consequence, Malgarini and Margani (2008) highlight that agents might deviate from the rational expectations hypothesis and formulate heterogeneous expectations because (i) they are using different models, (ii) they have different information sets, or (iii) they have different capabilities for processing information.

In fact, there has been a tradition in macroeconomic research of exploring this topic. Mankiw and Reis (2002), for instance, propose a sticky-information model, in which economic agents only update their expectations periodically because of the costs of collecting and processing information. As a result, the model displays properties that are more consistent with accepted views about the effects of monetary policy in comparison with models of sticky prices. Mankiw, Reis and Wolfers (2003) investigate whether the sticky-information model is capable of predicting the extent of disagreement in inflation expectations that are observed in the survey data, as well as its evolution over time. They conclude that the model is capable of explaining many features of the dispersion in inflation

[2] Curtin (2019, p. 151) argues that "of all the social sciences, economics has placed the greatest emphasis on explaining today's decisions by what may happen in the future rather than by what occurred in the past".

expectations in the US, but some features are still not explained. Motivated by some of the same insights but following a slightly different approach, Sims (2003) puts forward a model of rational inattention based on the well-accepted idea that individuals have limited capacity for processing information. When adding this reasoning to macroeconomic models, the author finds that it alters the behaviour implied by them in ways that seem to accord, along several dimensions, with observed macroeconomic behaviour. Both the sticky-information and the rational inattention models consider agents as utility optimizers subject to constraints. In the former case, the cost of information acquisition leads people to rationally update their information set only at regular intervals, whereas the latter assumes that the finite capacity to process information leads people to rationally not use all freely available information[3].

However, instead of focusing on the *homo economicus*, one might assume that a clearer path is to centre attention on the actual economic agent: the *homo sapiens*. Ball, Mankiw and Reis (2005) broadly define behavioural economics as the subfield that incorporates into economic theory the flaws in human decision-making that are ignored in the standard model of rational man. Also, they add that behavioural economics finds its roots in Herbert Simon's suggestion that people are *satisficers* rather than rational maximisers. In his Nobel lecture, Simon (1979) broadly defines *satisficing* as the selection mechanism by which the decision maker forms some aspiration as to how good an alternative he should find and, as soon as he discovers an alternative meeting his level of aspiration, he terminates the search and chooses that alternative. Moreover, the author highlights that the importance of satisficing theory is that it showed how choices could actually be made with reasonable amounts of calculation, and using very incomplete information. Thus, the theory of economic agents not behaving as utility maximisers, what Simon (1979) broadly calls *bounded rationality*, found its first robust formulation.

The extent to which people's choices are biased in comparison with what one might expect that a fully rational agent would choose was largely investigated by Amos Tversky and Daniel Kahneman. As Kahneman puts it in his Nobel Lecture (Kahneman, 2003), people exhibit two modes of thinking, roughly corresponding with intuition and reasoning. Reasoning is done deliberately and effortfully, whereas intuitive thoughts are spontaneous and effortless. The author argues that the rational agent of economic theory would be described as endowed with a single cognitive system that has the logical ability of a flawless reasoning and the low computing costs of intuition. Moreover, the author points that "behavioural economics have generally retained the basic architecture of the rational model, adding assumptions about cognitive limitations designed to account for specific anomalies"(Kahneman, 2003, p. 1469).

[3] That is, the rational use of scarce resources leads people to be inattentive to all new information during some time interval in the former case, whereas it leads people to be inattentive to some information at all times in the latter case.

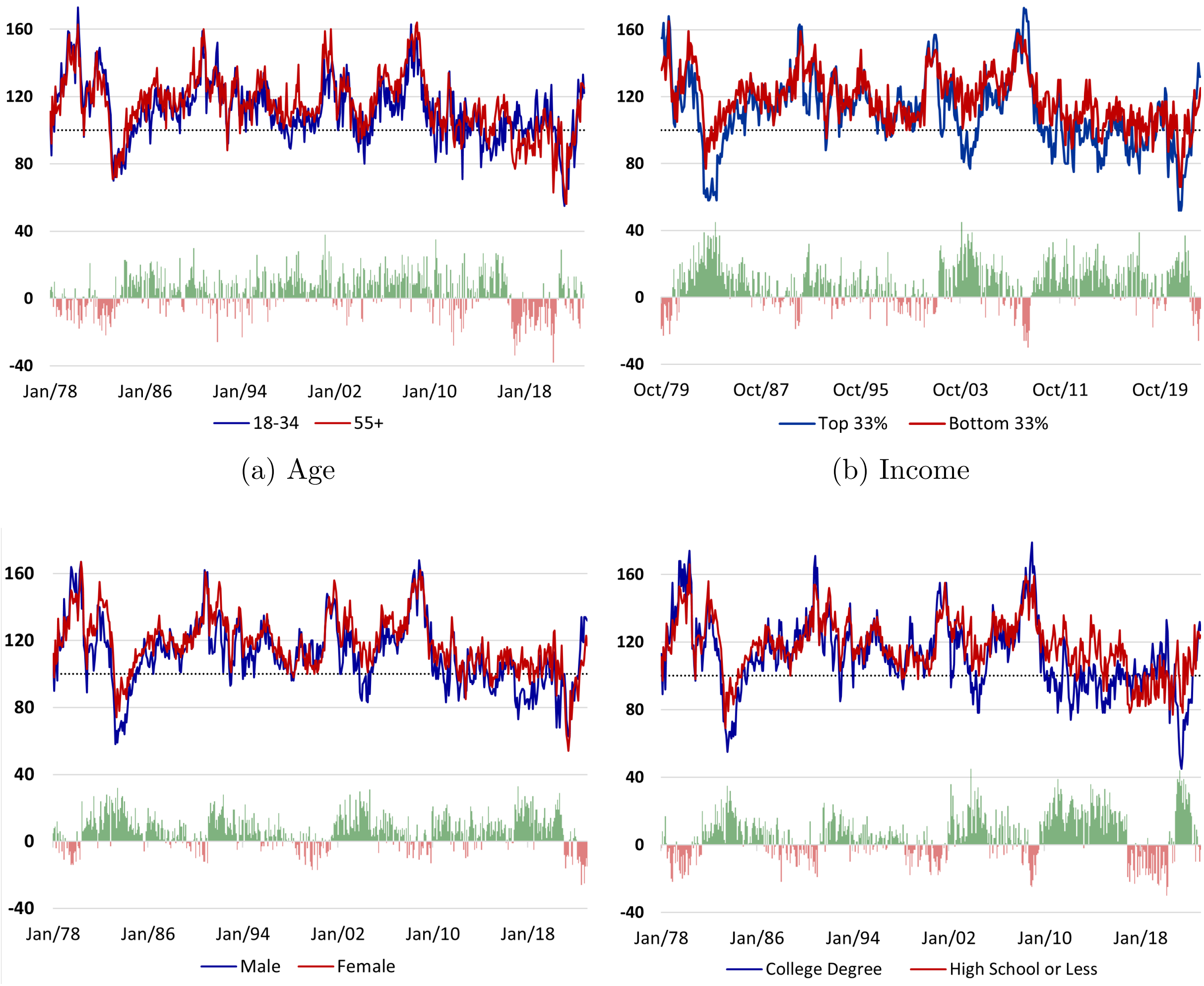
To find which assumptions to add about the cognitive limitations of human beings, it is necessary to first understand how reasoning and intuition work. Even though most decisions are usually intuitive, some monitoring of their quality is also performed. However, Kahneman (2003) highlights that this monitoring is normally lax, allowing some erroneous intuitive judgements to be expressed. Even so, the author points out that intuitive thinking can also be quite accurate, as is the case, for instance, when a skill is acquired after prolonged practice. Then, how good a decision will be depends on how tuned the intuitions are and how proficient and/or frequent the monitoring of reason is. According to Kahneman (2003), the ability to avoid errors of intuitive judgement is impaired by (i) time pressure, (ii) concurrent involvement in a different cognitive task, (iii) performing the task in the morning for "evening people" and vice-versa, and (iv) being in a good mood, among others. On the other hand, the facility of reasoning is positively correlated with (i) intelligence, (ii) the "need for cognition"(a psychological trait that consists basically of people finding that thinking is fun), (iii) and exposure to statistical thinking, among others. Given the mental process and capabilities that people actually face when making decisions, Tversky and Kahneman (1974) find that people usually rely on a limited number of heuristic principles (i.e., simple decision making rules) that reduce the complexity of the task at hand. In this sense, Visco and Zevi (2020) argue that heuristics can be an efficient tool for making choices when the cost of acquiring and processing information is high. However, as highlighted by Tversky and Kahneman (1974), they can sometimes lead to severe and systematic errors.

2.3 HETEROGENEITY IN UNEMPLOYMENT EXPECTATIONS

Once we leave the world of rational expectations and full information, it is reasonable to presume that the diversity among consumers' characteristics and situations will imply different sentiments regarding the future developments of unemployment. This presumption can be observed in Figure 2.1, where the BS was plotted for different demographic groups in the US using monthly data from the Michigan survey[4]. In all panels, the red line represents the group whose expectations were predominantly more pessimistic than the other (the differences are shown in the green and red bars). In panel (a), the BS is calculated for different age groups. From the 519 observations, the group comprising people between 18 and 34 years old were more pessimistic than the group comprising people over 55 years old on 328 occasions (63% of the time). In turn, panel (b) reveals that the people from the bottom 33% of the income distribution held more pessimistic expectations in comparison with the top 33% on 73% of the time. When divided by gender, females held more pessimistic expectations 89% of the time, and, when divided by education, the people holding a high school degree or less were more pessimistic 86% of the time in comparison

[4] All panels start in January 1978 and finish in January 2023, with the exception of panel (b), which starts in October 1979.

Figure 2.1 – Plot of the balance score of unemployment expectations for different demographic groups in the US



(a) Age (b) Income

(c) Gender (d) Education

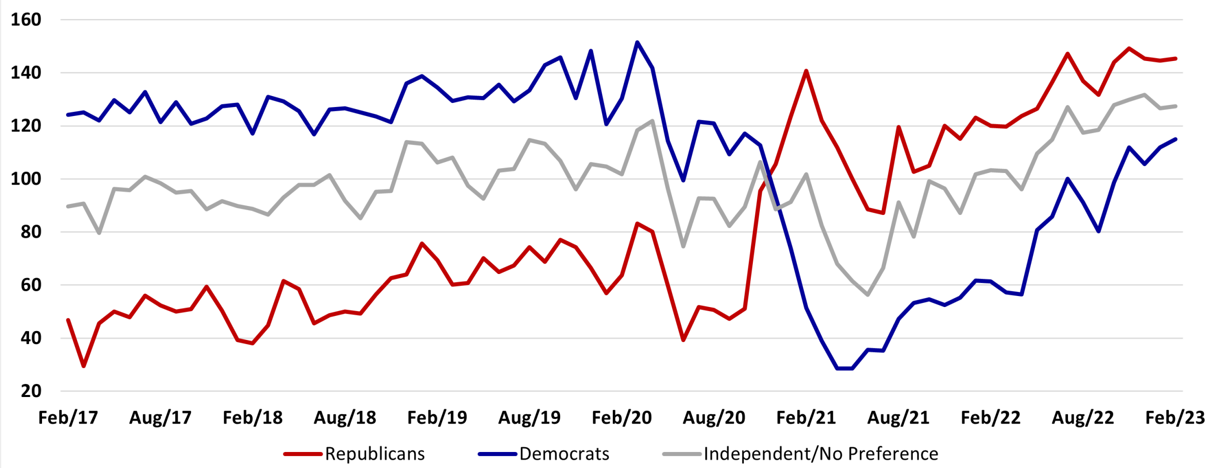
Source: University of Michigan Survey of Consumers.

to the people holding college degrees. Another suggestive case is depicted in Figure 2.2, which shows the BS for Republicans, Democrats and Independents/No Preference. Even though the sample is much smaller[5], we can see that Democrats were substantially more pessimistic than Republicans during the Trump administration, and this pattern reverted in November 2020, the exact month that Joe Biden won the elections. Of course, the observation that one group was on average more pessimistic than the other does not imply causality[6], it simply shows that people with different characteristics might hold different expectations.

One similar feature held by all demographic groups depicted in Figure 2.1, as well

[5] Only 73 observations, going from February 2017 to February 2023.
[6] For example, there might be a confounding in each relationship.

Figure 2.2 – Plot of the balance score of unemployment expectations for different political affiliations in the US



Source: University of Michigan Survey of Consumers.

as the general BS depicted in Figure 1.2, is that the ratio of pessimists was on average greater than the ratio of optimists, that is, the BS was on average above 100 (dotted lines in Figure 2.1). This can be seen in the grey column in Table 2.1, which shows that even the least pessimistic group (top 33% of the income distribution) was still pessimistic 69% of the time, whereas the most pessimistic group (bottom 33% of the income distribution) was pessimistic 89% of the time. For instance, Garz (2013) empirically investigates whether this observed pessimism indicates a link to negativity in economic news coverage in Germany. Using monthly data from the European Business and Consumer Surveys from 2001 to 2009 and controlling for alternative sources of information, their estimates suggest that the cumulative effects of repeated media coverage affect long-run attitudes. That is, if expectations change in reaction to an increase in news output, they do not necessarily return to their initial level but instead tend to shift permanently. Their results, which are robust over time and across demographic groups, also suggest that a single negative report has a long-run effect that is similar to the influence of a positive report. However, the highest occurrence of negative reports causes an asymmetric reaction in unemployment expectations, which promotes pessimism.

Going beyond the reasons for the overall negativity in unemployment expectations and investigating the potential causes for heterogeneity, one might assume that the specific circumstances that each person faces in the labour market affect their perceptions of overall unemployment. Malgarini and Margani (2008), for instance, use data on unemployment expectations for Italy and find that employees are unable to correctly incorporate the effects of a 2003 law that allowed new forms of temporary jobs (the so-called Biagi Law). According to the authors, this evidence suggests that these agents are more likely to form unemployment expectations based on their own idiosyncratic experiences, and do

Table 2.1 – Periods of pessimism for each demographic group and in total

Group	Subgroup	Pessimism		Neutral		Optimism	
		Total	Perct.	Total	Perct.	Total	Perct.
Age	18-35	415	77%	12	2%	114	21%
	55+	447	83%	8	1%	86	16%
Education	High School or Less	463	86%	9	2%	69	13%
	College Degree	389	72%	11	2%	141	26%
Gender	Female	472	87%	7	1%	62	11%
	Male	396	73%	12	2%	133	25%
Income	Bottom 33%	463	89%	5	1%	52	10%
	Top 33%	357	69%	10	2%	153	29%
	All	436	81%	5	1%	99	18%

Source: University of Michigan Survey of Consumers.

not observe overall labour market dynamics when forecasting. In the same direction, Kuchler and Zafar (2019) use a rich panel of survey data from the US to investigate whether people who experienced unemployment became more pessimistic about future nationwide unemployment. Exploring the within-variation of individuals that experienced job transitions (people who were employed and lost their jobs and vice versa), they find that the pessimism about future nationwide unemployment is in fact associated with the experience of being unemployed. As a matter of fact, Krueger *et al.* (2011) use survey data collected in the fall of 2009 and winter of 2010 from a large sample of unemployed workers in the US and find that dissatisfaction and unhappiness with their lives were not only high, but increased the longer they remained unemployed. This is a potential explanation for the evidence showing that people extrapolate their current job situation to their expectations for the overall economy.

However, the extrapolation might also be in the opposite direction. Roth and Wohlfart (2020) use a representative online panel from the US to examine how individuals' macroeconomic expectations are updated with new information and how this revision affects their personal economic prospects. The authors find that people exposed to macroeconomic risks (such as individuals working in cyclical industries) tend to extrapolate their expectations on the overall economy to their own situation (i.e, they tend to expect a higher chance of being unemployed in the future). The relation between consumer‘s unemployment expectations and the cyclical movements in unemployment was investigated by Tortorice (2012), comparing unemployment expectations from the Michigan Survey of Consumers to movements in unemployment that are forecastable by a Vector Auto-Regressive (VAR) model containing GDP, the unemployment rate, the inflation rate, and the fed funds rate (a rough approximation of the "true" model and the "relevant" set of information). Looking at periods of recession, the author documents some interesting facts. First, a large amount of the population expects unemployment to rise when it is actually falling at the end of a recession. Second, this amount is greater than the amount of the population expecting a fall in unemployment when it is actually rising at the beginning of the recession. In both cases, the actual movement was predicted by the VAR. Third, the lag change in unemployment is almost as important as the VAR in predicting the fraction of the population that expects unemployment to rise. When comparing models of expectation formation, the author argues that models in which some agents form expectations by extrapolating current trends into the future can explain all the facts.

Given the bounds on consumers' rationality and how susceptible they are to their personal job conditions, the current state of the economy, and even news broadcast, one might expect that consumers' unemployment expectations would not carry much relevant information in order to forecast actual unemployment and future conditions of the economy. Leduc and Sill (2013), for instance, use data from the Livingston Survey and the Survey of Professional Forecasters for unemployment expectations in the US as a proxy for the future conditions of the economy[7]. Using a VAR, the authors find that a fall in unemployment expectations is typically followed by a rise in real economic activity. Since those surveys collect the expectations formulated by professionals, this result might not be surprising. However, the authors show that even when they use data from the University of Michigan Survey of Consumers, which collects expectations from regular people, the results remain substantially unchanged. Similar results were found for the Euro Zone by Girardi (2014), using data on unemployment expectations from the Joint Harmonised EU Program of Business and Consumer Surveys[8]. Lehmann and Weyh (2016) in turn, evaluate the forecasting performance of employment expectations for employment growth itself in 15 European states and find that for most of them, the survey-based indicator model outperforms the autoregressive benchmark. Instead of looking at the balance score of consumers' unemployment expectations (i.e., the difference between the ratio of pessimists and optimists), Claveria (2019) investigates the contribution of the degree of consensus in consumers' expectations to forecast unemployment in 8 European countries. To do so, he creates a new measure of consensus that is equal to zero when the disagreement is maximum (i.e. the ratio of optimists, pessimists, and neutrals is equal to one third each) and equal to 100 when the disagreement is null (i.e., all consumers are either optimists, pessimists or neutrals)[9]. He uses an Autoregressive Integrated Moving Average (ARIMA) model as a benchmark to produce out-of-sample forecasts and then adds the consensus measure as a predictor. As a result, the author finds that the proposed indicator leads to an improvement in forecast accuracy in most countries, especially for the prediction of turning points detected by agents in advance.

This empirical evidence on the predictive power of consumers' unemployment expectations is remarkable, since the bounds on rationality should cause persistent biases.

[7] The first survey starts in 1946, whereas the second one starts in 1968. Both surveys have been conducted by the Federal Reserve Bank of Philadelphia since 1990.

[8] The Survey is co-financed by the European Union and collects data from all member states.

[9] One clear advantage of this measure is that it allows us to use the information on neutrals in the analysis.

Curtin (2019) argues that evolutionary development has provided people with efficient means to form expectations by fully utilising both their *conscious* and *nonconscious* cognitive abilities. Even though the important role of the former has been focused on by many scientists, the latter has been generally dismissed. Curtin (2021) highlights two reasons for this dismissal. First, since nonconscious reasoning is by definition unknowable even to the decision maker, many scientists believe that unconscious processes can serve no useful role in the scientific analysis of economic behaviour. Second, since nonconscious reasoning cannot be directly observed, it has usually been classified by default as potentially irrational. Since the amount of information that the nonconscious mind can process is far greater then that of the conscious mind[10], Curtin (2019) argues that expectations serve a unique evolutionary role, which is to allow the human mind to maximise the use of its most precious resource: conscious cognitive deliberation. Hence, most economic expectations are formed by nonconscious cognitive activity. However, conscious deliberation is likely to dominate when people initially learn to form a specific expectation, when there is a sudden change in the underlying economic circumstances, or in other unusual situations. As a consequence, unless an unemployment rate changes in an unexpected manner, it typically receives little conscious attention. While the accuracy of the outcome is still a prime objective in forming expectations, accuracy is never absolute, but only as precise as allowed by a cost–benefit calculation. The benefits of accurate expectations are ultimately derived from the impact of the decision on a person's overall economic welfare. Since people in general possess no natural tendency to form expectations about economic events that have no direct impact on their lives, most effective economic expectations are always tailored to the specific decision needs of an individual in a given context (i.e., the economic conditions people actually face). For example, expectations of future changes in unemployment are not expectations about an economic statistic, but about a tangible fact of people's lives. Hence, the most relevant data does not come from official statistical agencies but from people's own interactions with the economy. According to Curtin (2019), this view on the formation of unemployment expectations allows us to understand why their formation is best explained by the theories of bounded rationality and yet they achieve a considerable level of accuracy.

[10] According to Curtin (2021), it has been estimated that the human brain can process around 11.2 million bits of information per second nonconsciously, compared to just 40 bits per second consciously.

3 A HETEROGENEOUS-EXPECTATIONS AUGMENTED EFFICIENCY WAGE MODEL

According to Akerlof and Yellen (1986), the efficiency wage models were developed to explain the cyclically varying involuntary unemployment. Yellen (1984) argues that these models present a convincing and coherent explanation of why firms may find it unprofitable to cut wages in the presence of involuntary unemployment. The basic argument is that the firm's productivity does not depend only on the hours of labour employed, but also on the workers' effort elicited by the wage compensation. As a consequence, if there is involuntary unemployment, the firm might not find it profitable to hire more employees because the corresponding lower wages would decrease the workers' effort by such an amount that the firm's profit would decrease. Since the wage compensation depends on the likelihood of receiving an alternative wage in case of being out of job, it is reasonable to assume that the different expectations that people have about future unemployment (as evidenced by survey data) will elicit different levels of effort provision. Drawing on this idea, Silveira and Lima (2021) put forward an efficiency wage model augmented with heterogeneous unemployment expectations. Since this model will be used as the description of the short-run equilibrium in Chapter 4, Section 3.1 will present its structure. Then, Section 3.2 will compute its equilibrium, discuss the comparative statics, and evaluate the results of the model in light of the empirically observed relationship between unemployment expectations and the paths of actual unemployment.

3.1 THE STRUCTURE OF THE MODEL

The workers' effort will be modelled as a non-linear function of the relative difference between the wage paid by the firm and the wage compensation associated with the labour market's expected conditions. Following Romer (2019, chapter 11), such function is given by

$$\varepsilon_\tau = \begin{cases} \left(\dfrac{w_\tau - \mu_\tau}{\mu_\tau}\right)^\gamma, & \text{for } w_\tau > \mu_\tau, \\ 0, & \text{otherwise,} \end{cases} \tag{3.1.1}$$

where ε_τ is the level of effort exerted by the worker of type $\tau = n, o, p$ (which stands for neutral, optimistic, and pessimistic, respectively), $w_\tau \in \mathbb{R}_{++}$ is the wage received by the worker of type $\tau = n, o, p$, μ_τ is an indicator of the wage compensation associated with the expected labour market conditions for a worker of type $\tau = n, o, p$ and the parameter $\gamma \in (0, 1) \subset \mathbb{R}$ denotes the measure of the effort-enhancing effect of paying a worker of type

$\tau = n, o, p$ a wage compensation that is higher than the wage compensation associated with her expected labour market conditions.

The model assumes that μ_τ is given by

$$\mu_\tau = (1 - u_\tau^e)w_{a,\tau}, \tag{3.1.2}$$

where $u_\tau^e \in (0, 1) \subset \mathbb{R}$ is the expected unemployment rate for workers of type $\tau = n, o, p$ and $w_{a,\tau} \in \mathbb{R}_{++}$ is the alternative wage the worker of type $\tau = n, o, p$ could receive in the market had he been working for another employer.

In accordance with the survey data presented in Chapter 2, such as the one collected by the University of Michigan, Silveira and Lima (2021) propose the following ordering of the unemployment expectation for each type $\tau = n, o, p$:

$$0 < u_o^e < (u_n^e = u) < u_p^e < 1, \tag{3.1.3}$$

where u is the current rate of unemployment.

Each firm is assumed to be small with respect to the economy, and therefore takes workers' expected cost of job loss as given. Moreover, firms are unable to perfectly detect both the type of each worker and her effort level. Thus, the firm will set a homogeneous wage, denoted by w, that minimizes the cost of labour per unit of average effort, denoted by ε. Similarly, the homogeneous wage w and the amount of labour, denoted by L, can be obtained as solution for the following maximization problem:

$$\max_{w,L} \pi = f(\varepsilon L) - wL \quad s.t. \quad \varepsilon = \varepsilon_n^\eta \varepsilon_o^\theta \varepsilon_p^\rho, \tag{3.1.4}$$

where $f(\cdot)$ is the production function, such that $f'(\cdot) > 0$ and $f''(\cdot) < 0$, and η, θ, ρ represents the proportions of neutral, optimistic, and pessimistic workers, respectively. The triple (η, θ, ρ), by its very definition, belongs to the simplex given by $\Sigma = \Big\{(\eta, \theta, \rho) \in \mathbb{R}_+^3 : \eta + \theta + \rho = 1\Big\}$[1]. Assuming $w > \mu_\tau$, the first-order conditions for an interior solution are the following:

$$\begin{cases} \dfrac{\partial \pi}{\partial w} = F'(\varepsilon L)L\dfrac{\partial \varepsilon}{\partial w} - L = 0, \\ \dfrac{\partial \pi}{\partial L} = F'(\varepsilon L)\varepsilon - w = 0. \end{cases} \tag{3.1.5}$$

The first-order conditions in (3.1.5) can be rearranged to yield the so called Solow condition[2], which states that the profit maximizing pair (w, L) implies a unitary wage-effort elasticity:

$$\frac{\partial \varepsilon}{\partial w}\frac{w}{\varepsilon} = 1. \tag{3.1.6}$$

[1] If we exclude the ratio of people that didn't answer or didn't know, the triple represents a given cross section depicted in Figure 1.1.

[2] This label was given by Akerlof and Yellen (1986).

Traditionally, firms face the trade-off between the revenue received when employing more workers and the increased costs with wage payments. Given that workers' effort is an argument of the production function in (3.1.4) and equation (3.1.1) implies that the effort depends on the wage, the firm now faces a second trade-off: paying higher wages to hire fewer workers that will be more efficient or paying lower wages to hire more workers that will be less efficient. The Solow condition in (3.1.6) states that the optimum choice is achieved when the wage-effort elasticity is equal to one.

Employing the definition of ε in (3.1.4), Silveira and Lima (2021) obtain what they dub the weighted Solow condition:

$$\eta \frac{\partial \varepsilon_n}{\partial w}\frac{w}{\varepsilon_n} + \theta \frac{\partial \varepsilon_o}{\partial w}\frac{w}{\varepsilon_o} + \rho \frac{\partial \varepsilon_p}{\partial w}\frac{w}{\varepsilon_p} = 1. \tag{3.1.7}$$

So, observing the average effort ε and setting the homogeneous wage w according to (3.1.6), the firm automatically satisfies condition (3.1.7).

3.2 STATIC EQUILIBRIUM AND COMPARATIVE STATICS

The symmetric Nash equilibrium features all firms paying the wage w that satisfies the weighted Solow condition in (3.1.7), so that $w_{a,\tau} = w > \mu$ for any $\tau = n, o, p$. Hence, the wage-effort elasticity for each type is given by γ / u_τ^e. Substituting on the Solow condition yields the following expression:

$$\gamma \left(\frac{\eta}{u_n^e} + \frac{\theta}{u_o^e} + \frac{\rho}{u_p^e} \right) = 1. \tag{3.2.1}$$

Silveira and Lima (2021) assume the following specific form for the well-defined ordering for the unemployment expectations of employed workers of type $\tau = n, o, p$:

$$u_\tau^e = \begin{cases} (1-\delta)u, & \text{for } \tau = o, \\ u, & \text{for } \tau = n, \\ (1+\delta)u, & \text{for } \tau = p, \end{cases} \tag{3.2.2}$$

where $\delta \in (0, 1-\gamma) \subset (0,1) \subset \mathbb{R}$ is the dispersion parameter. Since (3.2.1) represents the firm's optimum condition, it can be used, together with (3.2.2), to obtain the equilibrium rate of unemployment, denoted by u^* and given by

$$u^* = \gamma \left[1 + \left(\frac{\delta}{1-\delta} \right) \theta - \left(\frac{\delta}{1+\delta} \right) \rho \right] \in (0,1) \subset \mathbb{R}. \tag{3.2.3}$$

At the vertices of the simplex Σ, the equilibrium rate of unemployment is given by $u^* \,|_{\theta=1} = \gamma/(1-\delta)$, $u^* \,|_{\eta=1} = \gamma$, and $u^* \,|_{\rho=1} = \gamma/(1+\delta)$, which yields the following ordering: $u^* \,|_{\theta=1} > u^* \,|_{\eta=1} > u^* \,|_{\rho=1}$. As a matter of fact, the upper limit $1-\gamma$ of δ is derived from (3.2.3) as the condition such that even in the extreme case that $\theta = 1$, u^*

is still less than one. Using (3.2.2), we find that, in equilibrium and at the vertices, the unemployment rate expectations will be the same: $u_o^e|_{\theta=1} = u_n^e|_{\eta=1} = u_p^e|_{\rho=1} = \gamma$. Hence, in the monomorphic states, the only case in which expectations are confirmed is the one such that all the workers hold neutral expectations.

If we normalise the labour supply to one and use the first-order condition in (3.1.5), we have that the equilibrium wage will be given by $w^* = \varepsilon^* F'\big(\varepsilon^*(1-u^*)\big)$, where ε^* is given by

$$\varepsilon^* = \left[\left(\frac{u^*}{1-u^*}\right)^\gamma\right]^{1-\theta-\rho} \left[\left(\frac{(1-\delta)u^*}{1-(1-\delta)u^*}\right)^\gamma\right]^\theta \left[\left(\frac{(1+\delta)u^*}{1-(1+\delta)u^*}\right)^\gamma\right]^\rho > 0. \tag{3.2.4}$$

After the equilibrium is defined, the authors present some interesting results of comparative statics that connect the model proposed to the empirical observations discussed in Chapter 2. At first, one could calculate the derivative $\partial u^*/\partial\rho$ to find how changes in the proportion of pessimistic workers affect the equilibrium rate of unemployment. However, since $\eta + \theta + \rho = 1$, a change in ρ must be followed by either a change in η, a change θ or both. To address this issue, $\alpha = \theta/\rho$ can be defined as the optimistic to pessimistic ratio and (3.2.3) can be rewritten as

$$u^* = \gamma\left[1 + \left(\frac{\delta}{1-\delta}\right)\alpha\rho - \left(\frac{\delta}{1+\delta}\right)\rho\right]. \tag{3.2.5}$$

After (3.2.5), it is possible to get the following expression for $\partial u^*/\partial\rho$, in which the pessimistic to optimistic ratio is kept constant:

$$\left.\frac{\partial u^*}{\partial\rho}\right|_{d\alpha=0} = \gamma\left[\left(\frac{\delta}{1-\delta}\right)\alpha - \left(\frac{\delta}{1+\delta}\right)\right]. \tag{3.2.6}$$

The above derivative yields the following condition:

$$\text{if} \left\{\begin{array}{c} 0 < \alpha < \frac{1-\delta}{1+\delta} \\ \alpha = \frac{1-\delta}{1+\delta} \\ \alpha > \frac{1-\delta}{1+\delta} \end{array}\right\}, \text{ then } \frac{\partial u^*}{\partial\rho} \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} 0. \tag{3.2.7}$$

That is, an increase in the proportion of pessimistic workers, followed by a corresponding change in the proportion of optimistic ones such that their ratio is kept unchanged, will decrease the equilibrium rate of unemployment as long as the ratio of pessimistic to optimistic workers is less than $(1-\delta)/(1+\delta)$.

The impact of changes in the dispersion parameter on the equilibrium rate of unemployment is given by

$$\frac{\partial u^*}{\partial\delta} = \frac{\rho}{(1-\delta)^2}\left[\alpha - \left(\frac{1-\delta}{1+\delta}\right)^2\right]\gamma, \tag{3.2.8}$$

which yields the following condition:

$$\text{if}\left\{\begin{array}{c} 0<\alpha<\left(\frac{1-\delta}{1+\delta}\right)^2 \\ \alpha=\left(\frac{1-\delta}{1+\delta}\right)^2 \\ \alpha>\left(\frac{1-\delta}{1+\delta}\right)^2 \end{array}\right\}, \text{ then } \frac{\partial u^*}{\partial \delta}\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}0. \tag{3.2.9}$$

Thus, if we combine conditions (3.2.7) and (3.2.9), we get that if $\partial u^*/\partial \rho > 0$, then $\partial u^*/\partial \delta > 0$. Moreover, the limit given by $(1-\delta)/(1+\delta)$ varies negatively in relation to δ, that is,

$$\frac{\partial}{\partial \delta}\left(\frac{1-\delta}{1+\delta}\right)=\frac{-2}{(1+\delta)^2}<0. \tag{3.2.10}$$

Hence, the larger the dispersion parameter δ is, the smaller will be the limit ratio between pessimistic and optimistic workers such that increases in the pessimism (compensated by changes in optimism) will give rise to an increase in the equilibrium rate of unemployment.

The model proposed by Silveira and Lima (2021) can finally connect the BS to the equilibrium rate of unemployment. Following the framework presented so far, the BS can be defined as

$$BS=100(\rho-\theta+1)=100\left[(1-\alpha)\rho+1\right], \tag{3.2.11}$$

such that

$$\left.\frac{\partial BS}{\partial \rho}\right|_{d\alpha=0}=100(1-\alpha). \tag{3.2.12}$$

Combining equation (3.2.12) with condition (3.2.7) yields the following condition:

$$\text{if}\left\{\begin{array}{c} 0<\alpha<\frac{1-\delta}{1+\delta} \\ \alpha=\frac{1-\delta}{1+\delta} \\ \frac{1-\delta}{1+\delta}<\alpha<1 \end{array}\right\}, \text{ then } \frac{\partial BS}{\partial \rho}>0 \text{ and } \frac{\partial u^*}{\partial \rho}\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}0. \tag{3.2.13}$$

That is, if $\alpha \in \left(\frac{1-\delta}{1+\delta}, 1\right) \subset \mathbb{R}$, then an exogenous increase in the proportion of pessimistic workers yields a positive relationship between the BS and the equilibrium rate of unemployment. Moreover, an increase in the dispersion among pessimists and optimists both (i) decreases the limit $\frac{1-\delta}{1+\delta}$ (which causes the above relationship to be more likely to happen) and (ii) increases the equilibrium rate of unemployment. However, if $\alpha \in \left(0, \frac{1-\delta}{1+\delta}\right) \subset \mathbb{R}$, then the greater pessimism will work as a disciplinary mechanism, causing the equilibrium rate of unemployment to fall.

4 PERSISTENCE OF HETEROGENEITY IN UNEMPLOYMENT EXPECTATIONS

According to the recent literature on the heterogeneity of expectations presented in Chapter 2, one might wonder whether workers' unemployment expectations, considered to be exogenous in the model set forth by Silveira and Lima (2021) and presented in Chapter 3, are in fact endogenous. If people are assumed to tailor their expectations to specific decision needs based on the economic reality they face following a process aimed at maximising their mental resources, it is only reasonable to presume that the unemployment conditions that workers face affect their expectations on future changes in unemployment. Drawing on this view on the formation of expectations, this Chapter puts forward a dynamic model of workers' unemployment expectations, where the temporary (or short-run) equilibrium is given by the model set forth by Silveira and Lima (2021). To do so, Section 4.1 proposes a behavioural rule to determine how workers revise their expectations in face of changes in the unemployment rate. As a consequence, expectations will be determined by an evolutionary dynamics which will interact with the macroeconomic state, so that the latter will coevolve with the frequency distribution of unemployment expectations across workers. Then, Section 4.2 shows that there will be no monomorphic microeconomic state (frequency distribution of unemployment expectations across workers), or simply microstate, that is an equilibrium of the dynamic system. As a consequence, we will conclude that, if the dynamic system has an equilibrium, then it will be heterogeneous (workers will hold more than one different expectation). Going further, Section 4.3 demonstrates that, if the dynamic system has an equilibrium, then it is fully polymorphic (all three types of expectations will be held by at least one worker). In addition, Section 4.3 also shows that such fully polymorphic equilibrium not only exists, but is both unique and stable. All mathematical proofs are presented in Appendix A.

4.1 AN EVOLUTIONARY DYNAMICS FOR UNEMPLOYMENT EXPECTATIONS

Let us assume a single-population game whose agents are the workers from the short-run equilibrium model put forward by Silveira and Lima (2021). Each agent has a set of available strategies denoted by $S = (o, n, p)$, which stands for optimism, neutrality, and pessimism, respectively. Let (θ, η, ρ) denote a microstate, where θ, η, and ρ are the proportions of optimistic, neutral, and pessimistic workers, respectively. Then, any given microstate belongs to the simplex $\Sigma = \left\{(\theta, \eta, \rho) \in \mathbb{R}^3_+ : \theta + \eta + \rho = 1\right\}$ and yields the temporary equilibrium rate of unemployment in (3.2.3).

Since firms set a homogeneous wage and optimistic workers are the ones performing the least effort, they will receive the greatest wage per unit of effort. In this situation, why would a worker remain performing a level of effort consistent with either a pessimistic or a neutral unemployment expectation in the face of an inferior wage per unit of effort? The answer relies on the fact that effort is not determined by workers as a means to maximise wage per unit of effort. Instead, the effort employed by each worker is determined as a response to the wage compensation payed by the firm, that, in turn, depends of the expected cost of job loss. Since people hold different unemployment expectations, as evidenced by survey data, the different levels of effort are optimal responses to the relative differences between the homogeneous wage and the heterogeneous cost of job loss (which depend on each worker's unemployment expectation). The question that remains is the following: why do workers hold different unemployment expectations?

Since all workers face the same actual unemployment, some expectations will turn out to be more accurate than others. Since accuracy is a prime goal of forming expectations, one might think that workers holding inaccurate expectations would immediately change their position. However, as discussed in Section 2.3, accuracy is never absolute, but only as precise as allowed by a cost-benefit calculation. At the end, the accuracy of an expectation is not measured by its departure from national data but by its impact on the decision for which it was formed. According to Curtin (2021), it should be no surprise that more timely decisions maximise utility even if the decision accuracy could be improved by devoting more time to the decision. Then, it is reasonable to presume that rather than optimising, people aim toward satisficing, which means finding a solution that will permit some level of satisfaction that is context-dependent even if it is not the optimal choice. In this situation, satisficing rather than decimal-point accuracy is not irrational, as it enables an efficient use of people's mental resources.

For instance, if the equilibrium rate of unemployment ends up being higher than the unemployment expectation held by the worker, this difference might not be perceived as a mistake that should be corrected and, as a consequence, would not trigger a change in expectations. However, as soon as this difference becomes perceptible, this change in unemployment receives conscious attention, which might trigger a change in expectations. Then, let us assume that each individual worker holds a target rate of unemployment that serves as a reference to classify the equilibrium rate of unemployment as either "high" or "low". Thus, if the equilibrium rate of unemployment is above the expectation but below the target rate, then this equilibrium is still perceived as low. In this case, if the worker initially held an optimistic expectation and is satisficing, then it would not trigger a change in expectation. On the other hand, if the worker was initially pessimistic and is satisficing, then the equilibrium rate of unemployment would not be perceived as high enough to keep him pessimistic, and it would trigger a change in expectation.

To formalise this reasoning, let us denote the target rate of unemployment of worker i by u_i' and let us assume that the target rate u_i' is determined stochastically by a cumulative distribution function $F : (0,1) \subset \mathbb{R} \longrightarrow [0,1] \subset \mathbb{R}$, which is continuously differentiable and strictly increasing. Hence, the probability that the short-run unemployment rate u^* is greater than u_i' (i.e., the probability that u^* is perceived as "high" by worker i) is given by

$$\text{Prob}(u_i' \leq u^*) = F(u^*). \tag{4.1.1}$$

Similarly, the probability that the short-run unemployment rate u^* is less than u_i' (i.e., the probability that u^* is perceived as "low" by worker i) is given by

$$\text{Prob}(u_i' > u^*) = 1 - F(u^*). \tag{4.1.2}$$

If we assume that $F(u^*)$ is independent from the distribution of expectations in the population, then the ratio of optimistic workers in the subpopulation for whom the equilibrium rate of unemployment is perceived as too high is the same as the ratio of optimists in the whole population. Hence, the ratio of optimists for whom the equilibrium rate of unemployment is perceived as too high is given by the product $\theta F(u^*)$. For example, let us assume that there are 30% of optimists in the whole population and that 40% of the whole population considers the equilibrium rate of unemployment too high. If we assume independence, then the ratio of optimists in the subpopulation that considers the equilibrium rate of unemployment too high is also 30%. In this case, the ratio of optimists for whom the equilibrium rate of unemployment is perceived as too high would be 12%. As previously discussed, this is the proportion of optimistic workers that will switch their expectations either to pessimism or to neutrality. However, to what type of expectation will they change?

Since a jump from optimism to pessimism would be a more extreme change than from optimism to neutrality (meaning that the change in the provision of effort would be greater), it is reasonable to assume that this choice bears some risks that are higher than a moderate change to neutrality. Then, let us further assume that the probability that a worker is willing to incur such risks is given by $\xi \in [0,1] \subset \mathbb{R}$, and also that this probability is independent from both the frequency distribution of expectations among workers and the probability that a worker considers the equilibrium rate of unemployment either too high or too low. In consequence, if $\xi \in (0, 0.5) \subset \mathbb{R}$, then the probability of moderate changes in expectations is larger than extreme changes. Similarly, if $\xi \in (0.5, 1) \subset \mathbb{R}$, then the probability of extreme changes in expectations is larger than moderate changes. At last, if $\xi = 0.5$, then both the extreme and moderate changes are equiprobable. Even though the situations we are interested in describing are those allowing for both moderate and extreme changes, it is illustrative to understand what happens when ξ is either zero (there is no possibility of extreme changes) or one (there are only extreme changes). Hence, we will keep this possibility as a matter of discussion.

As a result, the ratio of optimistic workers becoming pessimistic would be given by the product $\theta F(u^*)\xi$, and the ratio of optimistic workers becoming neutral would be given by the product $\theta F(u^*)(1-\xi)$. Similarly, the ratio of pessimists becoming either optimistic or neutral would be given by $\rho[1-F(u^*)]\xi$ and $\rho[1-F(u^*)](1-\xi)$, respectively. Moreover, since neutral workers do not face any change that might be conceived as either extreme or moderate (they can either increase or decrease their expectations), the ratio of neutrals becoming either optimistic or pessimistic would not depend on ξ and would be simply given by $\eta[1-F(u^*)]$ and $\eta F(u^*)$, respectively.

Hence, the change in the ratio of optimists at a given point in time, given by the difference between those becoming optimists (i.e., the inflow of optimists) and those ceasing to be optimists (i.e., the outflow of optimists), will be given by

$$\dot{\theta} = \eta\Big[1 - F(u^*)\Big] + \rho\Big[1 - F(u^*)\Big]\xi - \theta F(u^*). \tag{4.1.3}$$

Following the same reasoning, one obtains the subsequent system of differential equations:

$$\begin{cases} \dot{\theta} = & \eta\Big[1 - F(u^*)\Big] + \rho\Big[1 - F(u^*)\Big]\xi - \theta F(u^*), \\ \dot{\rho} = & \eta F(u^*) + \theta F(u^*)\xi - \rho\Big[1 - F(u^*)\Big], \\ \dot{\eta} = & \theta F(u^*)(1-\xi) + \rho\Big[1 - F(u^*)\Big](1-\xi) - \eta. \end{cases} \tag{4.1.4}$$

Rearranging the terms and using the fact that $\eta = 1 - \theta - \rho$ yields the following dynamic system:

$$\begin{cases} \dot{\theta} = \Big[1 - (1-\xi)\rho\Big]\Big[1 - F(u^*)\Big] - \theta, \\ \dot{\rho} = \Big[1 - (1-\xi)\theta\Big]F(u^*) - \rho, \end{cases} \tag{4.1.5}$$

whose state space is $\Theta = \Big\{(\theta, \rho) \in \mathbb{R}^2_+ : \theta + \rho \leq 1\Big\}$, which is a projection of unit simplex Σ.

4.2 COEXISTENCE OF AT LEAST TWO TYPES OF UNEMPLOYMENT EXPECTATIONS ACROSS WORKERS

Let us show that no vertex of the state space Θ will be an equilibrium of (4.1.5). In other words, let us show that if the system starts in a state comprised only by workers holding the same unemployment expectation, then the resulting short-run equilibrium rate of unemployment will generate an outflow of workers holding such expectation and the system will move away from this monomorphic state.

Evaluating (4.1.5) at the state $(\theta, \rho) = (1, 0)$ comprised only of optimistic workers, yields

$$\begin{cases} \dot{\theta}\,|_{\theta=1} = -F(u^*\,|_{\theta=1}), \\ \dot{\rho}\,|_{\theta=1} = F(u^*\,|_{\theta=1})\xi. \end{cases} \tag{4.2.1}$$

From (3.2.3), we have that $u^*\,|_{\theta=1} = u^*_{max} = \gamma/(1-\delta)$. Since $F(u^*)$ is strictly increasing and ranges between 0 and 1, $F(u^*\,|_{\theta=1}) = F(u^*_{max}) > 0$. Hence, at the pure state $(\theta, \rho) = (1, 0)$ comprised only of optimistic workers, there will be a mass of optimistic workers for whom the corresponding temporary equilibrium rate of unemployment (given by $u^*\,|_{\theta=1}$) is greater than the target level of unemployment that triggers a change in expectations (given by u'). As a consequence, there will be a net outflow of optimists (i.e., $\dot{\theta} < 0$), with a proportion of ξ becoming pessimists and $1 - \xi$ becoming neutral. In the extreme case where $\xi = 0$, the system would move away from the vertex but along the frontier where $\rho = 0$, as can be seen from Panel (a) in Figure 4.1. However, as the ratio of optimists decreases and the ratio of neutrals increases, there would be an outflow of neutrals becoming pessimists, and the system would move away from the frontier given by $\rho = 0$. On the other extreme case where $\xi = 1$, the result is quite different. In this case, the system would move away from the vertex but along the frontier where $\eta = 0$, as can be seen in panel (c) of Figure 4.1. However, since $\xi = 1$, we have that no pessimist and no optimist would ever become neutral. As a consequence, the system would move away from the vertex but along the frontier where $\eta = 0$ and not towards the interior of the state space. For the general case where $\xi \in (0, 1) \subset \mathbb{R}$, the vector field generated by the evolutionary dynamics will point towards the interior of the state space (away from both the vertex and any other frontier), as can be seen from Panel (b) of Figure 4.1. In other words, $(\theta, \rho) = (1, 0)$ is not an equilibrium of (4.1.5).

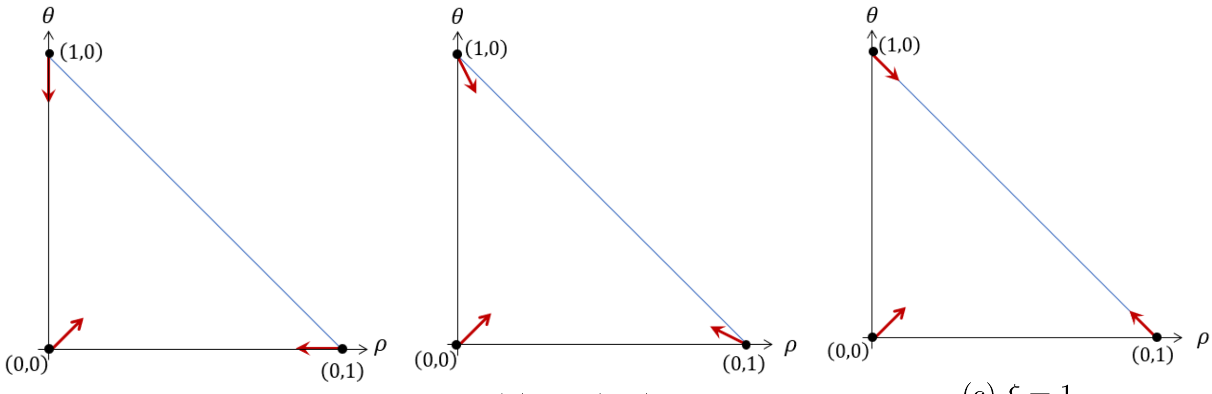
Now, let us evaluate (4.1.5) at the state $(\theta, \rho) = (0, 1)$ comprised only by pessimistic workers. Then,

$$\begin{cases} \dot{\theta}\,|_{\rho=1} = \Big[1 - F(u^*\,|_{\rho=1})\Big]\xi, \\ \dot{\rho}\,|_{\rho=1} = -\Big[1 - F(u^*\,|_{\rho=1})\Big]. \end{cases} \tag{4.2.2}$$

From (3.2.3), we have that $u^*\,|_{\rho=1} = u^*_{min} = \gamma/(1 + \delta)$. Again, since $F(u^*)$ is strictly increasing and ranges between 0 and 1, $F(u^*\,|_{\rho=1}) = F(u^*_{min}) < 1$. Hence, at the pure state $(\theta, \rho) = (0, 1)$ there will be a net outflow of pessimists (i.e., $\dot{\rho} < 0$), with a proportion of ξ becoming optimists and $1-\xi$ becoming neutral. If $\xi \in (0, 1)$, then the vector field generated by the evolutionary dynamics will, again, immediately point towards the interior of the state space[1], as can be seen from Panel (b) of Figure 4.1. In other words, $(\theta, \rho) = (0, 1)$ is not an equilibrium of (4.1.5) as well.

[1] The extreme cases where $\xi = 0$ or $\xi = 1$ would generate situations similar to the ones discussed for $\theta = 1$. Specifically, if $\xi = 0$, then the system would first move along the frontier where $\theta = 0$ but eventually move to the interior of the state space (Panel (a) of Figure 4.1). However, if $\xi = 1$, then the system would move along the frontier where $\eta = 0$ (Panel (c) of Figure 4.1).

Figure 4.1 – The vector field at the each vertex for different values of ξ in the state space Θ



(a) $\xi = 0$ (b) $\xi \in (0,1) \subset \mathbb{R}$ (c) $\xi = 1$

Source: Created by the author.

At last, let us evaluate (4.1.5) at the pure state $(\theta, \rho) = (0,0)$ comprised only by neutral workers. Then,

$$\begin{cases} \dot{\theta}\,|_{\eta=1} = 1 - F(u^*\,|_{\eta=1}), \\ \dot{\rho}\,|_{\eta=1} = F(u^*\,|_{\eta=1}). \end{cases} \tag{4.2.3}$$

From (3.2.3), we have that $u^*\,|_{\eta=1} = \gamma$ and, since $u^*_{min} < u^*\,|_{\eta=1} < u^*_{max}$, then $0 < F(u^*\,|_{\eta=1}) < 1$. Hence, at the pure state $(\theta, \rho) = (0,0)$ there will be a net outflow of neutrals, and the vector field generated by the evolutionary dynamics will immediately point towards the interior of the state space for all $\xi \in [0,1]$, as can be seen from all panels of Figure 4.1. In other words, $(\theta, \rho) = (0,0)$ is not an equilibrium of (4.1.5) as well. This conclusion allows us to state the following Proposition:

Proposition 1 (Heterogeneity). *Let $X \in \Theta$ be the set of all equilibria of system (4.1.5). Then, no element of X will be a vertex of the state space Θ. In other words, all equilibria will be polymorphic, that is, microstates characterised by the coexistence of at least two types of unemployment expectations.*

4.3 NO EXTINCTION OF ANY TYPE OF UNEMPLOYMENT EXPECTATION

Even though no equilibrium will be a monomorphic state, it could still be the case that one of the strategies would vanish in equilibrium. For example, Proposition 1 excludes the possibility that the triple $(\theta, \eta, \rho) = (1,0,0)$ is an equilibrium, but it doesn't exclude the possibility that $(\theta, \eta, \rho) = (0.5, 0.5, 0)$ is an equilibrium, in which case there would be a combination of optimists and neutrals describing a heterogeneous microstate, but with no pessimistic worker. Therefore, let us show that no boundary (or frontier) points of the state space Θ will be an equilibrium of (4.1.5). In other words, let us show that if the system starts in a state such that one of the expectations is not held by any worker, then the resulting temporary equilibrium rate of unemployment will generate an inflow of workers into that strategy, and the system will move towards fully polymorphic states (states in which all strategies are held by at least one worker).

Evaluating (4.1.5) at the frontier given by $\Theta_1 = \{(\theta, \rho) \in \Theta : \theta = 0\}$, yields

$$\begin{cases} \dot{\theta}\,|_{\theta=0} = \Big[1 - (1-\xi)\rho\Big]\Big[1 - F(u^*\,|_{\theta=0})\Big], \\ \dot{\rho}\,|_{\theta=0} = F(u^*\,|_{\theta=0}) - \rho. \end{cases} \tag{4.3.1}$$

From (3.2.3), we have that $u^*\,|_{\theta=0} = \gamma - [\gamma\delta/(1+\delta)]\rho$. Hence, $u^*_{min} \leq u^*\,|_{\theta=0} \leq u^*\,|_{\eta=1}$ and, as a consequence, $F(u^*\,|_{\theta=0}) < 1$. So, $\dot{\theta}\,|_{\theta=0}$ will only be equal to zero (and the system will only remain in the frontier given by Θ_1) in the extreme case where both $\xi = 0$ and $\rho = 1$. However, as stated before, at this pure state the system would immediately move along the frontier given by $\theta = 0$ and, as ρ diminishes, θ would increase, moving the system away from the frontier. For the general case where $\xi \in (0, 1) \subset \mathbb{R}$, if the initial state describes a situation where there are both neutrals and pessimists but no optimist, the vector field would move away from the frontier and towards the interior of the state space. In other words, no $(0, \rho) \in \Theta_1$ is an equilibrium of (4.1.5). But what about $\dot{\rho}$? At the pure state $(\theta, \rho) = (0, 0)$, we know that $\dot{\rho} > 0$. On the other hand, at the pure state $(\theta, \rho) = (0, 1)$, we know that $\dot{\rho} < 0$[2]. Because $u^*\,|_{\theta=0}$ is strictly decreasing in ρ[3] and $F(u^*\,|_{\theta=0})$ is strictly increasing in u^*, we know that $F(u^*\,|_{\theta=0})$ will be monotonically decreasing in ρ. Hence, by the Intermediate Value Theorem we know that there will be a single critical point $(0, \rho_{c1})$ such that $F(u^*\,|_{\theta=0}) = \rho_{c1}$ and, as a consequence, $\dot{\rho} = 0$. Simple inspection reveals that all points $(0, \rho) \in \Theta_1$ above $(0, \rho_{c1})$ yield $\dot{\rho} < 0$ and all points below yield $\dot{\rho} > 0$. This can be seen in all panels of Figure 4.2.

Now, if we evaluate (4.1.5) at the frontier given by $\Theta_2 = \{(\theta, \rho) \in \Theta : \rho = 0\}$, we have

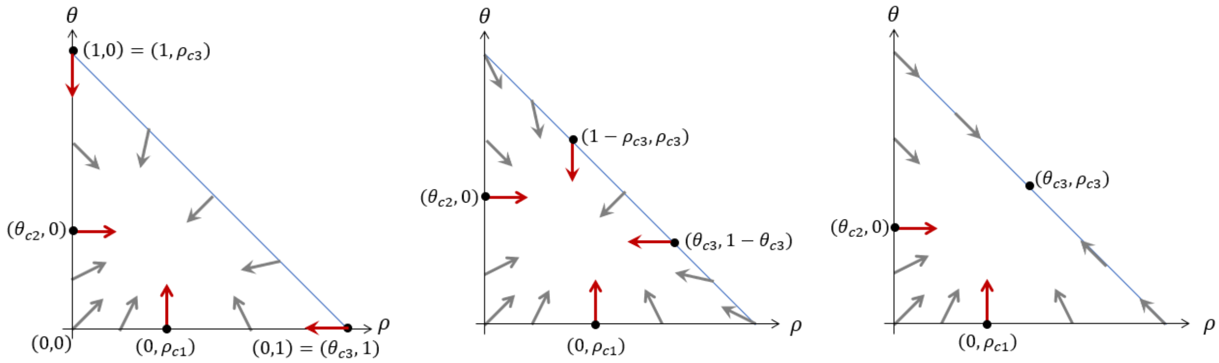
$$\begin{cases} \dot{\theta}\,|_{\rho=0} = \Big[1 - F(u^*\,|_{\rho=0})\Big] - \theta, \\ \dot{\rho}\,|_{\rho=0} = \Big[1 - (1-\xi)\theta\Big]F(u^*\,|_{\rho=0}). \end{cases} \tag{4.3.2}$$

From (3.2.3), we have that $u^*\,|_{\rho=0} = \gamma + [\gamma\delta/(1-\delta)]\theta$. Hence, $u^*\,|_{\eta=1} \leq u^*\,|_{\rho=0} \leq u^*_{max}$ and, as a consequence, $F(u^*\,|_{\rho=0}) > 0$. So, in the general case where $\xi \in (0, 1) \subset \mathbb{R}$, if the initial state describes a situation where there are both neutrals and optimists but no pessimist, the vector field would move away from the frontier and towards the interior of the state space. In other words, no $(\theta, 0) \in \Theta_2$ is an equilibrium of (4.1.5). Following the same reasoning as before, we find that there will be a single critical point $(\theta_{c2}, 0) \in \Theta_2$

[2] Both results regarding the pure states were discussed in the previous Section

[3] $\partial u^*\,|_{\theta=0} / \partial\rho = -\gamma\delta/(1-\delta) < 0$ for all $\gamma \in (0, 1) \subset \mathbb{R}$ and $\delta = (0, 1-\gamma) \subset \mathbb{R}$.

Figure 4.2 – The vector field at the boundary of the state space Θ for different values of ξ (grey arrows are auxiliary vectors)



(a) $\xi = 0$ (b) $\xi \in (0,1) \subset \mathbb{R}$ (c) $\xi = 1$

Source: Created by the author.

such that $1 - F(u^* |_{\rho=0}) = \theta_{c2}$ and, as a consequence, $\dot{\theta} = 0$. Simple inspection reveals that all points $(\theta, 0) \in \Theta_2$ above $(\theta_c, 0)$ yield $\dot{\theta} < 0$ and all points below yield $\dot{\theta} > 0$, as can be seen from all panels in Figure 4.2.

At last, if we evaluate (4.1.5) at the frontier given by $\Theta_3 = \{(\theta, \rho) \in \Theta : \theta + \rho = 1\}$, we have

$$\begin{cases} \dot{\theta} |_{\eta=0} = \Big[1 - (1-\xi)(1-\theta)\Big]\Big[1 - F(u^* |_{\eta=0})\Big] - \theta, \\ \dot{\rho} |_{\eta=0} = \Big[1 - (1-\xi)(1-\rho)\Big] F(u^* |_{\eta=0}) - \rho, \end{cases} \tag{4.3.3}$$

alongside with

$$\dot{\eta} = \theta F(u^* |_{\eta=0})(1-\xi) + \rho[1 - F(u^* |_{\eta=0})](1-\xi). \tag{4.3.4}$$

By simple inspection, we conclude that $\dot{\eta} = 0$ at all points in the frontier Θ_3 if $\xi = 1$. As a consequence, if $\xi = 1$, then the vector field would not move away from the frontier and the system would never move towards a fully polymorphic state. The logic behind this result is quite simple: since $\xi = 1$ means that all pessimists will change directly to optimism (if so) and vice-versa, if the system starts in a state with no neutrals, then it will remain without any neutral worker.

For all $\xi \in [0, 1) \subset \mathbb{R}$, we know from our previous discussion that $\dot{\eta}$ is greater than zero at the vertices $(\theta, \rho) = (1, 0)$ and $(\theta, \rho) = (0, 1)$. Let us analyse $\dot{\eta}$ outside the vertices $(1, 0)$ and $(0, 1)$ for all $\xi \in [0, 1) \subset \mathbb{R}$. Since

$$u^* |_{\eta=0} = \gamma \left[1 + \left(\frac{\delta}{1-\delta}\right)(1-\rho) - \left(\frac{\delta}{1+\delta}\right)\rho\right], \tag{4.3.5}$$

then $u^*_{min} < u^* |_{\eta=0} < u^*_{max}$ for all $(\theta, \rho) \in \Theta_3$ that is not a vertex, and, as a consequence, $0 < F(u^* |_{\eta=0}) < 1$. So, both in the general case where $\xi \in (0, 1) \subset \mathbb{R}$ and the extreme case where $\xi = 0$, if the initial state describes a situation where there are both optimists and pessimists (outside the vertices), but no neutral, the vector field would move away from the frontier and towards the interior of the state space. In other words, if $\xi \in [0,1) \subset \mathbb{R}$, then no $(\theta,\rho) \in \Theta_3$ is an equilibrium of (4.1.5). But what about $\dot{\theta}$ and $\dot{\rho}$? Let us first analyse the extreme cases for ξ before we evaluate the general case where $\xi \in (0,1) \subset \mathbb{R}$.

If $\xi = 1$, then system (4.3.3) becomes

$$\begin{cases} \dot{\theta}\,|_{\eta=0} = (1-\theta) - F(u^*\,|_{\eta=0}), \\ \dot{\rho}\,|_{\eta=0} = F(u^*\,|_{\eta=0}) - \rho. \end{cases} \tag{4.3.6}$$

So, if $\xi = 1$, then both $\dot{\theta}$ and $\dot{\rho}$ will be equal to zero at the point $(\theta_{c3}, 1-\theta_{c3}) = (1-\rho_{c3}, \rho_{c3}) \in \Theta_3$, such that $\rho_{c3} = F(u^*\,|_{\eta=0})$ and $\theta_{c3} = 1 - F(u^*\,|_{\eta=0})$. So, in this extreme case we would have an equilibrium at the frontier where $\eta = 0$. Simple inspection of (4.3.6) reveals that for any $(\theta,\rho) \in \Theta_3$ such that $\rho > \rho_c$, we have that $\dot{\theta} > 0$ and $\dot{\rho} < 0$. Similarly, for any $(\theta,\rho) \in \Theta_3$ such that $\rho < \rho_c$, we have that $\dot{\theta} < 0$ and $\dot{\rho} > 0$, as can be seen from panel (c) of Figure 4.2.

Now, let us consider the opposite extreme case where $\xi = 0$. Then, system (4.1.5) becomes

$$\begin{cases} \dot{\theta}\,|_{\eta=0} = -\theta F(u^*\,|_{\eta=0}), \\ \dot{\rho}\,|_{\eta=0} = -\rho\big[1 - F(u^*\,|_{\eta=0})\big]. \end{cases} \tag{4.3.7}$$

Since $\theta = 0$ implies that $u^*\,|_{\eta=0} = u^*_{min}$ and, as a consequence, $F(u^*\,|_{\eta=0}) = F(u^*_{min}) = 0$, we have that $\dot{\theta} = 0$ at the frontier where $\eta = 0$ only at the vertex $(0,1) \in \Theta_3$. For any pair $(\theta,\rho) \in \Theta_3$ such that $\theta < 1$, we have that $\dot{\theta} < 0$. Conversely, since $\rho = 0$ implies that $u^*\,|_{\eta=0} = u^*_{max}$ and, as a consequence, $F(u^*\,|_{\eta=0}) = F(u^*_{max}) = 1$, we have that $\dot{\rho} = 0$ at the frontier where $\eta = 0$ only at the vertex $(\theta,\rho) = (1,0) \in \Theta_3$. For any pair $(\theta,\rho) \in \Theta_3$ such that $\rho < 1$, we have that $\dot{\rho} < 0$. This can be seen on Panel (a) of Figure 4.2.

Finally, let us consider the general case where $\xi \in (0,1) \subset \mathbb{R}$. Then, system (4.1.5) becomes

$$\begin{cases} \dot{\theta}\,|_{\eta=0} = \Big[1 - (1-\xi)\rho\Big]\Big[1 - F(u^*\,|_{\eta=0})\Big] - (1-\rho), \\ \dot{\rho}\,|_{\eta=0} = \Big[1 - (1-\xi)\theta\Big]F(u^*\,|_{\eta=0}) - (1-\theta), \end{cases} \tag{4.3.8}$$

The condition for $\dot{\theta}\,|_{\eta=0} = 0$ is

$$F(u^*\,|_{\eta=0}) = \frac{\xi(1-\theta)}{1-(1-\xi)(1-\theta)}. \tag{4.3.9}$$

From (4.3.5), we have that $u^*\,|_{\eta=0}$ is strictly increasing in θ. Hence, the left-hand side (LHS) of (4.3.9) will be strictly increasing in θ. Conversely, the right-hand side (RHS) of (4.3.9) is strictly decreasing in θ. Since $\dot{\theta} > 0$ at $(\theta,\rho) = (0,1)$ and $\dot{\theta} < 0$ at $(\theta,\rho) = (1,0)$, we have by the Intermediate Value Theorem that there will be a single critical point $(\theta_{c3}, 1-\theta_{c3}) \in \Theta_3$ such that $\dot{\theta} = 0$.

Similarly, the condition for $\dot{\rho}\,|_{\eta=0} = 0$ is

$$F(u^*\,|_{\eta=0}) = \frac{\rho}{1-(1-\xi)(1-\rho)}. \tag{4.3.10}$$

From (4.3.5), we have that $u^*\,|_{\eta=0}$ is strictly decreasing in ρ. Hence, the LHS of (4.3.10) will be strictly decreasing in ρ. Conversely, the RHS of (4.3.10) is strictly increasing in ρ. Since $\dot{\rho} > 0$ at $(\theta, \rho) = (1, 0)$ and $\dot{\rho} < 0$ at $(\theta, \rho) = (0, 1)$, we have by the Intermediate Value Theorem that there will be a single critical point $(1 - \rho_{c3}, \rho_{c3}) \in \Theta_3$ such that $\dot{\rho} = 0$. Both critical points can be seen in panel (b) of Figure 4.2.

In summary, starting from $\xi = 0$, the critical points will be at $(\theta, \rho_{c3}) = (0, 1)$ and $(\theta_{c3}, \rho) = (1, 0)$. As ξ increases, both ρ_{c3} and θ_{c3} decreases. Finally, as ξ approaches 1, $\rho_{c3} \to (1 - \theta_{c3})$ and $\theta_{c3} \to (1 - \rho_{c3})$. This discussion allows us to state the following Proposition:

Proposition 2 (**Fully Heterogeneous Expectations**). *Let $X \in \Theta$ be the set of all equilibria of system (4.1.5). If $\xi \neq 1$, then any element of X is in the interior of the state space Σ, that is, all equilibria will be fully polymorphic.*

It follows from Proposition 2 that, if $\xi \in (0, 1) \subset \mathbb{R}$, then the vector field will point towards the interior of the state space. However, it does not follow immediately that the vector field will converge to any equilibrium in the interior of the state space. For example, it could be the case that the system will follow a chaotic trajectory. Hence, now that we have shown that there will be no equilibrium on the frontier of the state space, let us finally prove that there will be in fact an equilibrium in the interior of the state space (describing a fully polymorphic state) that is both unique and asymptotically stable.

Let $\left(\overline{\theta}, \overline{\rho}\right) \in \Theta$ be the pair such that $\dot{\theta} = \dot{\rho} = 0$ and let us denote $u^*\left(\overline{\theta}, \overline{\rho}\right) = \overline{u}$. Then, system (4.1.5) becomes

$$\begin{cases} \Big[1-(1-\xi)\overline{\rho}\Big]\Big[1-F(\overline{u})\Big] - \overline{\theta} = 0, \\ \Big[1-(1-\xi)\overline{\theta}\Big]F(\overline{u}) - \overline{\rho} = 0. \end{cases} \tag{4.3.11}$$

From the first equation of system (4.3.11), we have

$$1 - F(\overline{u}) = \frac{\overline{\theta}}{1-(1-\xi)\overline{\rho}}, \tag{4.3.12}$$

and from the second equation of system (4.3.11), we have

$$F(\overline{u}) = \frac{\overline{\rho}}{1-(1-\xi)\overline{\theta}}. \tag{4.3.13}$$

Hence, the pair $\left(\overline{\theta}, \overline{\rho}\right)$ will be an equilibrium of (4.1.5) if it satisfies both (4.3.12) and (4.3.13) simultaneously.

Proposition 3 (**Existence and Uniqueness of a Fully Polymorphic Equilibrium**). *There will be a unique* $(\theta, \rho) \in \Theta = \left\{(\theta, \rho) \in \mathbb{R}_+^2 : \theta + \rho \leq 1\right\}$, *such that* (4.3.12) *and* (4.3.13) *are simultaneously true.*

Proof. See Appendix A.1.

Having proved that there is a unique equilibrium $\left(\overline{\theta}, \overline{\rho}\right)$, and also that this equilibrium is fully polymorphic for the general case where $\xi \in (0,1) \subset \mathbb{R}$, now we have to analyse its stability. Since (4.1.5) is a nonlinear system, its stability will be analysed using a first-order approximation. The equilibrium $\left(\overline{\theta}, \overline{\rho}\right)$ of the linear version of (4.1.5) will be asymptotically stable if the eigenvalues of the Jacobian matrix have negative real parts. If that is the case, then we can apply the Hartman-Grobman Theorem and conclude that $\left(\overline{\theta}, \overline{\rho}\right)$ will be a locally asymptotically stable equilibrium of the nonlinear system (4.1.5). This brief discussion allows us to state the following Proposition:

Proposition 4 (**Stability**). *The unique and polymorphic equilibrium* $\left(\overline{\theta}, \overline{\rho}\right)$ *is* ***locally asymptotically stable*** *for all values of the parameters* $\xi \in (0,1) \subset \mathbb{R}$, $\gamma \in (0,1) \subset \mathbb{R}$, *and* $\delta \in (0, 1-\gamma) \subset \mathbb{R}$ *and for all cumulative distribution functions* F.

Proof. See Appendix A.2.

That is, the model put forward in the present Chapter implies that there will be a fully polymorphic equilibrium of the dynamic system that is both unique and asymptotically stable. In other words, the model implies that the heterogeneity among workers' unemployment expectations is an evolutionary stable equilibrium, consistent with empirically observed results.

5 FINAL REMARKS

Drawing on the recent literature on how expectations are formed and the empirical evidences on the persistence of heterogeneity in workers' unemployment expectations, the present work put forward a dynamic model of endogenous fluctuations in workers' unemployment expectations where the short-run equilibrium is given by the model set forth by Silveira and Lima (2021). The dynamics of the model is given by a behavioural rule that describes how workers revise their expectations based on the temporary equilibrium rate of unemployment. This simple rule is consistent with both the theories of bounded rationality and empirical evidences that show that people form their unemployment expectations based on the reality they face in the labour market. As a consequence, the frequency distribution of expectations across workers will converge to a unique equilibrium that is both asymptotically stable and fully polymorphic. This result replicates the persistence in heterogeneity observed in empirical surveys on consumers' expectations, such as the one conducted by the University of Michigan.

More specifically, a initial frequency distribution of unemployment expectations will give rise to a short-run equilibrium that will either confirm or reject each worker's expectation. However, not all the differences between the observed and expected rate of unemployment will be perceived as a mistake that must be fixed. Even so, as soon as this difference becomes perceptible, a change in expectations will be triggered. Some workers might be willing to jump from one extreme expectation to the other (from pessimism to optimism, for example). Others will be more cautions and prefer to become neutrals first and only change to the other extreme after that. The ratio of workers that are willing to jump from one extreme to the other is given by an exogenous parameter in the model. This change in the frequency distribution of unemployment expectation across workers will give rise to a new short-run equilibrium that, again, will either confirm or reject each worker's expectations. This process will converge to a long-run equilibrium where both the inflow and outflow of each expectation will be the same and, as a consequence, the frequency distribution of unemployment expectations will remain the same. Then, the model concludes that the heterogeneity of unemployment expectations survive as an evolutionary stable equilibrium (no type of expectation will vanish in equilibrium).

The model put forward by the present work gives rise to many questions that could be answered by further research. First, the stylised fact that states that the frequency distribution of expectations varies considerably across different periods of time can be replicated by the model if we allow exogenous changes in the parameters. Hence, the comparative statics analysis of the model is a natural step of advancement. Second, it would be interesting to compare the results of the model to what would happen if we had assumed homogeneity. In other word, future research should investigate whether heterogeneity in expectations yield a different outcome then the representative agent hypothesis. Finally, future research should investigate how the model connects with general equilibrium models and if it affects the conduct of fiscal and/or monetary policy.

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APPENDIX A – MATHEMATICAL PROOFS

This Appendix presents the mathematical proofs for both the Proposition 3 (Section A.1) and Proposition 4 (Section A.2), as stated in Chapter 4.

A.1 PROPOSITION 3 (EXISTENCE AND UNIQUENESS OF A FULLY POLYMORPHIC EQUILIBRIUM)

The proof will be divided into two steps. The first one, presented in subsection A.1.1, will be devoted to prove the existence of the equilibrium. Then, subsection A.1.2 proves its uniqueness.

A.1.1 Proof of Existence

If we isolate $F(\overline{u})$ in (4.3.12) and set it equal to (4.3.13), we have the following equation:

$$\frac{\overline{\rho}}{1-(1-\xi)\overline{\theta}} = \frac{1-(1-\xi)\overline{\rho}-\overline{\theta}}{1-(1-\xi)\overline{\rho}}. \tag{A.1.1}$$

Rearranging the terms yields the following quadratic equation on $\overline{\rho}$:

$$(1-\xi)\overline{\rho}^2 - \overline{\rho}\Big[(2-\xi)-(1-\xi)^2\overline{\theta}\Big] + \overline{\theta}^2(1-\xi) - \overline{\theta}(2-\xi) + 1 = 0, \tag{A.1.2}$$

whose solution will be[1]

$$\overline{\rho}(\overline{\theta};\xi) \equiv \frac{(2-\xi)-(1-\xi)^2\overline{\theta} - \sqrt{\left[(2-\xi)-(1-\xi)^2\overline{\theta}\right]^2 - 4(1-\xi)\left[(1-\xi)\overline{\theta}^2-(2-\xi)\overline{\theta}+1\right]}}{2(1-\xi)}, \tag{A.1.3}$$

that is well defined for all $\xi \in (0,1) \subset \mathbb{R}$.

Let us define the following expression by substituting both $\overline{\theta}$ and $\overline{\rho}(\overline{\theta};\xi)$ in (3.2.3):

$$\overline{u}(\overline{\theta};\xi) \equiv \gamma\left[1+\left(\frac{\delta}{1-\delta}\right)\overline{\theta}-\left(\frac{\delta}{1+\delta}\right)\overline{\rho}(\overline{\theta};\xi)\right]. \tag{A.1.4}$$

Then, substituting $\overline{\rho}(\overline{\theta};\xi)$ in either one of expressions (4.3.12) and (4.3.13) yields the following condition:

$$F\Big(\overline{u}(\overline{\theta};\xi)\Big) = \frac{\overline{\rho}(\overline{\theta};\xi)}{1-(1-\xi)\overline{\theta}}. \tag{A.1.5}$$

[1] The solution where the square root is added, and not subtracted, would lead to values outside the simplex.

Hence, all that is left to prove is that there is some $\overline{\theta} \in (0,1) \in \mathbb{R}$ that solves equation (A.1.5) for some feasible values of parameter ξ. To do so, let us consider the following function:

$$\varphi(\overline{\theta};\xi) = F\Big(\overline{u}(\overline{\theta};\xi)\Big) - g(\overline{\theta};\xi), \tag{A.1.6}$$

where $g(\overline{\theta};\xi)$ represents the RHS of (A.1.5). As a consequence, if there is some $\overline{\theta} \in (0,1) \in \mathbb{R}$ such that $\varphi(\overline{\theta};\xi) = 0$ for feasible values of ξ, then the condition (A.1.5) holds, so that the pair $\Big(\overline{\theta},\overline{\rho}\Big)$ will be an equilibrium of System (4.1.5).

First, observe that

$$\overline{\rho}(0;\xi) = \frac{2-\xi-\sqrt{(2-\xi)^2-4(1-\xi)}}{2(1-\xi)} = 1. \tag{A.1.7}$$

Hence, $g(0;\xi) = 1$ for all $\xi \in (0,1) \subset \mathbb{R}$. Since $\overline{u}(0;\xi) = \gamma/(1+\delta)$, then $F\Big(\overline{u}(0;\xi)\Big) < 1$ and, in consequence, $\varphi(0;\xi) < 0$ for all $\xi \in (0,1) \subset \mathbb{R}$. Now, observe that

$$\overline{\rho}(1;\xi) = \frac{1+\xi-\xi^2-\sqrt{(1+\xi-\xi^2)^2}}{2(1-\xi)} = 0. \tag{A.1.8}$$

Hence, $g(1;\xi) = 0$ for all $\xi \in (0,1) \subset \mathbb{R}$. Since $\overline{u}(1;\xi) = \gamma/(1-\delta)$, then $F\Big(\overline{u}(1;\xi)\Big) > 0$ and, in consequence, $\varphi(1;\xi) > 0$ for all $\xi \in (0,1) \subset \mathbb{R}$. Therefore, since $\varphi(\overline{\theta};\xi)$ is continuous in all its domain, we can use the Intermediate Value Theorem to conclude that there will be some $\overline{\theta} \in (0,1) \in \mathbb{R}$ such that $\varphi(\overline{\theta};\xi) = 0$, which was to be demonstrated.

A.1.2 Proof of Uniqueness

To prove that $\Big(\overline{\theta},\overline{\rho}\Big)$ is a unique equilibrium, all we have to prove is that there is one, and only one, $\overline{\theta}$ such that $\varphi(\overline{\theta};\xi) = 0$. Since $\varphi(\overline{\theta};\xi)$ is continuous in all its domain, $\varphi(0;\xi) < 0$, and $\varphi(1;\xi) > 0$, this will be the case if $\varphi(\overline{\theta};\xi)$ is strictly monotonically increasing, that is, if $\partial\varphi(\overline{\theta};\xi)/\partial\overline{\theta} > 0$ for all $\xi \in (0,1) \subset \mathbb{R}$.

Since

$$\frac{\partial\varphi(\overline{\theta};\xi)}{\partial\overline{\theta}} = F'(\overline{u})\frac{\partial\overline{u}(\overline{\theta};\xi)}{\partial\overline{\theta}} - \frac{\partial g(\overline{\theta};\xi)}{\partial\overline{\theta}}, \tag{A.1.9}$$

let us analyse each term on the RHS of (A.1.9) separately. As $F'(\overline{u}) > 0$ by definition and

$$\frac{\partial\overline{u}(\overline{\theta};\xi)}{\partial\overline{\theta}} = \gamma\left[\left(\frac{\delta}{1-\delta}\right) - \left(\frac{\delta}{1+\delta}\right)\frac{\partial\overline{\rho}(\overline{\theta};\xi)}{\partial\overline{\theta}}\right], \tag{A.1.10}$$

we conclude that the first term of the RHS of (A.1.9) will be positive if, and only if, $\partial\overline{\rho}(\overline{\theta};\xi)/\partial\overline{\theta} < 0$. Evaluating this partial derivative using the command `Reduce[]` from the software Wolfram Mathematica, we have that $\partial\overline{\rho}(\overline{\theta};\xi)/\partial\overline{\theta}$ will be less than zero for all values of $\overline{\theta} \in (0,1) \subset \mathbb{R}$ and $\xi \in (0,1) \subset \mathbb{R}$. Hence, the first term of the RHS of (A.1.9) will be positive.

In turn, if we follow the same procedure with the second term of the RHS of (A.1.9) (i.e., calculate the partial derivative of $g(\overline{\theta};\xi)$ in relation with $\overline{\theta}$ and evaluate it with the command `Reduce[]` from Wolfram Mathematica), we find that the partial derivative will be strictly negative for all values of $\overline{\theta} \in (0,1) \subset \mathbb{R}$ and $\xi \in (0,1) \subset \mathbb{R}$. Hence, the second term of the RHS of (A.1.9) will also be positive. As a consequence, we have proved that $\varphi(\overline{\theta};\xi)$ is strictly monotonically increasing (i.e., $\partial\varphi(\overline{\theta};\xi)/\partial\overline{\theta} > 0$ for all values of $\overline{\theta} \in (0,1) \subset \mathbb{R}$ and $\xi \in (0,1) \subset \mathbb{R}$). Then, we can finally conclude that the equilibrium $\left(\overline{\theta};\overline{\rho}\right)$ will not only exist and be fully polymorphic, but will also be unique, which was to be demonstrated.

A.2 PROPOSITION 4 (STABILITY)

The partial derivatives of the Jacobian matrix, when evaluated at $\left(\overline{\theta},\overline{\rho}\right)$, are

$$\left.\frac{\partial\dot{\theta}}{\partial\theta}\right|_{(\overline{\theta},\overline{\rho})} = -\Big[1-(1-\epsilon)\overline{\rho}\Big]F'(\overline{u})\left(\frac{\gamma\delta}{1-\delta}\right) - 1 < 0, \tag{A.2.1}$$

$$\left.\frac{\partial\dot{\theta}}{\partial\rho}\right|_{(\overline{\theta},\overline{\rho})} = -(1-\epsilon)\Big[1-F(\overline{u})\Big] + \Big[1-(1-\epsilon)\overline{\rho}\Big]F'(\overline{u})\left(\frac{\gamma\delta}{1+\delta}\right), \tag{A.2.2}$$

$$\left.\frac{\partial\dot{\rho}}{\partial\theta}\right|_{(\overline{\theta},\overline{\rho})} = -(1-\epsilon)F(\overline{u}) + \Big[1-(1-\epsilon)\overline{\theta}\Big]F'(\overline{u})\left(\frac{\gamma\delta}{1-\delta}\right), \tag{A.2.3}$$

$$\left.\frac{\partial\dot{\rho}}{\partial\rho}\right|_{(\overline{\theta},\overline{\rho})} = -\Big[1-(1-\epsilon)\overline{\theta}\Big]F'(\overline{u})\left(\frac{\gamma\delta}{1+\delta}\right) - 1 < 0. \tag{A.2.4}$$

The trace of the Jacobian matrix, evaluated at the equilibrium, will be

$$\begin{aligned}
\operatorname{tr}(\mathcal{J}) &= -\Big[1-(1-\xi)\overline{\rho}\Big]F'(\overline{u})\frac{\gamma\delta}{(1-\delta)} - \Big[1-(1-\xi)\overline{\theta}\Big]F'(\overline{u})\left(\frac{\gamma\delta}{1+\delta}\right) - 2, && \text{(A.2.5a)}\\
&= -F'(\overline{u})\frac{\gamma\delta}{(1-\delta^2)}\Big\{2-(1-\xi)\Big[\overline{\rho}(1+\delta)+\overline{\theta}(1-\delta)\Big]\Big\} - 2, && \text{(A.2.5b)}
\end{aligned}$$

that is less than zero for all γ, δ, ξ, and $F(u^*)$. In turn, the determinant of the Jacobian matrix will be

$$\begin{aligned}
\det(\mathcal{J}) = F'(\overline{u})\gamma\delta\Bigg\{&\frac{1-(1-\xi)\overline{\theta}}{1+\delta} + \frac{1-(1-\xi)\overline{\rho}}{1-\delta} + (1-\xi)F(\overline{u})\frac{[1-(1-\xi)\overline{\rho}]}{1+\delta}\\
&+(1-\xi)[1-F(\overline{u})]\frac{[1-(1-\xi)\overline{\theta}]}{1-\delta}\Bigg\}\\
&-(1-\xi)^2F(\overline{u})[1-F(\overline{u})] + 1.
\end{aligned} \tag{A.2.6}$$

Since the term in braces is positive, as well as the term multiplying it, and since $(1-\epsilon)^2F(\overline{u})\Big[1-F(\overline{u})\Big] < 1$, we have that $\det(\mathcal{J}) > 0$. Hence, both conditions for stability are satisfied, which was to be demonstrated.