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Thiago Wilvert

**A METHODOLOGY TO MEASURE RESIDUAL STRESSES CLOSE TO THE
YIELD STRESS USING AN OPTICAL HOLE DRILLING METHOD**

Florianópolis
2025

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for the degree of Doctor in Mechanical Engineering.

Advisor: Prof. Armando Albertazzi Gonçalves Júnior,
Dr. Eng.

Co-advisor: Matias Roberto Viotti, Dr. Ing.

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composta pelos seguintes membros:

Prof. Gary Schajer, Dr. Ing.
University of British Columbia - UBC

Prof. Eduardo Alberto Fancello, D.Sc.
Universidade Federal de Santa Catarina - UFSC

Prof. Marco Beghini, Dr. Ing.
Università di Pisa – UniPi

Certificamos que esta é a versão original e final do trabalho de conclusão que foi julgado
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Prof. Henrique Simas, Dr. Eng
Coordenador do Programa de Pós-Graduação em Engenharia Mecânica



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Prof. Armando Albertazzi Gonçalves Júnior, Dr. Eng.
Orientador

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ABSTRACT

The demand for exploration of oil in deep areas has increased in the last few years. Deeper regions introduce higher requirements for flexible riser structures, especially the tensile armor layer. The tensile armor wires are helically wound around the riser. With the exploration of oil in deeper regions, engineers had to push the limit of the design. Understanding the introduction of residual stresses is a good strategy adopted when the design of structures demands a high performance. The hole-drilling method (HDM) is widely employed to measure residual stresses. It consists in the measurement of the strains generated around a small hole. The hole is drilled in small increments of usually 10 or 20 steps. Preliminary measurements of residual stresses in flexible risers indicated that high levels of residual stresses are present. Although the HDM is widely employed in the oil and gas industry, it has a limitation in the level of residual stress it can measure. Plastic strains near the hole result in deviation because it infringes the elastic assumption considered in the calculation of the calibration coefficients presented in the standard ASTM E837-20a. There are corrections available for the plasticity effects in uniform measurements. For the nonuniform method, corrections are very limited and assume not well-accepted premises. In this work, a strategy to measure uniform residual stresses is presented. The technique combines full-field optical measurements with an iterative algorithm to correct the plasticity effects of residual stress measurements in components under high levels of residual stresses. Full-field data acquired with an optical device provides higher details of the displacement around the hole. Modifications to an existing iterative algorithm were performed to incorporate the full-field data signature into the algorithm. The computation of the inclination of the partial derivative of the radial displacement with respect to the radius as a function of theta was performed for both the experimental and numerical values. This link between the full-field displacement data with the numerical displacement data allowed the algorithm to correct the residual stress measurements. The numerical trials indicated a good convergence rate for different depths and annular diameters. Experimental validations showed that the near case is capable of measuring high residual stress levels. The intermediate case presented a few limitations, while the far case did not suit well for experimental measurements.

Keywords: Oil industry, hole-drilling method, high level of stress, plasticity.

RESUMO

A demanda por exploração de petróleo em águas profundas aumentou nos últimos anos. Regiões mais profundas impõem exigências mais elevadas para algumas camadas da estrutura dos dutos flexíveis, em especial à armadura de tração. Os arames da armadura de tração são enrolados de forma helicoidal ao redor do duto. Com a exploração de petróleo em águas profundas, o projeto destes dutos flexíveis se tornou mais crítico, exigindo um melhor desempenho. Compreender o fenômeno de introdução de tensões residuais é uma ótima estratégia que auxilia no projeto de estruturas de alta performance. O Método do Furo Cego (MFC) é amplamente utilizado para medição de tensões residuais e consiste, basicamente, na medição das deformações geradas pela introdução de um pequeno furo. O furo incremental é comumente produzido com 10 ou 20 incrementos. Análises preliminares indicaram que tensões residuais elevadas estão presentes em dutos flexíveis. Apesar de ser bastante empregado na indústria de petróleo e gás, o método do furo cego possui limitação com relação às tensões que ele pode medir. Deformações plásticas no entorno do furo geram desvios no resultado pelo fato destas infringirem o regime de elasticidade considerado para obtenção dos coeficientes de calibração publicados na norma ASTM E837-20a. Correções para o efeito da plasticidade estão disponíveis no método uniforme. Entretanto, para o método não uniforme as correções são bastante limitadas e pouco aceitas no meio científico. Neste trabalho, é apresentada uma estratégia para medir tensões residuais uniformes. A técnica combina medições ópticas de campo completo (*full-field*) com um algoritmo iterativo para corrigir os efeitos da plasticidade em medições de tensões residuais elevadas (componentes sob altos níveis de tensões residuais). Dados de campo completo obtidos com um dispositivo óptico fornecem maiores detalhes do deslocamento ao redor do furo. Modificações em um algoritmo iterativo existente foram realizadas para incorporar a assinatura de dados de campo completo no algoritmo. O cálculo da inclinação da derivada parcial do deslocamento radial em relação ao raio como uma função de θ foi realizado para os valores experimentais e numéricos. A correlação entre os dados de deslocamento de campo completo com os dados de deslocamento numérico permitiu que o algoritmo corrigisse as medições de tensão residual. As avaliações numéricas indicaram uma boa taxa de convergência para diferentes profundidades e diâmetros anulares. Validações experimentais mostraram que o caso *near* é capaz de medir altos níveis de tensão residual. O caso *intermediate* apresentou algumas limitações, enquanto o caso *far* não se adequou bem para medições experimentais.

Palavas-chave: Dutos flexíveis, tensões residuais, Método do Furo Cego, plasticidade.

RESUMO EXPANDIDO

Introdução

Irregularidades no material, como um furo, geram concentração de tensões. Quando o componente é submetido a um elevado nível de tensão residual, a concentração de tensões promovida nas proximidades do furo cego induz não apenas deformações elásticas, mas também deformações plásticas. Neste caso, exceder o limite elástico pode resultar em maiores valores de tensão computados ao empregar o modelo elástico do Método do Furo Cego. As deformações plásticas mascaram o valor real das tensões, geralmente superestimando as tensões e atingindo, em algumas ocasiões, tensões maiores que a tensão de escoamento do próprio material. Certos níveis de deformações plásticas podem ser tolerados, mas deformações excessivas geram desvios consideráveis nos resultados de tensão.

Existem algumas metodologias disponíveis para corrigir as medições de tensão residual levando em consideração o efeito da plasticidade no método uniforme. Para o método não uniforme, as correções disponíveis são limitadas ao estado de tensão equibiaxial. Neste trabalho, é proposta uma metodologia para medir altos níveis de tensões residuais uniformes com o auxílio de um dispositivo óptico. O sistema de medição equipado com dispositivo óptico fornecerá dados de toda a região ao redor do furo (*full-field*) para alimentar o modelo. A grande quantidade de dados obtidos com técnicas ópticas fornece dados de deslocamento para diferentes raios e permite a avaliação completa de tensões ao medir altos níveis de tensões residuais envolvendo deslocamentos não lineares ao redor do furo.

Objetivos

O objetivo geral desta tese é desenvolver, validar e aplicar uma metodologia computacional para medir níveis elevados de tensão residual considerando os dados/deslocamentos adquiridos ao redor do furo (*full-field*). Alguns objetivos específicos foram estabelecidos para atingir este objetivo geral:

Identificar e testar as condições de contorno mais adequadas para trabalhar com modelos elastoplásticos. Indicar os desvios que podem ser gerados por eventuais considerações feitas de acordo com as condições de contorno adotadas.

Especificar as dimensões do modelo numérico de acordo com parâmetros associados ao Método do Furo Cego (MFC) como, por exemplo, deformação ou tensão na superfície ao redor do furo.

Processar dados experimentais de deslocamento obtidos ao redor do furo para obter uma assinatura mecânica com os valores de deslocamento. Obter uma metodologia para calcular um valor equivalente da assinatura mecânica para os dados numéricos.

Desenvolver uma metodologia para medir tensões residuais uniformes em componentes sob tensão residual próxima à tensão de escoamento considerando os dados obtidos por técnicas ópticas.

Validar numericamente as estratégias e também experimentalmente com o auxílio de um dispositivo de carregamento para induzir altos níveis de tensão residual:

- Avaliação preliminar em um dispositivo de carregamento manual;
- Testes finais em corpo de prova de geometria confiável e máquina de tração robusta.

Metodologia

A metodologia foi dividida em alguns capítulos para melhor detalhar as etapas de implementação do algoritmo iterativo e suas modificações. Por esse motivo, o desenvolvimento desta tese foi dividido em alguns capítulos com os seguintes temas:

- Modelo Elastoplástico de Elementos Finitos: Nesta seção, o comportamento do modelo elastoplástico de elementos finitos é testado para garantir que ele se comporte de acordo com a curva de tensão-deformação.

- Definição das Dimensões do Modelo de Elementos Finitos: este capítulo abrange a avaliação das dimensões do modelo de elementos finitos de acordo com os parâmetros mais empregados no Método do Furo Cego, como deformação e tensão.

- Abordagem Iterativa: neste capítulo, o algoritmo iterativo é apresentado em detalhes, juntamente com as modificações e as etapas de processamento para: a) dados experimentais adquiridos com o dispositivo óptico e b) os dados de validação numérica obtidos com o modelo de elementos finitos.

- Planejamento experimental: este capítulo descreve o planejamento experimental empregado para realizar as medições de tensão residual. É dividido em duas partes: 1) pré-testes realizados em um dispositivo de carga e 2) os testes finais realizados na máquina de tração.

Resultados e discussões

Os resultados e as discussões estão distribuídas em dois capítulos: capítulo de resultados numéricos e capítulo de resultados experimentais. Com relação aos resultados numéricos, primeiro é apresentada uma rápida visão geral do cálculo da derivada parcial do deslocamento radial em relação ao raio como função de θ , juntamente com a análise do espaçamento entre os pontos da malha. Em seguida, é apresentada uma validação numérica completa do algoritmo iterativo.

Na sequência, são apresentados os resultados experimentais. A seção de resultados experimentais é dividida em duas subseções: pré-teste e testes. O pré-teste se refere à análise preliminar realizada em um dispositivo de carregamento manual, enquanto a seção “testes” resume os resultados e discussões com base em medições de tensão residual mais robustas, tanto em termos de equipamento, quanto de corpo de prova. Neste último caso, eles foram realizados com o auxílio de uma máquina de tração para induzir elevados níveis de tensão. Os testes e pré-testes foram utilizados na avaliação do algoritmo iterativo.

A capacidade de convergência do algoritmo foi testada para regiões anulares com diferentes pares de diâmetros internos e externos. A região mais próxima do furo (caso “próximo”) apresentou uma boa capacidade de correção, enquanto a região denominada como caso “intermediário” teve um bom desempenho na maioria dos testes, e a região mais distante do furo (caso “distante”) não teve um bom desempenho e não foi capaz de corrigir os efeitos da plasticidade.

Conclusões

Foi desenvolvida uma metodologia para corrigir desvios gerados pelo efeito da plasticidade em medições de tensão residual sob altos níveis de tensões. Toda a área de medição ao redor do furo (*full-field*) foi levada em consideração. Testes experimentais com altos níveis de tensão ($f \approx 0,9$) foram realizados para validar a metodologia.

As validações numéricas realizadas indicam que, além do caso “próximo”, tanto o caso “intermediário” quanto o caso “distante” possuem potencial para corrigir os efeitos da plasticidade. No entanto, replicá-los com resultados experimentais apresentou alguns desafios. Melhorias no dispositivo óptico para reduzir a perda de correlação poderiam eventualmente melhorar o método e estender sua aplicabilidade ao caso “distante”.

LIST OF FIGURES

Figure 1-1 – (a) Typical installation of flexible risers connecting the platform to the seabed and (b) cross-section of a flexible riser.	22
Figure 2-1 – Cross-section of a glass plate illustrating the presence of residual stresses without any external load.	25
Figure 2-2 – Illustration of residual stress model with comments for manufacturing processes of steels.	26
Figure 2-3 – Workpiece under uniaxial loading before and after the hole-drilling procedure.	27
Figure 2-4 – Strain gauge rosettes employed in the HDM.	28
Figure 2-5 – Representation of a single grid of a strain gauge.	28
Figure 2-6 – ESPI data acquired before and after the drilling (Specklegrams) and the fringe pattern obtained after processing.	30
Figure 2-7 – Full-field DSPI portable device for measuring residual stresses.	30
Figure 2-8 – Residual stress profile: (a) uniform and (b) non-uniform.	31
Figure 2-9 – Physical meaning of the calibration coefficients.	34
Figure 2-10 – Sequence commonly employed in FE software.	35
Figure 2-11 – Superposition principle employed to find the stress relaxation: a) stress state before the hole drilling, b) stress relaxation after the drilling, and c) stress state after the drilling. d) application of loads at a specific depth.	36
Figure 2-12 – Stress-strain curve for a bilinear hardening model.	38
Figure 2-13 – Finite element model a) 3D and b) 2D.	40
Figure 2-14 – Iterative algorithm to obviate the plasticity effects on the measurement of residual stress values.	43
Figure 2-15 – Flowchart of the ANN.	47
Figure 2-16 – Different types of loading devices.	48
Figure 2-17 – Tensile testing machine employed to induce a nearly uniaxial stress state.	49
Figure 2-18 – Stress-strain curves presented by carbon steels.	50
Figure 3-1 – Iterative algorithm with proposed modifications in red.	53
Figure 3-2 – Representation of the non-uniform and uniform methods.	53
Figure 4-1 – (a) 1/8-cylinder model, (b) discretization, (c) boundary conditions, and (d) application of the displacement.	56
Figure 4-2 - Stress-strain curve (left) and amplification/detail “a” (right).	57
Figure -4-3 – Strain profile around the hole for different element sizes. The profile was obtained by rotating a single strain gauge around the hole.	58
Figure 4-4 – (left) Full hexahedral mesh and (right) hexahedral mesh with tetrahedral mesh near the boundaries.	59
Figure 4-5 – Strain profile obtained around the hole for different interpolating strategies and model shape.	60
Figure 4-6 – Comparison of the strain profile around a through hole: a) current model and b) literature.	61
Figure 4-7 – Stress transformation for the equibiaxial stress state.	62
Figure 4-8 – Stress transformation for the uniaxial stress state.	62
Figure 4-9 – Stress transformation for the pure shear stress state.	63
Figure 4-10 – Methodology employed to apply the load on the hole wall.	64
Figure 4-11 – Strain profile around the hole for two load application strategies considering the uniaxial case.	66
Figure 4-12 – Strain profile around the hole for two load application strategies considering the equibiaxial case.	66

Figure 4-13 – Strain profile around the hole for two load application strategies considering the pure shear case.....	67
Figure 4-14 – Extension of the plastic strains around the hole for two different cases: (left) application of the load directly at the hole wall and (right) application of the load at the extreme boundaries of the model.....	68
Figure 5-1 – Strain profile variation X and nonuniform stress profile variation Φ for a) the current model and b) results obtained from the literature.	70
Figure 5-2 – Influence of the model geometry on the final residual stress values. Where, for example, "Th6" means thickness of 6 mm and "10x10" stands for FEM model size of 10 mm x 10 mm. SymZ stands for the model with a symmetric boundary condition applied at $Z=0$	72
Figure 5-3 – Maximum and minimum stresses for five different thicknesses ($f = 1$). Where, for example, "Th6" means thickness of 6 mm and "20x20" stands for FEM model size of 20 mm x 20 mm. SymZ stands for the model with a symmetric boundary condition applied at $Z=0$	72
Figure 5-4 – Nonuniform stress profile variation Φ for the current model ($f = 0.96$) together with the results obtained in the literature.....	73
Figure 6-1 - Modifications to the iterative algorithm.	75
Figure 6-2 (a) Representation of the annular area divided into sectors evenly distributed around the hole, a few points were captured along the radius with an interval of $\Delta r = 0.05$ mm, (b) partial derivative of the radial displacement with respect to the radius (inclination) for a single value of θ and (c) lines adjusted representing the radial displacement for each sector producing the complete profile.....	77
Figure 6-3 – Diagram of the modified iterative algorithm.	79
Figure 7-1 – Schematics of the trials planned to be performed in this work.....	80
Figure 7-2 – Long dog-bone-shaped steel specimen instrumented with 10 strain gauges.	81
Figure 7-3 – Loading device employed to induce a uniform residual stress profile.	81
Figure 7-4 - True stress vs strain curve for three aluminum specimens.....	82
Figure 7-5 - True stress vs strain curve for four 3/8" steel specimens. Curve "Test" was obtained without the use of a clip gauge.	84
Figure 7-6 – Specimen cross-section for thicknesses of (a) 9.5 mm (3/8 in) and (b) 15.9 mm (5/8 in).	85
Figure 7-7 – Specimens for residual stress measurement.....	86
Figure 7-8 – Simulation of the experimental setting within the frame of the tensile machine.	87
Figure 7-9 – Experimental setting within the tensile machine.	87
Figure 8-1 – Nodes located within the inner and outer annular area (near case) together with the marginal nodes located near the inner and outer limits. Annulus diameter: near ($R_1 = 1.5 \cdot RF$; $R_2 = 3 \cdot RF$; $R_x = 2.7 \cdot RF$).	89
Figure 8-2 – Nodes located within the inner (red dots) and outer (black dots) annular area (intermediate). The circle with the smallest diameter illustrates the approximate dimensions of the hole. Annulus diameter: medium ($R_1 = 2 \cdot RF$; $R_2 = 4 \cdot RF$; $R_x = 3.6 \cdot RF$).	90
Figure 8-3 – Nodes located within and nearby the inner and outer annular area (near case) employed in calculating the partial derivative. The black lines represent the approximate displacement profile for the annular region.....	91
Figure 8-4 Residual stress computation for different grid (node) spacing following the strain gauge formalism for the optical technique. The term 'Ref' represents the reference stress values, 'Map' the stresses computed considering the mapped grid, 'FE' the original mesh from the finite element model, and 'SG' the stress computation according to the ASTM standard procedures (Type B).....	92

Figure 8-5 – Nodes located within and nearby the inner and outer annular area (near case) employed in the calculation of the partial derivative. Straight black lines represent the approximate displacement profile for the annular region.	93
Figure 8-6 – Comparison between the inclination of the radial displacements (Near case) or partial derivative of the radial displacement with respect to the radius as a function of theta and the strains measured with strain gauges Type B.	94
Figure 8-7 - Partial derivative of the radial displacement with respect to the radius as a function of theta for two different fitting strategies: connecting the first and last points and linear lest-square fitting method considering every point on the line.	94
Figure 8-8 - Iterations required by the near case to achieve full convergence for depths of 0.1, 0.2, 0.3, and 1.0 mm. The expected value is represented by ‘Ref’.....	95
Figure 8-9 - Stresses in the x and y direction for depth 0.3 mm, indicating convergence issue and their respective average for the last two points for the intermediate (Int. avg) and far (Far avg) cases, together with the expected (reference) values for the simulation.....	96
Figure 8-10 - Partial derivative of the radial displacement with respect to the radius as a function of the angle theta for the data collected within the annular areas far, medium, and near, as well as two different hole depths of 0.3 mm and 1.0 mm. Annulus diameter: near ($R1=1.5*RF$; $R2=3*RF$; $Rx=2.7*RF$), medium ($R1=2*RF$; $R2=4*RF$; $Rx=3.6*RF$), and far ($R1=2.5*RF$; $R2=5*RF$; $Rx=4.5*RF$).	98
Figure 8-11 - Partial derivative of the radial displacement with respect to the radius as a function of the angle theta obtained with the numerical model for each iteration performed by the iterative algorithm for the a) near, b) intermediate, and c) far cases.	99
Figure 9-1 – Near case: radial displacement around the hole together with the partial derivative of the radial displacement with respect to the radius.	102
Figure 9-2 – Intermediate case: radial displacement around the hole together with the partial derivative of the radial displacement with respect to the radius.	103
Figure 9-3 – Far case: radial displacement around the hole together with the partial derivative of the radial displacement with respect to the radius.	103
Figure 9-4 - Partial derivative of the radial displacement with respect to the radius as a function of theta.	104
Figure 9-5 – Convergence process of the iterative algorithm for the near, intermediate, and far cases (depth of 1 mm). UB stands for Upper Bound and LB for Lower Bound of the reference value uncertainty. The first iteration indicates the initial guess computed by the iterative algorithm.	104
Figure 9-6 – Case: Medium, Iteration: 2 nd . Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.....	105
Figure 9-7 – Case: Medium, Iteration: 3 rd . Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.....	105
Figure 9-8 – Case: Medium, Iteration: 4 th . Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.....	106
Figure 9-9 – Case: Medium, Iteration: 5 th . Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.....	106
Figure 9-10 – Case: Medium, Iteration: 6 th . Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data, and its rounded slope “Inclination”.	107

Figure 9-11 – Case: Near, Depth= 1 mm, T1 stands for first test, T2 for the second test, and T3 for the third test. Partial derivative of the radial displacement with respect to the radius as a function of theta for a) the traditional and b) the incremental method.	108
Figure 9-12 – Iterative process obtained for experiments T1 (red), T2 (black), and T3 (blue). Annular regions: near, medium, and far. The dotted lines represent the upper and lower bounds and the expected values (Ref.).	109
Figure 9-13 – Iteration behavior for two initial guesses multiplier: 0.80 and 0.95.	109
Figure 9-14 – Phase difference for the first increment of experiment T1 indicating a) the center of the area sensitive to radial displacements, annular regions: b) near, c) medium, and d) far.	110
Figure 9-15 – Radial displacements and partial derivative of the radial displacement with respect to the radius. x- and y-axis are not equal for different annulus radii.	111

LIST OF TABLES

Table 2-1 – Summary of works and ANN input variables (features) employed in the HDM.	46
Table 3-1 – Comparison between the two strategies.	52
Table 4-1 – Strain variation for hexahedral and tetrahedral mesh.	60
Table 4-2 - Comparison between two strategies of load application for the uniaxial, equibiaxial, and pure shear cases ($D_0=1.8\text{mm}$, $h=1\text{mm}$).	65
Table 8-1 Iterative algorithm stress values output for the depth of 1 mm. Reference values: $\sigma_x=1027.8\text{ MPa}$ $\sigma_y=0\text{ MPa}$	99
Table 9-1 – Summary of the algorithm's performance for the pre-test and final tests for each annulus diameter: near ($R1=1.5*RF$; $R2=3*RF$; $R_x=2.7*RF$), medium ($R1=2*RF$; $R2=4*RF$; $R_x=3.6*RF$), and far ($R1=2.5*RF$; $R2=5*RF$; $R_x=4.5*RF$).	112

LIST OF ABBREVIATIONS

APDL	Ansys Parametric Design Language
ANN	Artificial Neural Network
ASTM	American Society for Testing and Materials
DIC	Digital Image Correlation
DM	Drilling Module
CNPQ	National Council for Scientific and Technological Development
ESPI	Electronic Speckle Pattern Interferometry
FE	Finite element
FEM	Finite element method
HDM	Hole-Drilling Method
IM	Integral Method
LABMETRO	Laboratório de Metrologia e Automatização
MFC	Método do Furo Cego
MM	Measurement Module
MTRES	Medição de Tensões Residuais
PETROBRAS	Brazilian Petroleum Corporation
POSMEC	Graduate Program in Mechanical Engineering
RS	Residual Stress
SB	Supporting Base
TRRiFlex	Residual Stress in Flexible Risers
UFSC	Universidade Federal de Santa Catarina

LIST OF SYMBOLS

Capital Letters

B	Clockwise angle from the x-axis to σ_1 (maximum principal stress direction) [°]
C_h	Fourier terms [-]
E	Young's Modulus [GPa]
K^{ep}	Tangent modulus [GPa]
P, Q, T	Combination stresses [MPa]
W	Bivariate polynomial variable [-]

Lowercase Letters

$\bar{a}, \bar{b}$	Calibration constants [-]
c	Tri-diagonal 'second derivative matrix' [-]
f	Dimensionless loading intensity factor or Beghini's factor [-]
f_{el}	Elastic loading intensity factor [-]
j	Number of hole depth increments performed [-]
k	Sequence number for the hole depth increment [-]
m	Dimensionless loading factor [-]
Δx	Length of the grid [mm]
p, q, t	Combination strains [$\mu\text{m}/\text{m}$]
u_r	Radial displacement [mm]
u_θ	Tangential displacement [mm]
u_{x1}, u_{x2}	Displacements at the ends of the grid [mm]
w	Width of the grid [mm]

Greek Letters

α_P	Regularization factor for P stresses [-]
α_Q	Regularization factor for Q stresses [-]
α_T	Regularization factor for T stresses [-]
β	Angle between $\sigma_{\max}$ and the strain gauge ε_1 [°]
γ	Angle between strain gauge 3 and σ_x [°]
ε	Total strain [$\mu\text{m/m}$]
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Strains measured with strain gages [-]
ε_e	Elastic strain [$\mu\text{m/m}$]
ε_p	Plastic strain [$\mu\text{m/m}$]
θ	Angle between the strain gauge (or any other point) and the x-axis [°]
μ	Bivariate polynomial variable [-]
σ	Stress [MPa]
σ_0	Stress profile coefficient [MPa]
$\sigma_{10\%}$	True stress at a true plastic strain of 10% [MPa]
σ_{PD}	Predefined stress profile [MPa]
σ_{IM}	Stress profile obtained with the integral method [MPa]
σ_{eqv}	Von mises equivalent stress [MPa]
$\sigma_{\max}$	Maximum principal stress [MPa]
$\sigma_{\min}$	Minimum principal stress [MPa]
τ	Shear stress [MPa]
ν	Poisson's ratio [-]
Φ_1, Φ_2	Local angles [°]
Ω	Stress ratio [-]

CONTENTS

1 INTRODUCTION.....	22
1.1 OBJECTIVES.....	24
1.1.1 General objective.....	24
1.1.2 Research steps	24
2 STATE OF THE ART	25
2.1 RESIDUAL STRESS	25
2.2 THE HOLE-DRILLING METHOD.....	27
2.2.1 Optical device for measuring residual stress.....	29
2.2.2 Stress computation	31
2.3 FINITE ELEMENT METHOD	35
2.3.1 Acquisition of calibration coefficients (Elastic regime)	36
2.3.2 Plasticity.....	37
2.3.3 Boundary conditions employed in the HDM	39
2.4 STRATEGIES FOR MEASURING HIGH RESIDUAL STRESSES	40
2.4.1 The iterative procedure developed by Vangi.....	42
2.5 ARTIFICIAL NEURAL NETWORKS.....	45
2.5.1 Neural Network in the HDM.....	46
2.6 VALIDATION METHODS EMPLOYED IN THE HDM	47
3 METHODOLOGY.....	51
3.1 RESIDUAL STRESS IN FLEXIBLE RISERS.....	51
3.2 ITERATIVE ALGORITHM	52
3.2.1 Modification of an existing algorithm	52
3.3 General overview of the methodology	53
4 ELASTOPLASTIC FINITE ELEMENT MODEL	55
4.1 STRESS-STRAIN CURVE.....	55
4.2 ANALYSIS OF THE MESH TYPE.....	57
4.2.1 Tetra vs hexa mesh.....	59
4.3 VALIDATION OF THE ELASTOPLASTIC MODEL.....	61
4.4 LOADING STRATEGIES: HOLE SURFACE OR EXTREME BOUNDARIES	61
4.4.1 Evaluation of the load application strategies.....	64
5 REFINING THE FINITE ELEMENT MODEL	70

5.1	EVALUATION OF THE GEOMETRY EFFECTS CONSIDERING PLASTICITY	71
5.1.1	Validation of the non-uniform profile	73
6	ITERATIVE APPROACH	75
6.1	MODIFIED ITERATIVE ALGORITHM	75
7	EXPERIMENTAL PLANNING	80
7.1	Pre-test.....	80
7.2	FINAL TESTS	83
8	NUMERICAL RESULTS AND DISCUSSIONS.....	89
8.1	COMPUTATION OF THE PARTIAL DERIVATIVE.....	89
8.2	NUMERICAL VALIDATION OF THE ITERATIVE ALGORITHM	95
9	EXPERIMENTAL RESULTS AND DISCUSSIONS	102
9.1	PRE-TEST.....	102
9.2	TESTS.....	10
7		
10	CONCLUSIONS.....	113
11	FUTURE WORK SUGGESTIONS	115
	REFERENCES	116
	APPENDIX A - EVALUATION OF A 2D MODEL CONSIDERING PLASTICITY..	120
	APPENDIX B - INTRODUCTION OF A RESIDUAL STRESS PROFILE WITH MECHANICAL APDL	127
	APPENDIX C - TEMPERATURE GRADIENT.....	133
	APPENDIX D – DOG-BONE-SHAPED SPECIMEN LOADING	136
	APPENDIX E – CONVERGENCE BEHAVIOR WITH MOVING AVERAGE.....	140

1 INTRODUCTION

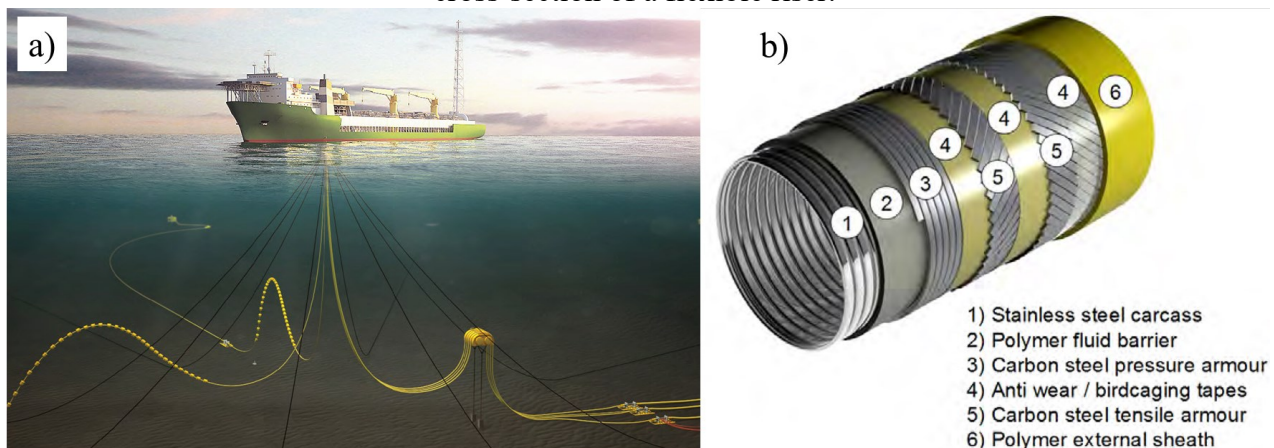
Residual stresses are present in almost any mechanical component. These stresses are generated by manufacturing processes and may change during service life. Additionally, the residual stresses are combined with service loads which can result in the increase of the active stress in the component. These stresses can be measured with experimental techniques.

The Brazilian Petroleum Corporation (PETROBRAS) has a long-term cooperation with the Metrology and Automatization Laboratory (LABMETRO) to develop new technologies related to the experimental measurement of residual stresses. PETROBRAS currently uses the Hole-Drilling Method to perform residual stress measurements in pipelines located on land. The technique is employed to evaluate the stress state of these pipelines. Reliable techniques to measure residual stress for an attractive price, easy operation, and execution in field become suitable to industrial applications. Indeed, it is indispensable for the midstream businesses focused on the transportation of oil, natural gas, and derivatives.

The most recent cooperation established between PETROBRAS and LABMETRO aimed to investigate issues related to the failure of flexible risers. More specifically, the laboratory was in charge of the development of techniques to evaluate the residual stresses in the wires of flexible risers.

Connecting the seabed to the floating platform is a critical task in offshore structures. Petroleum and service fluids are transferred with the aid of risers. Risers are a special type of pipe positioned vertically or near the vertical portion [1]. They connect the facilities above the water to the seabed and can be either rigid or flexible. The use of flexible risers is mandatory in pre-salt fields located in deep water to bring flexibility to projects and reduce the time to first oil. A typical installation of flexible risers is shown in Figure 1-1a. This environment requires pipes with low bending resistance in comparison to their axial stiffness. This characteristic is promoted by its numerous layers [2]. The main layers are presented in the cross-section of the flexible riser shown in Figure 1-1b.

Figure 1-1 – (a) Typical installation of flexible risers connecting the platform to the seabed and (b) cross-section of a flexible riser.



Source: Fergstad and Lotveit [3].

Risers are exposed to cyclic loads, including tensile, bending, and internal stress variation. This increases attention to the operational safety regarding the fatigue life. Besides, the extreme metal forming employed to produce the wires of flexible risers negatively impacts the residual stress levels. Reducing the levels of unwanted residual stresses at the final product contributes positively to the durability of the flexible riser. Understanding the introduction of residual stresses during the manufacturing process of flexible risers was the main goal of the cooperation between PETROBRAS and LABMETRO.

The wires undergo a series of manufacturing processes and, therefore, it is not straightforward to identify the residual stress levels. They are helically wound around the riser, which contributes to increasing the levels of stresses present in this component. Besides, initial tests indicated that the residual stress state in such wires can be close to the residual stress levels allowed to be measured with the HDM. The HDM consists in the measurement of strains or displacements near a small hole drilled in increments. The stress relief around the hole results in deformations. The strains around the hole are usually measured with strain gauges and then the stresses at the location of the hole are computed based on the elastic assumption of the material [4].

Irregularities in the material, such as a hole, naturally cause a stress concentration. When the component is subjected to a high residual stress level, the stress concentration promoted around the blind hole induces plastic deformations. In this case, exceeding the elastic limit can result in higher computed stress values when employing the hole-drilling elastic model. The plastic deformation masks the real value of the stresses, usually overestimating the stresses reaching, on some occasions, stresses higher than the yield stress [5]. Certain levels of plastic strains can be tolerated, but excessive strains can generate considerable deviations in the stress results.

There are a few methodologies available to correct the residual stress measurements taking into account the plasticity effect in the uniform method. For the nonuniform method, the corrections available are limited to the equibiaxial stress state and were obtained taking into account the questionable premise of the Superposition Principle. In this work, a methodology is proposed to measure high levels of uniform residual stresses with the aid of an optical device. The measurement system equipped with an optical device provides full-field data to feed the model. The huge amount of data obtained with optical techniques gives displacement data at different radius locations and allows the complete evaluation of strains when measuring high levels of residual stresses involving nonlinear displacements around the hole.

1.1 OBJECTIVES

In this section, the general and specific objectives will be presented in detail.

1.1.1 General objective

The objective of this thesis is to develop, validate, and apply a computational methodology to measure high levels of residual stress taking into account the full-field data acquired around the hole.

1.1.2 Research steps

A few research steps were established to achieve the general objective:

Identify and test the most appropriate boundary conditions for working with elastoplastic analysis in finite element simulations. Indicate the deviations that can be generated by eventual considerations made according to the adopted boundary conditions.

Specify the dimensions of the numerical model according to HDM parameters such as strain or stress values.

Process experimental full-field data to obtain a mechanical signature with the displacement values. Obtain a methodology to compute an equivalent value of the mechanical signature for the numerical data.

Develop a methodology to measure uniform residual stresses in components under residual stress near the yield stress considering the full-field data captured by optical techniques.

Validate the strategies numerically and also experimentally with the aid of a loading device to induce high levels of residual stress:

- Preliminary evaluation on a manual loading device with high availability;
- Final tests with a reliable workpiece geometry and robust material testing machine.

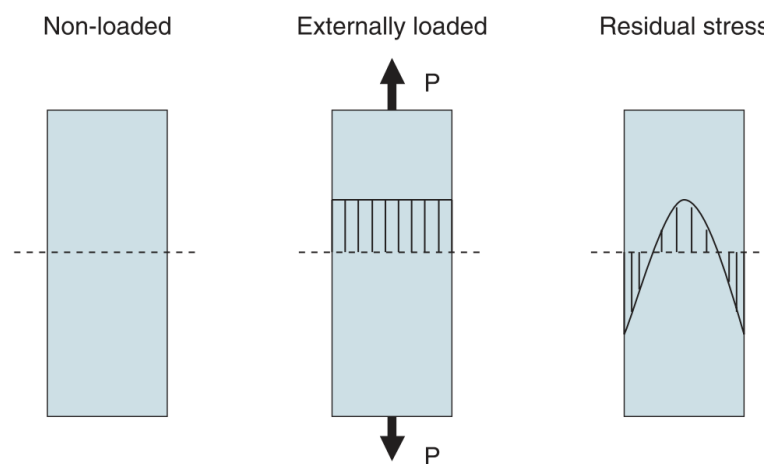
2 STATE OF THE ART

In this chapter, an overview about residual stress and the fundamentals of the hole drilling method will be presented. The limitations regarding the measurement of high levels of residual stresses, together with strategies employed to obtain reliable numerical simulations as well as strategies used to reduce the deviations of the final residual stress values will also be presented.

2.1 RESIDUAL STRESS

Residual stresses represent the stress state of a component when there is no other external load acting on the material. The expression ‘residual stress’ refers to the creation of this type of stress as a residue from a previous processing. The nature of self-equilibrated residual stresses is shown in Figure 2-1.

Figure 2-1 – Cross-section of a glass plate illustrating the presence of residual stresses without any external load.



Source: Schajer [6].

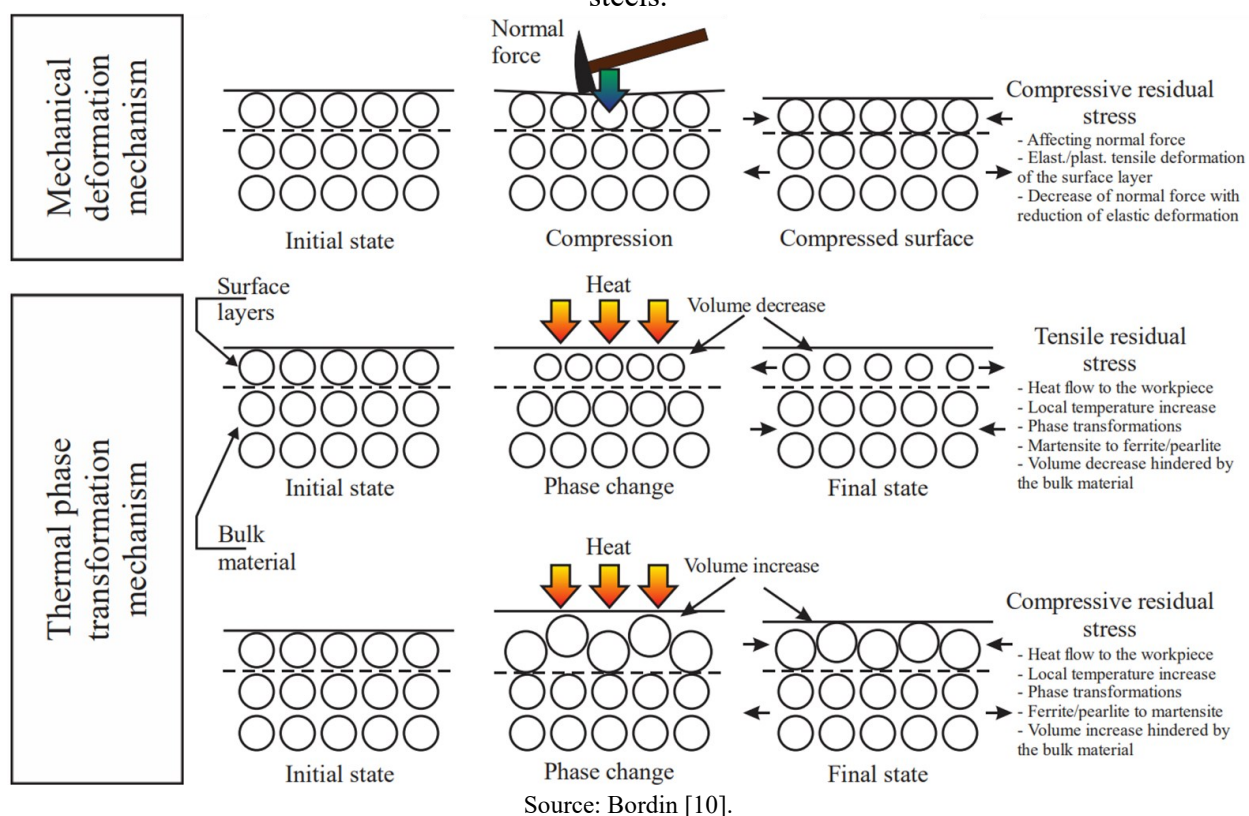
A variety of manufacturing methods can generate residual stresses, such as forming, surface machining, surface treatments, and welding. Certain levels of residual stresses are present in mechanical components and larger structures. These stresses, when combined with service loads, may result in the rupture of the component. Taking residual stresses into account when estimating total stresses can, in some cases, result in a significant difference in the real stress value applied to the component.

Design in engineering is trending toward lighter and less conservative structures. Smaller safety factors naturally increase the importance of taking into account the residual stresses [7]. In this way, it is possible to design safe structures even with less conservative safety factors. However, it is difficult to predict the residual stress level of a sample. To do that, the complete history from the production of the raw material up to the final component in service should be known. Besides, the

residual stresses can evolve during the in-service life of the component [8]. Consequently, the most reliable method for determining residual stresses is through experimental methods [9].

Residual stresses are created by the following mechanisms: 1) non-uniform plastic deformation which occurs in manufacturing processes such as forging, rolling, bending, drawing, and extrusion; 2) surface modification which also occurs in manufacturing processes such as machining and peening, as well as during service by corrosion or oxidation; 3) material phase and/or density changes, which usually occurs associated with large thermal gradients in manufacturing processes such as welding, casting and phase transformation [6]. These mechanisms are simplistically illustrated in Figure 2-2, with resulting residual stresses varying from compressive to tensile. The mechanical and thermal phase mechanisms can act at the same time, depending on the manufacturing process.

Figure 2-2 – Illustration of residual stress model with comments for manufacturing processes of steels.



Residual stresses are usually divided into three categories: Type I, II, and III. The categories are divided according to the length necessary to equilibrate the stress. Residual stress type I may extend to a length above 1mm and is commonly present in manufactured components. Type II are microscopic residual stresses with a length within the micrometer range. It can act, for instance, at the grain interface of the metal. Residual stresses from Type II can be generated by Type I stresses. Type III are residual stresses on the atomic scale around the displacement of crystalline planes.

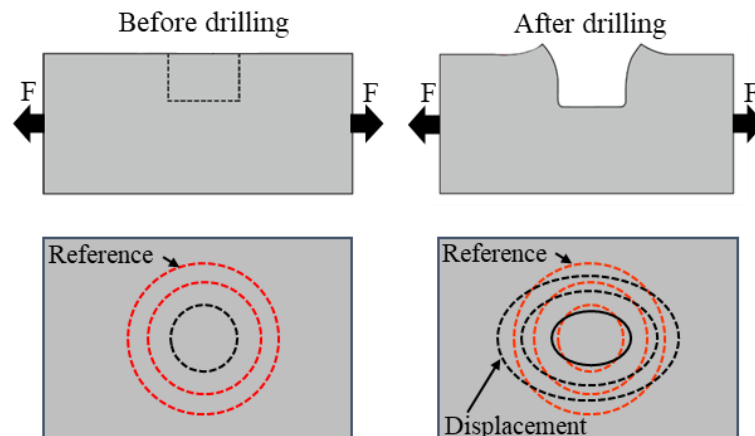
There are different techniques for measuring residual stresses. The two most common are the mechanical relaxation techniques and diffraction methods. For instance, the HDM, the slitting

method, and contouring are classified as mechanical relaxation techniques. The HDM is one of the most employed techniques to measure residual stress [6]. This work focuses on the HDM and, therefore, a general view will be presented in the next section.

2.2 THE HOLE-DRILLING METHOD

The Hole-Drilling Method is a semi-destructive technique with normalized procedures described in the standard ASTM E837 – 20a [11]. The experimental procedure, as well as the computation steps of the residual stress measurement, are described in the standard. This technique consists in the drilling of a hole in a predefined number of steps. It can be either a blind hole or a through-hole. As a consequence, there is a localized stress relief that induces strains around the hole. After the deformation, the component again reaches the equilibrium state. A representation of the workpiece before the experiment and the final equilibrium state for a uniaxial loading case is shown in Figure 2-3. The measurement of the deformation occurs at the surface. After the acquisition of the deformation, the computation of the stress value is performed.

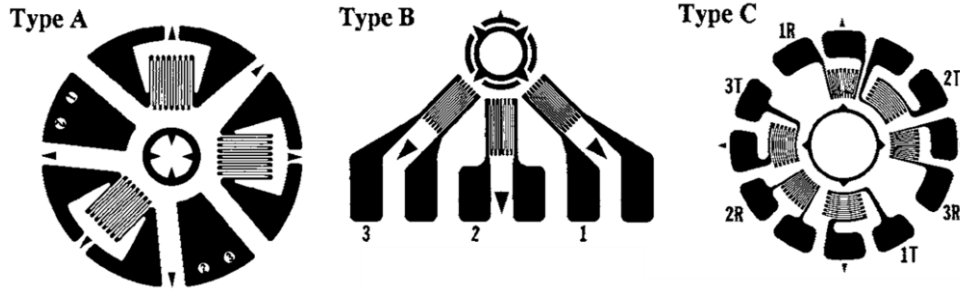
Figure 2-3 – Workpiece under uniaxial loading before and after the hole-drilling procedure.



Source: Blödorn *et al.* [12].

The strains around the hole are commonly measured with strain gauge rosettes. Many types of rosettes can be employed. However, a few of them are standardized rosettes used in the HDM. The standard presents the calibration coefficients for these rosettes. Calibration coefficients are employed to compute the residual stress values [4]. Rosette type C is shown in Figure 2-4, which was developed to achieve higher sensitivity and precision. The improvement came from the increase of the measurement area and the radial and circumferential gauges connected in half-bridge [13].

Figure 2-4 – Strain gauge rosettes employed in the HDM.



Source: ASTM E837 13a [4].

Strain gauges are employed to experimentally measure strains. However, in some occasions it is necessary to acquire strain data from numerical models. Simplifications can be employed considering gages with zero gage length at the location of the mean gage diameter (center of the gage) [14]. However, strain gauges are commonly modeled considering individual grids. The strain measured by each grid can be simplified by considering the displacements of its two ends. In addition, the total strain response of the gage takes into account the contribution of each grid. Consequently, the overall strain response is given by Eq.(2-1) [13, 15].

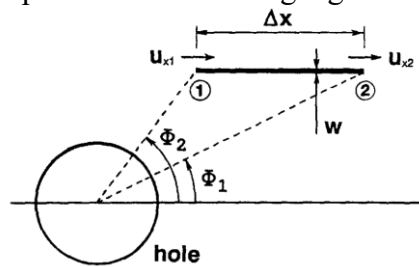
$$\varepsilon = \frac{\sum \frac{(u_{x2} - u_{x1})}{w}}{\sum \frac{\Delta x}{w}} \quad (2-1)$$

$$u_{x1} = u_{r1} \cos \Phi_1 - u_{\theta 1} \sin \Phi_1 \quad (2-2)$$

$$u_{x2} = u_{r2} \cos \Phi_2 - u_{\theta 2} \sin \Phi_2 \quad (2-3)$$

Where the displacements at both ends of the grid are given by u_{x1} in Eq.(2-2) and u_{x2} in Eq.(2-3). u_r and u_θ represent the radial and tangential displacement, respectively. Additionally, Δx represents the length and w the width of the grid. The local angles Φ_1 and Φ_2 are represented in Figure 2-5.

Figure 2-5 – Representation of a single grid of a strain gauge.



Source: Schajer and Tootoonian [13].

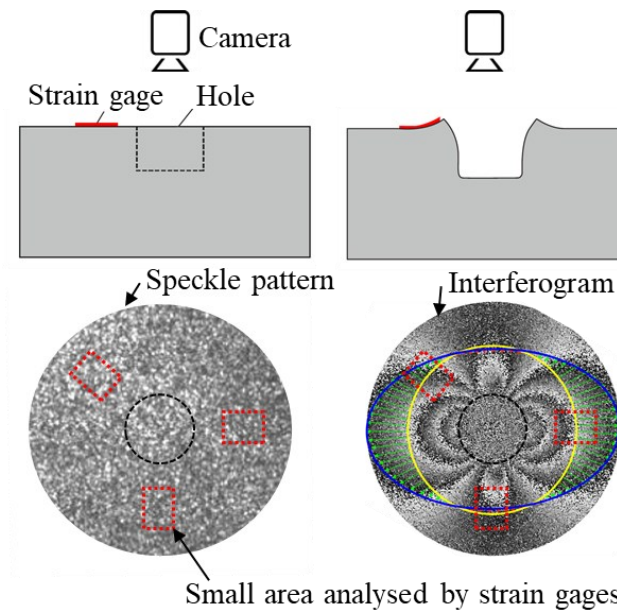
Besides the use of strain gauges, optical techniques are also employed to capture the strains/displacements. In this work, an optical technique will be used to measure the displacements/strains. Therefore, a summary of this method will be presented in the next section.

2.2.1 Optical device for measuring residual stress

The surface deformation generated by the hole-drilling started being measured using optical techniques in the 1980s. These techniques acquire full-field data in a much larger area, generating a huge amount of data. The data can be useful for error checking, data averaging, and allows the analysis of detailed information within the area of measurement. There are a few optical techniques capable of measuring displacement/strains such as Digital Image Correlation (DIC), Moiré interferometry, and Electronic Speckle Pattern Interferometry (ESPI) [7]. These techniques have been used in some institutions, replacing the traditional strain gauge. So far, the ASTM's standard has not presented recommendations regarding optical techniques.

A comparison between the measurement area of a strain gauge rosette and the ESPI technique is shown in Figure 2-6. On the left side, a specklegram is obtained before the hole drilling (reference). Additionally, another specklegram is obtained after the hole drilling procedure. On the right side, a fringe pattern is shown. It is obtained with the aid of image processing techniques. The processing employs both specklegrams acquired before and after the drilling. This fringe pattern represents the surface displacement. This configuration is also sensitive to pure translation, which results in fringes of translation. Translation fringes are undesirable, which pushes the development of these devices toward a rigid design.

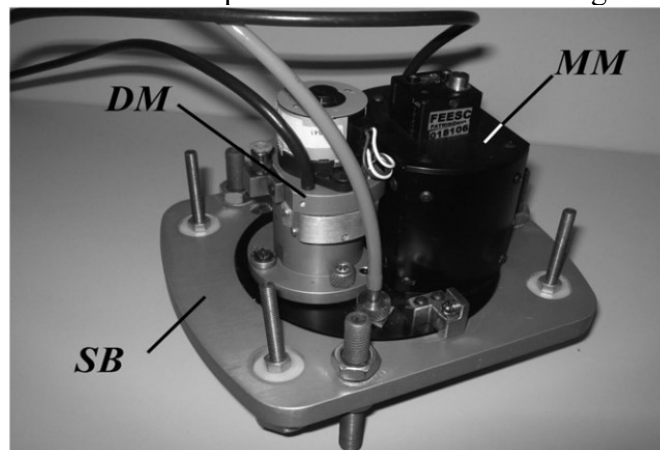
Figure 2-6 – ESPI data acquired before and after the drilling (Specklegrams) and the fringe pattern obtained after processing.



Source: Adapted from Schajer [6] and Albertazzi and Pinto [16].

The standard procedure described in ASTM E837 13a refers to the use of a strain gauge rosette and a drilling device. The devices became more complex with the use of full-field optical techniques to measure the surface displacements [9, 17]. A modular configuration allowed the acquisition of a huge amount of full-field data within a short period of time. The entire measuring procedure, after setup, was completely automated [18]. One example of a compact sensor developed by LABMETRO is presented in Figure 2-7. The device is composed of two modules: a hole-drilling module (DM) and a measurement module (MM). The base (SB) supports both modules and features a high-precision rotation mechanism for the repositioning of the modules.

Figure 2-7 – Full-field DSPI portable device for measuring residual stresses.



Source: Viotti and Albertazzi [18].

The measurement module of this optical device double illuminates the surface with a laser to obtain the interference speckle pattern. The illumination produces a granular pattern with a random distribution of scattered light, which is known as the speckle effect. The random distribution is modified by changes in the surface, such as deformation. The identification of the points before and after the drilling enables the measurement of the deformation [19]. The device is connected to a computer, which obtains the strain values from the full-field data, computes the stresses, and provides the stress results to the user.

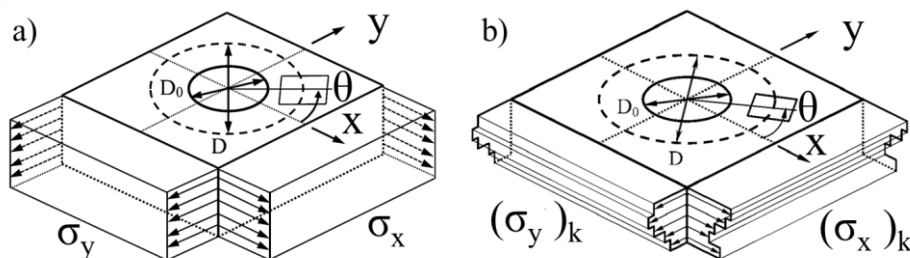
Two different approaches can be employed to compute the residual stresses: initially referenced data or prior referenced data. The initially referenced data technique refers to the computation of the total deformation data. The initially referenced data strategy is traditionally employed with the strain gauge technique. In this case, prior referenced data does not present any improvement in the measurement results. On the other hand, the prior referenced approach computes the incremental deformation data between two increments. It has been shown in the literature that the prior referenced strategy yields better results for optical techniques, especially when random noise is present. It is due to the excess data available in optical techniques, as opposed to the just necessary amount of data provided by the traditional strain gauge technique [20].

An overview of the stress computation methods (uniform and nonuniform) will be presented in the next section.

2.2.2 Stress computation

There are two different computation methods: the uniform (Figure 2-8a) and the non-uniform method (Figure 2-8b). The uniform method was largely employed before the development of the non-uniform method. Basically, the stresses at the hole location are computed from the experimentally measured strains. Mathematical relations are considered in this process based on the theory of elasticity. The uniform method provides an average of the stresses along the depth of the hole. An improvement in the computational procedure resulted in the non-uniform method. This method provides a stress profile with a single stress value for each increment. Both methods are described in the standard ASTM E837 – 13a [4].

Figure 2-8 – Residual stress profile: (a) uniform and (b) non-uniform.



Source: ASTM E837 – 13a [4].

The computation of the uniform method requires the calculation of the combination strains p , q , and t using Eq. (2-4) to (2-6). In this case, ε_1 , ε_2 , and ε_3 represent the strains relieved and measured by each strain gauge during the experimental procedure. Then, it is necessary to obtain the combination stresses P , Q , and T with Eq. (2-7) to (2-9). P , Q , and T represent the uniform equibiaxial stress, uniform 45° shear stress, and the uniform x-y shear stress, respectively. The calibration coefficients $\bar{a}$ and $\bar{b}$ are tabulated in the ASTM standard. They represent the strain relief caused by the removal of material with a unit stress at each increment. These coefficients depend on the rosette type and hole diameter. Additionally, the principal stresses $\sigma_{\max}$ and $\sigma_{\min}$ can be obtained with Eq. (2-10). The angle between $\sigma_{\max}$ and the strain gauge ε_1 is represented by β and can be obtained with Eq. (2-11). Finally, the axial stresses can be obtained with Eq. (2-12)-(2-14).

$$p = \frac{(\varepsilon_3 + \varepsilon_1)}{2} \quad (2-4)$$

$$q = \frac{(\varepsilon_3 - \varepsilon_1)}{2} \quad (2-5)$$

$$t = \frac{(\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)}{2} \quad (2-6)$$

$$Q = -E \frac{\sum(\bar{b} \cdot q)}{\sum(\bar{b}^2)} \quad (2-7)$$

$$P = -\frac{E}{1 + \nu} \frac{\sum(\bar{a} \cdot p)}{\sum(\bar{a}^2)} \quad (2-8)$$

$$T = -E \frac{\sum(\bar{b} \cdot t)}{\sum(\bar{b}^2)} \quad (2-9)$$

$$\sigma_{\max}, \sigma_{\min} = P \pm \sqrt{Q^2 + T^2} \quad (2-10)$$

$$\beta = \frac{1}{2} \text{atan} \left(\frac{-T}{-Q} \right) \quad (2-11)$$

$$\sigma_x = P - Q \quad (2-12)$$

$$\sigma_y = P + Q \quad (2-13)$$

$$\tau_{xy} = T \quad (2-14)$$

The computation of non-uniform stresses through the integral method uses information of strain for each increment. The combination strain is obtained for each increment with the strain values ε_1 , ε_2 , and ε_3 (Eq. (2-15)-(2-17)). The subscript 'j' represents the hole depth increment performed so far. The combination stresses for each hole depth are obtained by solving three matrix equations in Eq. (2-18)-(2-20). The combination stresses $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{T}$ can be represented as a function of the axial stresses, obtained through Eq. (2-21)-(2-23). In this case, the subscript 'k' represents the sequence

number for hole depth increments. Different from the uniform case, these equations refer to vectors and matrices. For instance, Eq. (2-18) for a hole with three increments would resemble Eq. (2-24).

$$p_j = \frac{(\varepsilon_3 + \varepsilon_1)_j}{2} \quad (2-15)$$

$$q_j = \frac{(\varepsilon_3 - \varepsilon_1)_j}{2} \quad (2-16)$$

$$t_j = \frac{(\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)_j}{2} \quad (2-17)$$

$$\bar{a}\mathbf{P} = \frac{E}{1+\nu}\mathbf{p} \quad (2-18)$$

$$\bar{b}\mathbf{Q} = E\mathbf{q} \quad (2-19)$$

$$\bar{b}\mathbf{T} = E\mathbf{t} \quad (2-20)$$

$$P_k = \frac{(\sigma_y)_k + (\sigma_x)_k}{2} \quad (2-21)$$

$$Q_k = \frac{(\sigma_y)_k - (\sigma_x)_k}{2} \quad (2-22)$$

$$T_k = (\tau_{xy})_k \quad (2-23)$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} (\sigma_y)_1 + (\sigma_x)_1 \\ (\sigma_y)_2 + (\sigma_x)_2 \\ (\sigma_y)_3 + (\sigma_x)_3 \end{bmatrix} \frac{1}{2} = \frac{E}{1+\nu} \begin{bmatrix} (\varepsilon_3 + \varepsilon_1)_1 \\ (\varepsilon_3 + \varepsilon_1)_2 \\ (\varepsilon_3 + \varepsilon_1)_3 \end{bmatrix} \frac{1}{2} \quad (2-24)$$

This procedure works well for a small number of increments. For a large number of increments (i.e, 10 or 20 increments), the calibration coefficient matrices become numerically ill-conditioned. Small errors in the measured strains result in large errors in the stress values. The standard recommends the use of the Tikhonov regularization to reduce the errors generated in the calculation process.

The Tikhonov second-derivative regularization is implemented by augmenting Eq. (2-18)-(2-20), resulting in Eq. (2-25)-(2-27).

$$(\bar{a}^T \bar{a} + \alpha_P \mathbf{c}^T \mathbf{c})\mathbf{P} = \frac{E}{1+\nu} \bar{a}^T \mathbf{p} \quad (2-25)$$

$$(\bar{b}^T \bar{b} + \alpha_Q \mathbf{c}^T \mathbf{c})\mathbf{Q} = E \bar{b}^T \mathbf{q} \quad (2-26)$$

$$(\bar{b}^T \bar{b} + \alpha_T \mathbf{c}^T \mathbf{c})\mathbf{T} = E \bar{b}^T \mathbf{t} \quad (2-27)$$

Where matrix $\mathbf{c}$ represents a tri-diagonal “second derivative” matrix with rows filled with [-1 2 -1], except for the first and last rows. The number of rows is equal to the number of hole increments employed. For instance, four increments would require the use of the matrix $\mathbf{c}$ shown in Eq. (2-28). The amount of regularization (smoothing) is controlled by the factors α_P , α_Q , and α_T . These factors have a specific interval recommended by the standard. An iterative algorithm is suggested to update these values and perform the convergence criterion. Finally, the regularized stresses for each increment j are obtained from the combination stresses P , Q , and T .

$$\mathbf{c} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2-28)$$

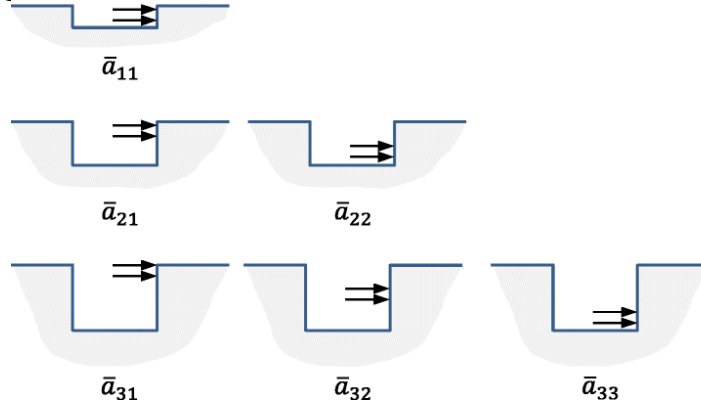
The calibration coefficients are specified in the standard for the three rosette types: A, B, and C. The mathematical representation of the relieved strains around the hole employs the calibration coefficients. The relieved strains at the surface after the drilling operation can be represented by Eq. (2-29) for the uniform method and by Eq. (2-30) for the non-uniform method, considering a hole with three increments.

$$\varepsilon = \frac{1+\nu}{2E} \bar{a}(\sigma_x + \sigma_y) + \frac{1}{2E} (\bar{b}(\sigma_x - \sigma_y) \cos(2\theta) + 2\bar{b}(\tau_{xy}) \sin(2\theta)) \quad (2-29)$$

$$\begin{pmatrix} \varepsilon_1(\theta) \\ \varepsilon_2(\theta) \\ \varepsilon_3(\theta) \end{pmatrix} = \frac{1+\nu}{2E} \begin{bmatrix} \bar{a}_{11} & 0 & 0 \\ \bar{a}_{21} & \bar{a}_{22} & 0 \\ \bar{a}_{31} & \bar{a}_{32} & \bar{a}_{33} \end{bmatrix} \begin{pmatrix} (\sigma_y)_1 + (\sigma_x)_1 \\ (\sigma_y)_2 + (\sigma_x)_2 \\ (\sigma_y)_3 + (\sigma_x)_3 \end{pmatrix} \\ + \frac{1}{2E} \begin{bmatrix} \bar{b}_{11} & 0 & 0 \\ \bar{b}_{21} & \bar{b}_{22} & 0 \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} \end{bmatrix} \left(\begin{pmatrix} (\sigma_y)_1 - (\sigma_x)_1 \\ (\sigma_y)_2 - (\sigma_x)_2 \\ (\sigma_y)_3 - (\sigma_x)_3 \end{pmatrix} \cos(2\theta) + 2 \begin{pmatrix} (\tau_{xy})_1 \\ (\tau_{xy})_2 \\ (\tau_{xy})_3 \end{pmatrix} \sin(2\theta) \right) \quad (2-30)$$

The physical representation of the calibration coefficients can be explained by the representation of the cross-section of the hole. In Figure 2-9, a hole-drilling operation performed in three increments illustrates the physical meaning of the calibration constants $\bar{a}_{jk}$ and $\bar{b}_{jk}$. These constants represent the strain relief in a hole with j increments (total depth) and a unity pressure applied at increment k (incremental).

Figure 2-9 – Physical meaning of the calibration coefficients.



Source: Alegre *et al.* [21].

The calibration coefficients are dimensionless and present little dependence on the material type [4]. It has been recently shown that the calibration data can be represented more compactly. Calibration coefficients can be well represented by fifth-degree polynomials. In this case, only 15 numerical coefficients are sufficient instead of the 231 coefficients employed in the traditional procedure with 20 increments [22].

2.3 FINITE ELEMENT METHOD

The ability to find approximate solutions to real-world problems has made the finite element method (FEM) a useful tool in various fields of study. Its main application started with structural analysis by numerically solving differential equations. Softwares employed nowadays usually have a graphic user interface that facilitates the implementation of the four main steps of a FE analysis: 1) space discretization dividing it into finite elements with the definition of elements employed, the number of nodes and interpolation functions; 2) determination of element equations; 3) combination of these equations into a global matrix together with the boundary conditions; 4) solution of the system of algebraic equations to obtain the numerical results. For the user, these four steps are performed following the sequence shown in Figure 2-10[23].

Figure 2-10 – Sequence commonly employed in FE software.

/PREP7
1. Define the Solid Model Geometry
2. Select the Element Types
3. Define the Material Properties
4. Mesh
/SOLUTION
5. Define the Boundary Conditions
6. Define the Loads
7. Set the Solution Options
8. Solve
/POST1 or /POST26
9. Plot, View, and Export the Results
10. Compare and Verify the Results

Source: Thompson [23].

The first step of an FE analysis impacts the precision of the model, as well as the appropriate choice of the boundary conditions. Finer elements improve the precision and are commonly employed preferentially only in higher gradient regions to reduce the computational time and improve the domain representativeness [24]. A mesh refinement analysis can be performed to identify the influence of the element size as part of the validation process [25]. However, during the initial testing period, coarse meshes can be employed to save time and computational cost [26].

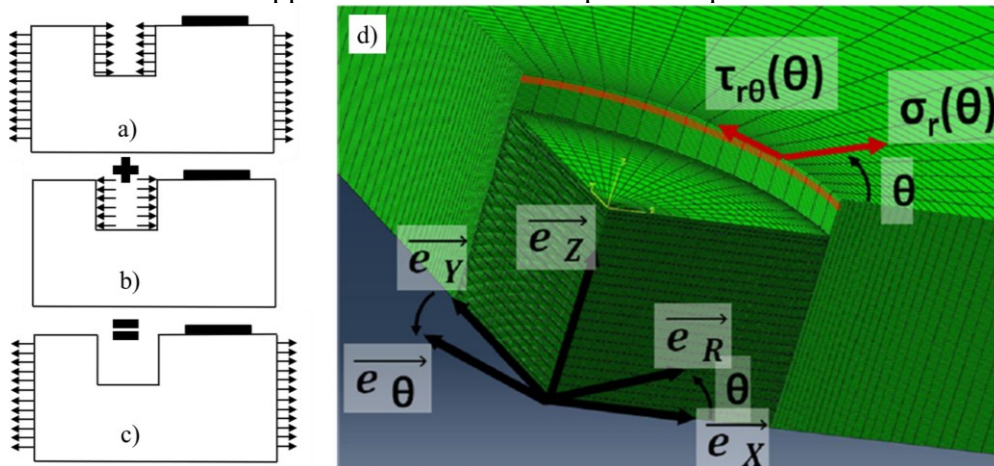
The text-based interface is more attractive for researchers working with the HDM because of the large number of geometry and load variations involved in the calculations. It certainly takes a much longer time to set up, but saves time during the solution and post-processing.

Besides the traditional solid volumes, a huge number of elements are available in commercial software. Elements with 1,2, or 3 dimensions are available. Either bidimensional or tridimensional elements are employed by hole drilling researchers according to their necessity. Another technology became of great importance to the HDM. For full three-dimensional models, applying the loads at the wall requires the use of surface elements in the Ansys Mechanical APDL. Surface elements extend the possibilities for load application, letting the user apply shear loads at the surface [27]. A similar strategy is adopted in the Abaqus software: application of a load named ‘surface traction’ [28]. The ‘surface traction load’ allows the application of a general load freely chosen, which can also be a shear – condition required for the calculation of the calibration coefficients ‘b’.

2.3.1 Acquisition of calibration coefficients (Elastic regime)

There are two possibilities to obtain numerically the strains around the hole: a) applying the stresses at the boundaries of the model and subtracting the model with the hole from the model without the hole, and b) applying the stresses directly at the hole wall considering the superposition principle (Figure 2-11 a-c). This principle considers the assumption of an elastic material. For the non-uniform method, the loads are applied directly at the hole wall due to its complexity.

Figure 2-11 – Superposition principle employed to find the stress relaxation: a) stress state before the hole drilling, b) stress relaxation after the drilling, and c) stress state after the drilling. d) application of loads at a specific depth.



Source: a-c) Zhu and Lanza di Scalea [29] and d) Magnier *et al.* [28].

Two load cases need to be applied to obtain the coefficient matrix **a** and **b**. The calibration matrix for isotropic stresses (**a**) is obtained based on the application of an equi-biaxial stress state. The calibration matrix for 45° shear stresses (**b**) requires the application of a 45° shear case ($\Omega = -1, \sigma_1 = -\sigma_2$). The application of this condition at the boundary of the model is straightforward.

However, the 45° shear case must be carefully applied at the hole wall. This stress state is generated by the application of two distinct loads at the hole wall: a radial load varying with $\cos(2\theta)$ and a tangential load varying with $\sin(2\theta)$, as shown in Figure 2-11c [28]. This stress state is obtained by converting the stresses at the boundaries with the aid of the Mohr circle.

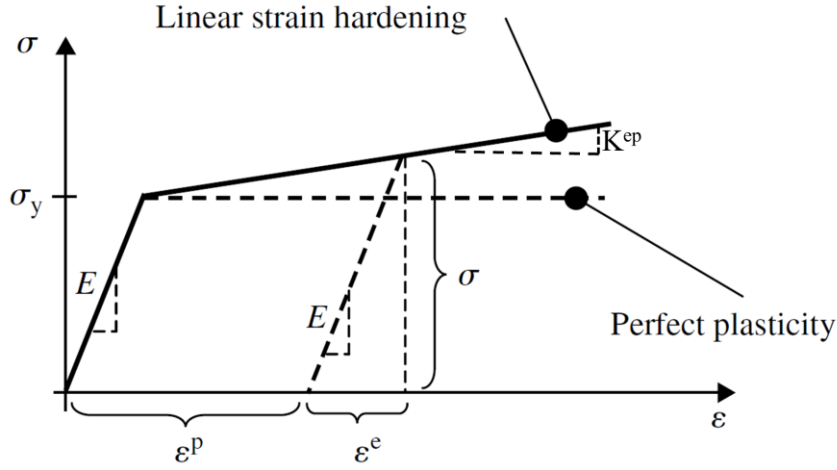
The loads are applied according to the location on the coefficient matrix. Each matrix number corresponds to a specific region where stress is applied at the hole wall. The row subscript (a_{ij}) represents the increment where the load is applied, while the column subscript (a_{ij}) represents the depth of the hole. The displacements are captured at the surface near the rim of the hole. Displacements are usually obtained from FE software and converted to an approximate strain value measured by each grid line of the strain gauge.

2.3.2 Plasticity

In elastoplastic analysis, the stress σ and strain ϵ have a nonlinear relationship. This relationship depends on the history of the strain and, for this reason, external forces are sometimes applied incrementally to identify the variation of strains and stresses. This type of analysis is solved iteratively and is part of the nonlinear group of numerical simulations (time-independent and rate-dependent) [30].

A uniaxial tensile test can be performed to generate a stress-strain curve. Under a uniaxial load, the material strain hardens after σ_y , as shown in Figure 2-12. From this point on, plasticity effects take place in the material (ϵ_p). Before the plasticity effects started, the strains were purely elastic (ϵ_e). The total strain (ϵ) can be decomposed into two components, as shown in Eq.(2-31). This relation is known as the additive decomposition of the strain. Considering both elastic and plastic strains, the stresses are obtained according to Eq.(2-32). For working conditions within the elastic region, the plastic strain is equal to zero $\epsilon_p=0$. On the other hand, in processes that induce large strains such as forging and forming, it is reasonable to assume $\epsilon_p \approx \epsilon$ and $\epsilon_e \approx 0$ [31].

Figure 2-12 – Stress-strain curve for a bilinear hardening model.



Source: Dunne and Petrinic [31].

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad (2-31)$$

$$\sigma = E \cdot \varepsilon_e = E(\varepsilon - \varepsilon_p) \quad (2-32)$$

For multiaxial stress, the yield condition relies on different methodologies. Two common methodologies are the Tresca and von Mises criteria. Hardening is usually defined as either isotropic or kinematic. In isotropic hardening, the yield stress changes with the total plastic strain. The kinematic hardening refers to a translation of the yield function in the stress space Neto (2000) *apud* Pockszevnicki [32].

There are a few isotropic hardening models. HDM researchers commonly employ the bilinear isotropic hardening [5, 33, 34]. The typical stress-strain curve for the one-dimensional case is shown in Figure 2-12. The Young's Modulus or elastic modulus (E) governs the strain until the yield stress (σ_y). After this point, the material acquires an elastoplastic behavior governed by the tangent modulus (K^{ep}). A tangent modulus equivalent to the dotted line would represent a perfectly plastic material behavior (It does not have strain hardening) [30].

In the traditional measurement, either strains or displacements are recorded after the execution of the hole. At the same time, the geometry of the hole creates a stress concentration around it. For instance, the stress concentration factor around through-holes can achieve values ranging from -4 to 4, depending on the load ratio acting in the workpiece [35]. For blind holes, the stress concentration is smaller [14]. For high loads, however, the stress near the blind hole reaches the yield point, resulting in plastic strains. When the user is not aware of the stress level, the elastoplastic strain can be interpreted as elastic. The strains are converted to stresses under the elastic assumption. This results in deviations between the measured stresses and the real stresses previously acting in the workpiece [36]. The closer the stresses approach the yield point, the higher the deviations.

There are a few possibilities to reproduce the drilling operation considering an elastoplastic analysis: a) increase the load in a plate with a hole, b) increase the diameter of the hole in a loaded plate or c) incrementally drill the hole on a loaded plate. It was already shown that the loading sequence little influenced the final results for simulations of the hole-drilling procedure [37, 38].

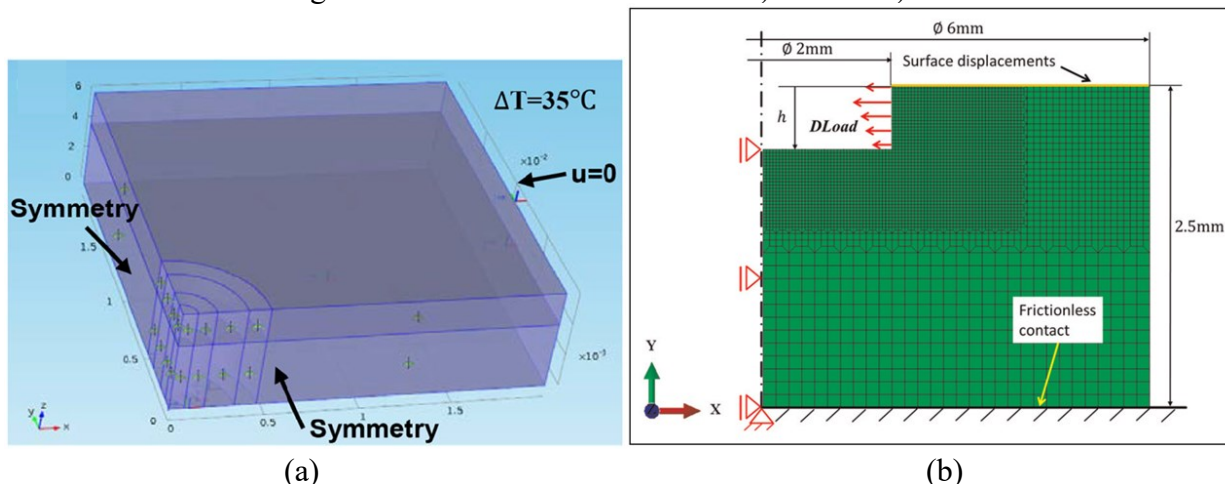
2.3.3 Boundary conditions employed in the HDM

There are a few different configurations employed in the HDM to generate the displacements/strains around the hole. According to the superposition principle, the loads can either be applied at the end of the plate (boundaries) or at the hole wall [39]. The selection of linear or non-linear material behavior, as well as the desired stress profile, influences the strategies employed by researchers. An analytical model was already employed in the HDM to analyze the plasticity effect on a plate with a through hole. However, it required a few limiting assumptions such as perfectly plastic materials and the use of the Tresca yield criterion [14]. For this reason, FEM is usually employed, allowing fewer restrictions in the model.

Axisymmetric and harmonic elements are commonly employed in models to obtain coefficients under the elastic assumption [22, 40]. On the other hand, 3D models are also employed by some researchers [41, 42]. However, these models usually have a higher computational cost. In particular, 3D models are necessary to simulate obstacles near the hole or when the material behavior is not linear elastic [7]. There is only one exception in this regard: it is possible to work with plasticity in 2D axisymmetric models under an equibiaxial stress field [33, 34]. For any other stress state, axisymmetric models were not employed in the literature when plasticity was taken into account.

Nobre *et al.* [33] employed the Ansys Mechanical APDL to generate a non-uniform equibiaxial stress state on a 2D axisymmetric model. This was achieved by constraining the model and applying a temperature gradient varying with depth. The application of different temperature gradients resulted in the desired residual stress profile, allowing the analysis of the elastoplastic effect. Following the same idea, Zhu and Lanza di Scalea [29] obtained the stress values based on a uniform temperature variation restricting normal displacement on two faces of the rectangular model (Figure 2-13a), resulting in uniaxial stresses. On the other hand, Chupakhin *et al.* [34] applied the stresses directly at the hole wall of a 2D axisymmetric model. A specific command from Abaqus was employed to apply a variable stress profile along the wall, as shown in Figure 2-13b. Plasticity effects were also considered in this case.

Figure 2-13 – Finite element model a) 3D and b) 2D.



Source: a) Zhu and Lanza di Scalea [29] b) Chupakhin *et al.* [34].

Besides the boundary conditions employed to obtain a numerical model that reproduces the HDM conditions, there are also a few recommendations regarding the precision of the model. The application of a mesh representing the strain gauges is a strategy to increase the precision of the model [22]. The mathematical model considers infinite plates, an assumption that also influences the precision of the result. To deal with this problem in models of finite size, two strategies can be employed [43]: (a) to apply precomputed displacements at the boundaries of the model (by replacing the stresses) or (b) to insert a ring with specific properties at the outer portion of the model. For a workpiece with finite dimensions, it is also possible to apply the real dimensions to the numerical model. In this case, the width and thickness of the workpiece are employed in the finite element model [44].

2.4 STRATEGIES FOR MEASURING HIGH RESIDUAL STRESSES

The measurement of high levels of stress is affected by plastic yielding near the hole. In this case, the elastic assumption is no longer valid. Additionally, the coefficients presented in the standard are not applicable to stress magnitudes larger than 80% of the yield stress for blind holes and 50% for through holes. The effect of plasticity on the measurement of high residual stresses and its quantification was first analyzed in the 90s.

Several studies have investigated the upper stress limit for residual stress measurement. Lower deviations from the expected stress results were obtained for a loading level of approximately 60% of the yield stress for thin workpieces. This result was obtained based on analytical analysis considering plasticity for a perfectly plastic material under an equibiaxial stress field [14].

For thick workpieces, deviations of 3-5% were obtained for residual stresses of approximately 70% of the yield stress, with a hole depth of 0.4 times the rosette mean diameter. In this case, shallow holes of 0.2 times the rosette mean diameter allowed the increase of the residual stress up to 90% of

the yield stress [38]. In other words, the residual stresses measured with shallow holes are less affected by the plasticity effect. A maximum variation of approximately 8% was obtained for a loading level of 80% of the yield stress after analyzing different loading ratios [45]. For non-uniform stress profiles and a loading level of 80% of the yield stress, an error of 22% was obtained for a maximum hole depth of 0.5mm and a hole diameter of 1.2mm [46].

There are a few works that present correction factors for residual stress measurements near the yielding stress. The first work evaluating the plasticity consequences in the HDM computations proposed a correction factor for uniform measurements. Additionally, it recommended the use of a four-gauge rosette to reduce the errors from 20% to 5% for the evaluated residual stresses. A dimensionless loading intensity factor f (Eq. (2-33)) was also introduced to compare different residual stress levels and loading factors [37]. This dimensionless loading factor is also known as Beghini's factor. In this equation, σ_{eq} represents the Von Mises equivalent stress (Eq. (2-34)), $\sigma_{eq,i}$ represents the theoretical stresses at the onset of plasticity for the 2D case, and σ_Y represents the yield stress of the material.

$$f = \frac{\sigma_{eq} - \sigma_{eq,i}}{\sigma_Y - \sigma_{eq,i}} \quad (2-33)$$

$$\sigma_{eq} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y} \quad (2-34)$$

Besides this loading intensity factor, another dimensionless loading factor was also found in the literature. It is obtained directly by dividing the stress applied at infinity by the yield stress $m = \sigma_\infty / \sigma_Y$ [14].

Considering the recommendations presented by Beghini in Ref. [37], Vangi and Tellini [5] developed an iterative procedure to compute uniform residual stresses using three or four strain gauges for residual stresses higher than $0.9\sigma_Y$ (90% of the yield strength). The performance of the iterative procedure was excellent for the case of a through-hole on a thin plate under a high residual stress level.

Another procedure for measuring high residual stresses was developed by Beghini *et al.* [38]. The procedure considers the plasticity induced by the drilling of the blind hole in four uniform increments. Corrections are applied in the evaluation of the residual stresses based on relations extracted from an extensive database covering various strain hardening ratios, hole diameters and depth, biaxiality ratios, plasticity factors, and rosette types. In this case, a more general equation was adopted to relate the elastic loading intensity factor f_{el} and the loading intensity factor, as shown in Eq. (2-35). The elastic intensity factor is obtained through an elastically evaluated plasticity factor.

Variables W and μ are obtained with polynomials as a function of the ratio D_0/D and the biaxiality ratio Ω .

$$f_{el} = f + Wf^{\mu} \quad (2-35)$$

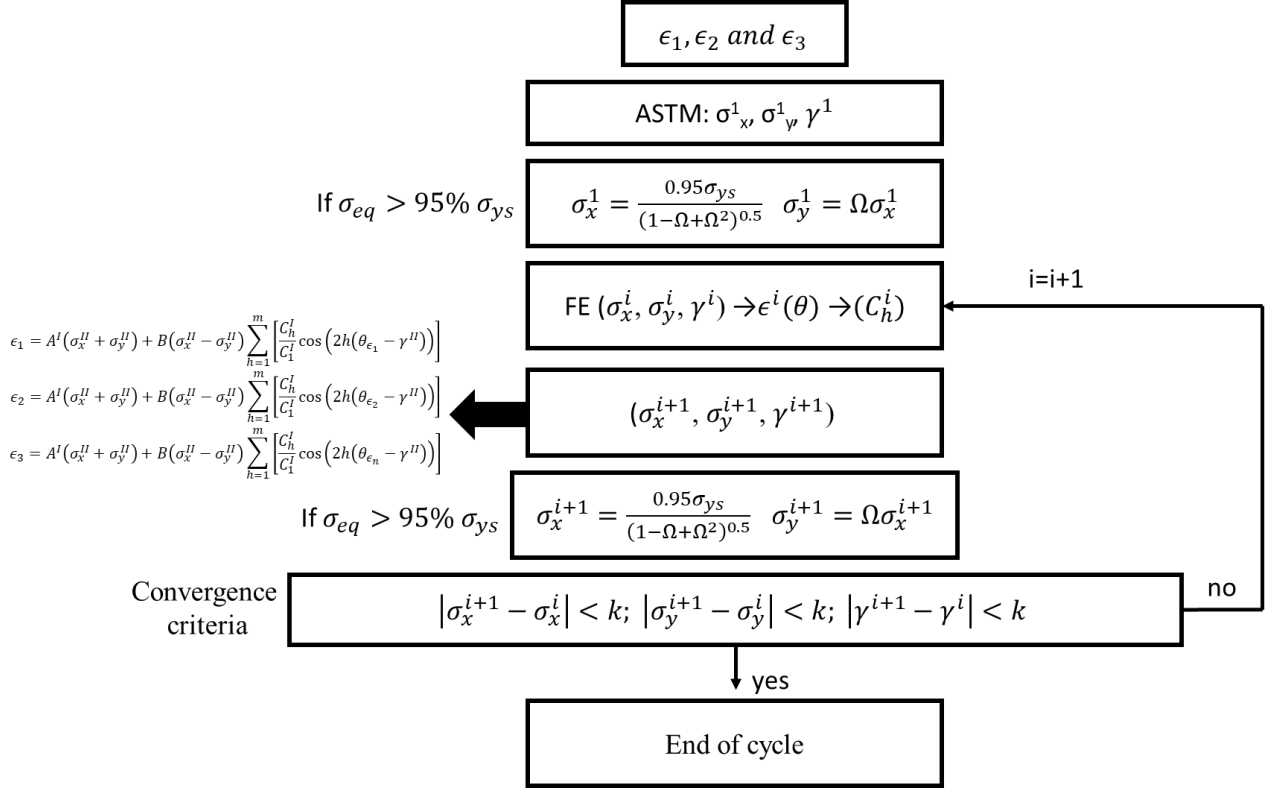
These authors firmly state that it is not possible to apply a load directly at the hole wall considering plasticity due to the violation of the superposition principle. On the other hand, Chupakhin *et al.* [34] applied the load directly at the hole wall for an equibiaxial stress analysis.

Nobre *et al.* [33] performed a deep analysis of the plasticity effects on the hole-drilling measurement results for the non-uniform method. They proposed a correction factor considering the strain hardening in the region where the hole is drilled. The yield strength near the surface on workpieces treated by shot-peening or laser peening may considerably deviate from the bulk material's value. The strain hardening is identified by hardness measurements along the depth, and the data is plugged into the equation.

2.4.1 The iterative procedure developed by Vangi

An iterative procedure was developed to calculate the residual stresses with the aid of an FE model, which was solved iteratively until convergence. In the last iteration of this algorithm, measured strains had to match with the strains obtained numerically, as represented by the convergence criteria shown in Figure 2-14. This algorithm was developed to estimate residual stresses according to the uniform method.

Figure 2-14 – Iterative algorithm to obviate the plasticity effects on the measurement of residual stress values.



Source: Vangi and Tellini [5].

This iterative process has a FE code integrated into it and works in the following way: 1) the acquisition of the strains ϵ_1, ϵ_2 and ϵ_3 ; 2) the computation of the stresses considering the elastic assumption described in the ASTM standard (it works as an initial guess vector); 3) the judgment of the equivalent stress ($\sigma_{eq} > 95\% \sigma_{ys}$) to bring overestimated stress values back to realistic values; 4) the beginning of the iterative cycle where the initial stress guess feeds the FEM elastoplastic model, obtaining the strain profile around the hole and the Fourier coefficients representing this profile; 5) the finding of the solution for the equation system; 6) repeat the correction from 'step 3' and 7) the output is the final result if the convergence criteria is met or it becomes the input for the i th+1 iteration in case the convergence criteria are not met.

The main variable of the algorithm was based on the strains around the hole. To work with an analytical equation, Vangi modeled the strains (obtained numerically) with a cosine even-term Fourier series with five terms. In this way, it was possible to estimate the stresses for the next iteration by forcing the experimental strains with equations (2-37) that represent the strains around the hole.

$$\epsilon_1 = A^I(\sigma_x^{II} + \sigma_y^{II}) + B^I(\sigma_x^{II} - \sigma_y^{II}) \sum_{h=1}^m \left[\frac{C_h^I}{C_1^I} \cos(2h(90 \text{ deg} - \gamma^{II})) \right] \quad (2-36)$$

$$\varepsilon_2 = A^I(\sigma_x^{II} + \sigma_y^{II}) + B^I(\sigma_x^{II} - \sigma_y^{II}) \sum_{h=1}^m \left[\frac{C_h^I}{C_1^I} \cos(2h(225 \text{ deg} - \gamma^{II})) \right] \quad (2-37)$$

$$\varepsilon_3 = A^I(\sigma_x^{II} + \sigma_y^{II}) + B^I(\sigma_x^{II} - \sigma_y^{II}) \sum_{h=1}^m \left[\frac{C_h^I}{C_1^I} \cos(2h(0 \text{ deg} - \gamma^{II})) \right] \quad (2-38)$$

In this equation, ε_1 , ε_2 , and ε_3 represent the experimental strain points, A^I and B^I are coefficients obtained from the previous iteration or from the elastic evaluation in the case of the first iteration, according to Eq. (2-39) and (2-40). For these variables, as well as the Fourier coefficients, ‘ I ’ represents the previous iteration and ‘ II ’ the next iteration. The solution for the current iteration is obtained for three terms: σ_x^{II} , σ_y^{II} and γ^{II} .

$$A^I = \frac{C_0^I}{\sigma_x^I + \sigma_y^I} \quad (2-39)$$

$$B^I = \frac{C_1^I}{\sigma_x^I - \sigma_y^I} \quad (2-40)$$

Besides that, a test was conducted by increasing the number of extensometers (strain points) from three to four. This allowed the iterative procedure to converge in cases where the residual stresses were near the yield stress. The test used a through hole in a thin workpiece [5].

This algorithm was later modified to speed up the convergence process. Instead of modeling the strain values and iterating on the stress values, the principal stresses are computed, and then the angle β is obtained in iterations based on the strain values. First, the main stresses are computed according to Eq. (2-41). ε_1 , ε_2 , and ε_3 represent the strain values obtained experimentally.

$$\sigma_{1,2} = \frac{\varepsilon_1 + \varepsilon_3}{4A} \pm \frac{\sqrt{(\varepsilon_3 - \varepsilon_1)^2 + (\varepsilon_1 + \varepsilon_3 - 2\varepsilon_2)^2}}{4B} \quad (2-41)$$

$$\beta = \frac{1}{2} \tan^{-1} \left(\frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_3 - \varepsilon_1} \right) \quad (2-42)$$

Where A and B represent the calibration coefficients, computed with Eq. (2-43) and (2-44). Where: a (radius of the hole), E (Young Modulus), μ (Poisson ratio), and r_1 r_2 (proximal and distal ends of the strain gauges). The author suggests that calibration coefficients can be calibrated with different stress levels of 30%, 70% and 90% of the yield stress. This hierarchical calibration is supposed to generate an approximate value for A and B in this iterative process.

$$A = \frac{1 + \mu a^2}{2Er_1r_2} \quad (2-43)$$

$$B = -\frac{1}{E} \frac{2a^2}{r_1r_2} \left[1 - \frac{1 + \mu a^2 (r_1^2 + r_1r_2 + r_2^2)}{4 r_1^2 r_2^2} \right] \quad (2-44)$$

Then, the angle β is computed with an optimization algorithm that adjusts the measured strains with the simulated strains and the angle β , obtaining the correct direction of the previously obtained principal values. The convergence criterion is based on the difference between the strain values in the current iteration and the measured values.

A different approach for solving plasticity problems is presented in the next section, employing Artificial Neural Networks.

2.5 ARTIFICIAL NEURAL NETWORKS

Artificial Neural Networks (ANN) models have been widely employed in engineering. In some occasions, the attempt to find an explicit formula for modeling a problem fails; consequently, engineers are oriented to trying out ANN [47]. They become attractive for solving problems that are too complex to be answered using analytical models and for models with a large number of input variables. Publications are showing a promising use of ANN in many processes to predict residual stresses or identify their generating parameters [48–52]. In addition, ANNs are seen as an efficient tool for solving complex inverse problems related to computational mechanics [53].

Huber and Tsakmakis [54] employed both equidistant patterns and random values to generate data for an ANN, thereby reducing the chances of overlearning effects during supervised training. They also recommend that any type of a priori knowledge should be used to choose the input variables. In other words, the user should start employing known variables, ratios, or dimensionless quantities. It certainly reduces the time spent setting up the input parameters and training the ANN.

ANNs models have also recently been used in the HDM. Applications of ANNs in the HDM are related to the correction of eccentricity errors, the correction of strains measured on workpieces with a film of known thickness applied to it (thermally sprayed coating), and the prediction of residual stress for an equibiaxial stress state generated by laser peening [34, 47, 49, 55]. No publication was found that took into account as input data for an ANN or any other model the strain or displacement profile around the hole. This data will be obtained with the aid of an optical device. In addition, a stress state other than equibiaxial – such as the uniaxial stress state, which requires a full 3D FE model – will be employed.

2.5.1 Neural Network in the HDM

Artificial Neural Networks have already been employed in the HDM for decades. Some researchers have employed this technique in the HDM to quickly obtain reliable results using a trained neural network model over the simulation data (database). These works involved the correction of the presence of varying film thickness of different materials, the correction of deviations generated by the hole eccentricity, and the calculation of residual stresses near the yield stress [34, 47, 55]. A summary of these works and the variables employed to create the simulation data is presented in Table 2-1.

Table 2-1 – Summary of works and ANN input variables (features) employed in the HDM.

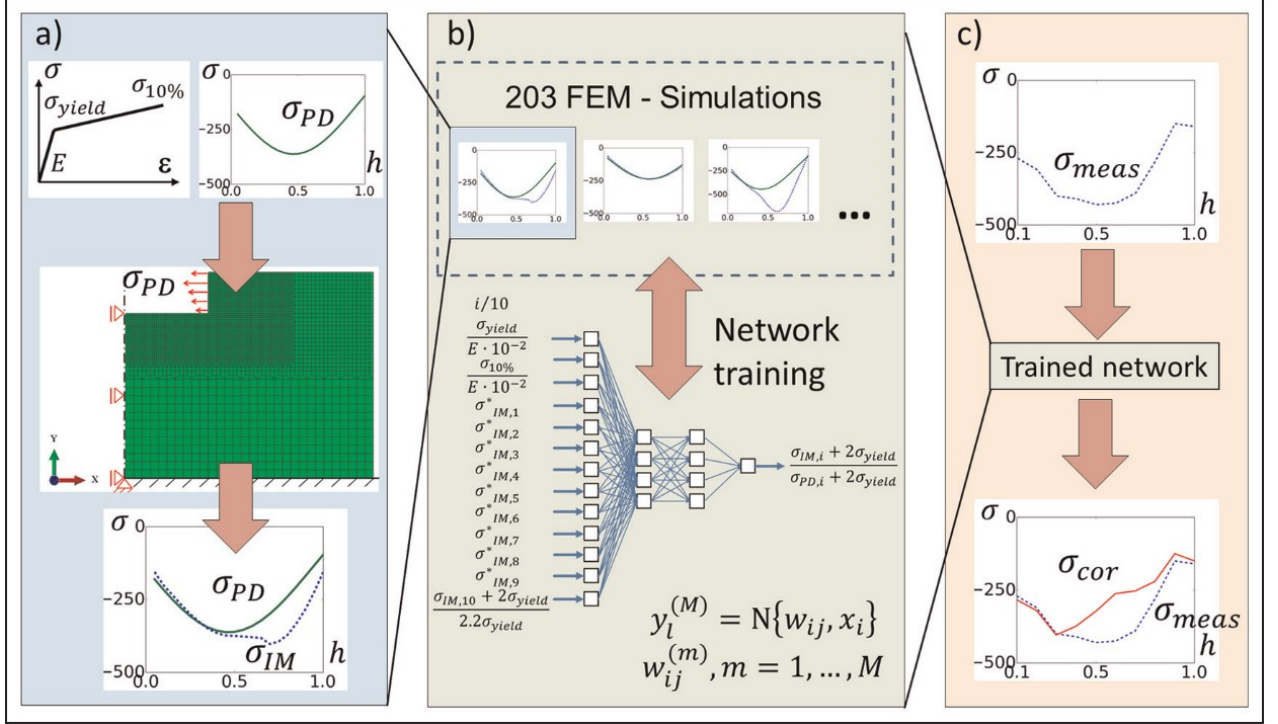
Objective (ANN type)	Lower limit	Variable	Upper limit	Cases	Total Combined
Varying coating thickness [47] (-)	0.1	$E_{\text{coating}}/E_{\text{substrate}}$	4	17	204
	0.005	t_c/D_0	5.5	12	
Excentricity [56] (Convolutional)	0.1	e	0.4	4	324
	0	α	360	9	
		k		9	
Excentricity [55] (Backpropagation)	0.025	e	0.375	4	480
	0	α	360	12	
	-5	σ_x/σ_y	5	10	
Plasticity [34] (Feedforward)	-	Stress profiles*	-	203	2030
	0.1	Increments	1	10	
Plasticity [44]	-1	σ_y/σ_x	1	41	7835100
	0.5	σ_{eq}^s/σ_Y	0.996	21	
	-45°	β^s	45°	91	
	0.01	E_t/E	1	5	
	0.35	D_0/D	0.43	5	
	-	Strain gauge rosette	-	6	

* In this study, a few coefficients were randomly chosen for each stress profile. Only the limit values of these coefficients were presented. The coefficients were kept constant while varying the increment for a specific stress profile.

Chupakhin *et al.* [34] employed an ANN model to obtain a correction factor for residual stress measurements near the yield stress. It was assumed that an equibiaxial stress profile was generated by the laser shock peening process. After an extensive search for input variables or features, the following scheme was employed to train and obtain the corrections with the ANN model to overcome the plasticity effects (Figure 2-15). Random stress profiles and material properties were chosen as features to feed the ANN model. Material properties and coefficients varied randomly within predefined limits. The material properties were the following: σ_0 150-360 MPa; σ_{yield} 300-500 MPa;

$\sigma_{10\%}$ 400-600 MPa, and E 70-210 GPa. Additional coefficients were employed to generate the stress profile with the aid of a trigonometric function. The ANN was then trained with the random stress profiles (predefined stress profiles σ_{PD}) and the stress profile obtained with the integral method (σ_{IM}), which is affected by plasticity for high loads.

Figure 2-15 – Flowchart of the ANN.



Source: Chupakhin *et al.* [34].

Regardless of whether it is an iterative method, analytical method, or artificial intelligence, all these methods for estimating residual stress correction due to plasticity effects require a reliable validation procedure, which will be presented in the next section.

2.6 VALIDATION METHODS EMPLOYED IN THE HDM

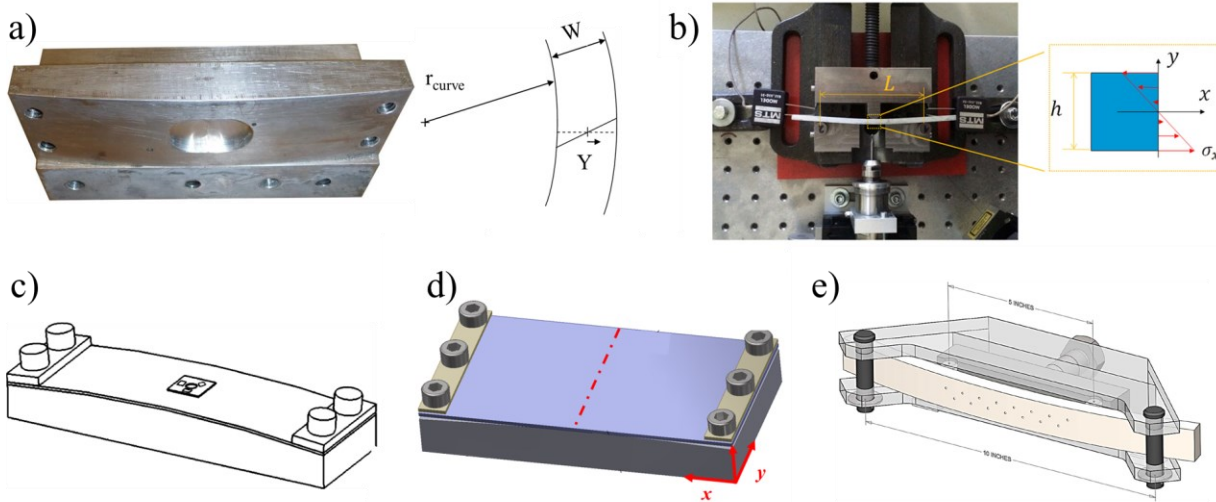
There are three strategies employed to validate new residual stress measurement methods: (a) to compare experimental measurements with another experimental method (reference method), (b) to employ a device to introduce a known stress profile, or (c) to validate the method by using numerical techniques. Nobre *et al.* [33] compared the HDM with X-ray diffraction. The latter was able to measure up to a depth of approximately 0.5 mm, and performed Vickers hardness measurement (HV_{01}) to estimate the local yield strength. Gore and Nobre [57] also employed X-ray diffraction as reference value while evaluating numerical methods using the integral method.

Vangi and Tellini [58] validated the iterative algorithm with strains obtained computationally. Besides the numerical validation of the neural network model achieved for strain correction of layered

material, Held and Gibmeier [47] also compared their results with X-ray diffraction and a micrograph to identify the thickness of the coating.

Another strategy employed in the HDM is to use a loading device. A few devices have already been designed to achieve different goals: loading device with a known curvature radius and a milled out section to reproduce the intermediate case (Figure 2-16a) [59]; three-point-bending-fixture employing only the central point of the beam to analyze the stress deviation generated varying the induced stresses from elastic limits up to values near the yield point (Figure 2-16b) [46]; loading device with a known curvature radius to obtain a steep stress gradient on a thin workpiece (Figure 2-16c-d) [60, 61]; four-point-bend-fixture controlled with a dial to induce a constant stress state on a thick specimen (Figure 2-16e) [62]. It is important to know the theoretical stress profile induced in the workpiece to validate the results.

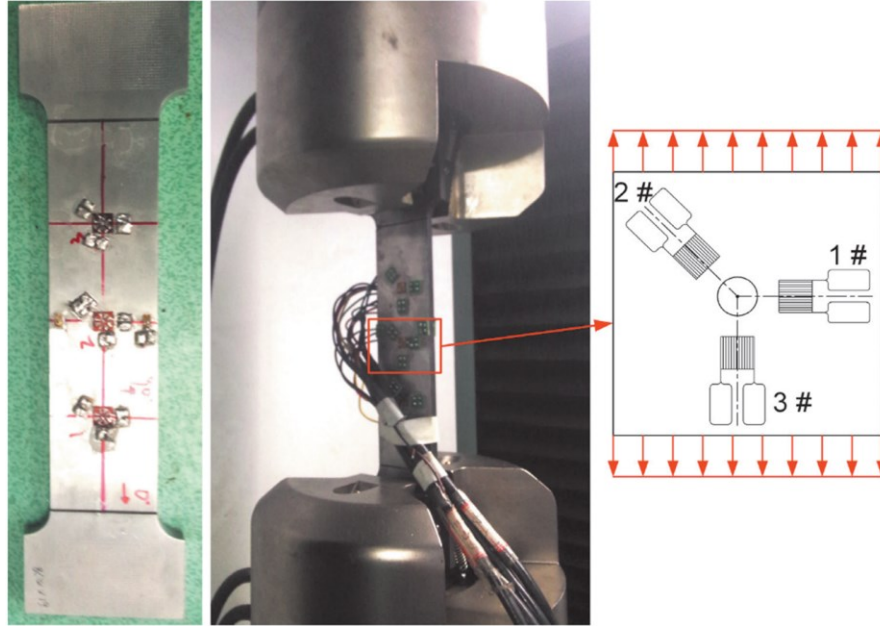
Figure 2-16 – Different types of loading devices.



Source: a) [59] b) [46] c) [60] d) [61] e) [62].

The four-point loading device shown in Figure 2-16e induces a relatively small stress-with-depth variation for thick specimens. Considering shallow holes, the stress variation is not huge when compared to the stress magnitude. Therefore, the expected stresses can be simplified. It is reasonable to assume that the average stress measured corresponds to the value at the mid-depth of the hole [62]. There is also another option for inducing a nearly uniaxial stress state. It can be induced with the aid of a tensile testing machine, as shown in Figure 2-17. This case requires a powerful machine to create a mechanical stress state that simulates a known uniaxial residual stress state on a considerable cross-sectional area. The specimens are more robust than those employed in tensile tests due to the minimum required dimensions specified in the standard.

Figure 2-17 – Tensile testing machine employed to induce a nearly uniaxial stress state.



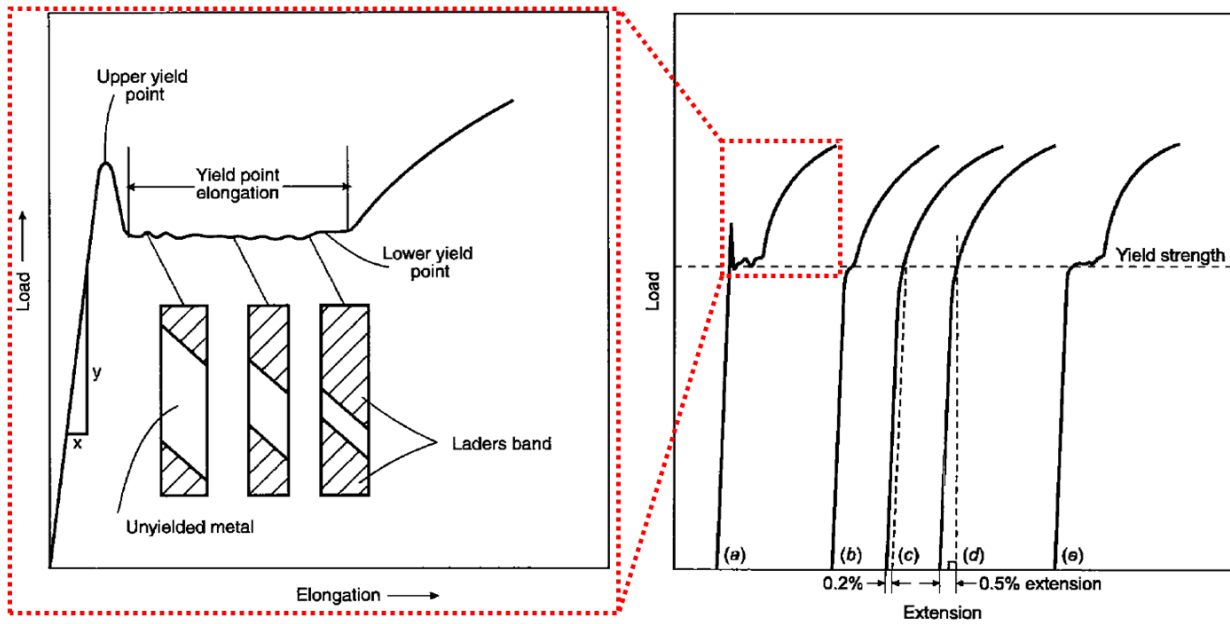
Source: Gu *et al.* [63]

Regardless of the loading method employed, it is important to ensure the residual stress level of the specimen is small. A method for reducing the residual stress levels is to conduct a heat treatment. Annealing is a common heat treatment procedure employed to relieve residual stresses on steels and remove hardness variation generated by manufacturing processes [64].

Material properties such as the yield strength can be obtained through tensile tests. The load-elongation measurements allow the generation of the stress-strain curve. Two types of stress-strain curves are usually employed: ‘engineering’ stress-strain curve (based on the original cross-sectional area) and ‘true’ stress-strain curve (instantaneous cross-sectional area).

Carbon steels may present different stress-strain curve profiles according to chemical composition and heat treatment, as shown in Figure 2-18. An approximation of the yield strength can be obtained from the stress-strain curve. More precise values are obtained using specific techniques, depending on the shape of the curve. The most commonly employed strategy is the 0.2% yield stress: plotting a curve parallel to the constant slope region with an offset of 0.002 strain.

Figure 2-18 – Stress-strain curves presented by carbon steels.



Source: [65].

Some materials, such as annealed low-carbon steel, present an irregular transition between the elastic and elastoplastic region, as shown in Figure 2-18. The load increases until the upper yield point and suddenly drops, fluctuating along the ‘yield point elongation’, and it rises again thereafter [65].

The manufacturing process of pipes influences the stress-strain curve. SMLS (Seamless) pipes are more inclined to present Lüders plateau, while UOE (U-pressing, O-pressing, and expanding), which is a type of longitudinally welded steel pipe, tends to have a stress-strain profile similar to a Ramberg-Osgood plasticity model [66].

3 METHODOLOGY

In this section, an overview of the thesis will be presented, together with a brief description of the next chapters.

3.1 RESIDUAL STRESS IN FLEXIBLE RISERS

This study proposal is part of the Residual Stresses in Flexible Risers (TRRiFlex) project. This acronym represents the Portuguese name of the project: *Tensões Residuais em Risers Flexíveis*. This project is coordinated by Prof. Armando Albertazzi [67], who is the advisor of this work. It started in 2018 and was extended until 2022.

The techniques developed in the TRRiFlex project helped to understand how the residual stresses were affected by the manufacturing processes. The primary objective of the project was to develop equipment for measuring residual stresses in flexible risers, as well as in individual wires of the tensile armor. The main goal was divided into three steps:

1. Develop equipment to measure residual stresses in flexible risers and wires of the tensile armor
2. Compare the measurement capability of the optical HDM device with other techniques
3. Characterize residual stresses in different steps of the manufacturing process of flexible risers.

This was a short-term project and encompassed only scholarships for master's students, but not for PhD students. However, after submitting a research proposal to a call from the National Council for Scientific and Technological Development (CNPq), the coordinator was awarded a PhD scholarship for his research group. This scholarship came from a partnership between CNPq and Equinor. Equinor is an international company headquartered in Norway. It operates in more than 30 countries and develops oil, gas, wind, and solar energy [68]. The call supporting research projects encompassed the topic of Submarine Engineering: rigid and flexible risers [69]. Therefore, the topic of the scholarship is in line with the planned work for the TRRiFlex project.

The TRRiFlex project had a few specific objectives. One of them was to measure high levels of residual stress with the aid of an optical device. High levels of residual stresses were identified in the tensile armor of flexible risers. The development, validation, and application of a methodology to measure stresses at the tensile armor was supposed to contribute to the operational safety of flexible risers. It will be useful to evaluate the condition of batches of flexible risers, reducing the chances of premature failures. However, the priority of measuring high levels of residual stresses was reduced and later removed from the project's scope. This decision was certainly based on the discovery of CO₂ permeation from the pressure carcass to the annulus. An eventual breach of the outer sheath

generates a mixture of gas and water in the annulus, resulting in a corrosive environment. This environment created a sour service environment for the tensile armor, driving it to failure.

The aforementioned specific objective is in line with the objective of this work: to develop, test, and validate a methodology to measure high levels of residual stresses. In the next sections, the strategy employed to measure high levels of residual stresses will be detailed.

3.2 ITERATIVE ALGORITHM

A few studies obtained correction factors to measure high residual stress levels considering the uniform method. The most flexible strategy was adopted to couple with the optical technique: the modification of an existing iterative algorithm for displacement fields.

3.2.1 Modification of an existing algorithm

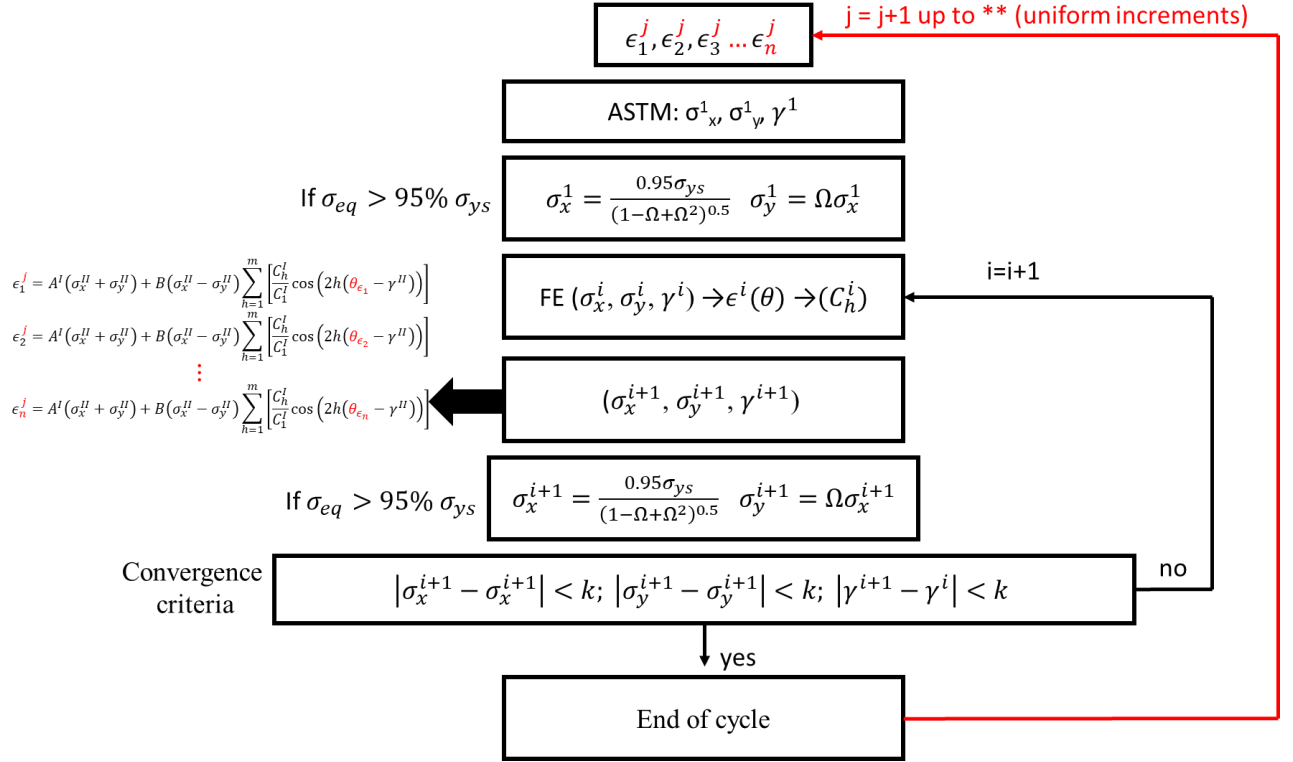
This work represents an extension of an existing algorithm originally developed by Vangi and Tellini [5]. The modifications are presented in Table 3-1 with a rough approximation of the number of points and increments.

Table 3-1 – Comparison between the two strategies.

Strategy	No. points (ϵ)	Method	N° increments
Original [58]	3 or 4	Uniform	Through hole
Modified	360	Uniform	adaptable

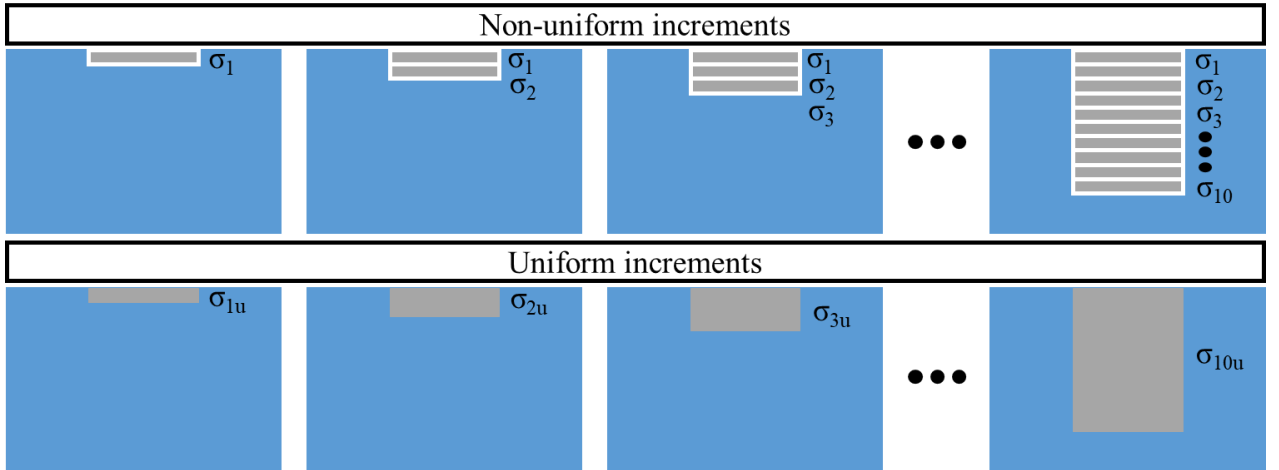
The proposed algorithm is shown in Figure 3-1, where the modifications from the original script are written in red. The superscripts in red represent the addition of the uniform depth increments, while the subscripts in red represent the strains at different locations acquired with the aid of the optical device (instead of the few points provided by the strain gauges).

Figure 3-1 – Iterative algorithm with proposed modifications in red.



It can be seen that this strategy results in uniform ('u') increments. Although they provide less information than the non-uniform method, they deliver information about the stresses with depth taking into account plasticity effects. Both cases are illustrated in Figure 3-2.

Figure 3-2 – Representation of the non-uniform and uniform methods.



3.3 General overview of the methodology

It was decided to divide the methodology into a few chapters to rigorously detail the implementation steps of the iterative algorithm and its modifications. For this reason, the development of this thesis was divided into a few chapters focusing on:

- **Elastoplastic Finite Element Model:** In this section, the behavior of the elastoplastic finite element model is tested to ensure it behaves according to the stress-strain curve.
- **Definition of the Finite Element Dimensions:** this chapter covers the evaluation of the finite element dimensions according to hole-drilling parameters such as strain and stress.
- **Iterative Approach:** in this chapter, the iterative algorithm is presented in detail, together with the modifications and the processing steps for both: a) experimental data acquired with the optical device and b) the numerical validation data obtained with the finite element model.
- **Experimental planning:** this chapter describes the experimental planning employed for performing the residual stress measurements. It is divided into two parts: 1) pre-tests performed on a loading device and 2) the final tests performed on the tensile machine.
- **Numerical results and discussion:** this chapter presents and discusses the numerical validation of the iterative algorithm and the main trends identified.
- **Experimental results and discussion:** this chapter presents and discusses the main results obtained from the experimental tests (pre-tests and final tests).
- **Conclusions and suggestions for future work:** the main findings of this work are summarized in this section, together with the final considerations and the suggestions for future work in this research field.

4 ELASTOPLASTIC FINITE ELEMENT MODEL

The steps necessary to obtain a reliable finite element model are presented in this section. A few validations need to be performed: a) validate the material model, b) the mesh type, c) the elastoplastic model, and d) the load application strategies.

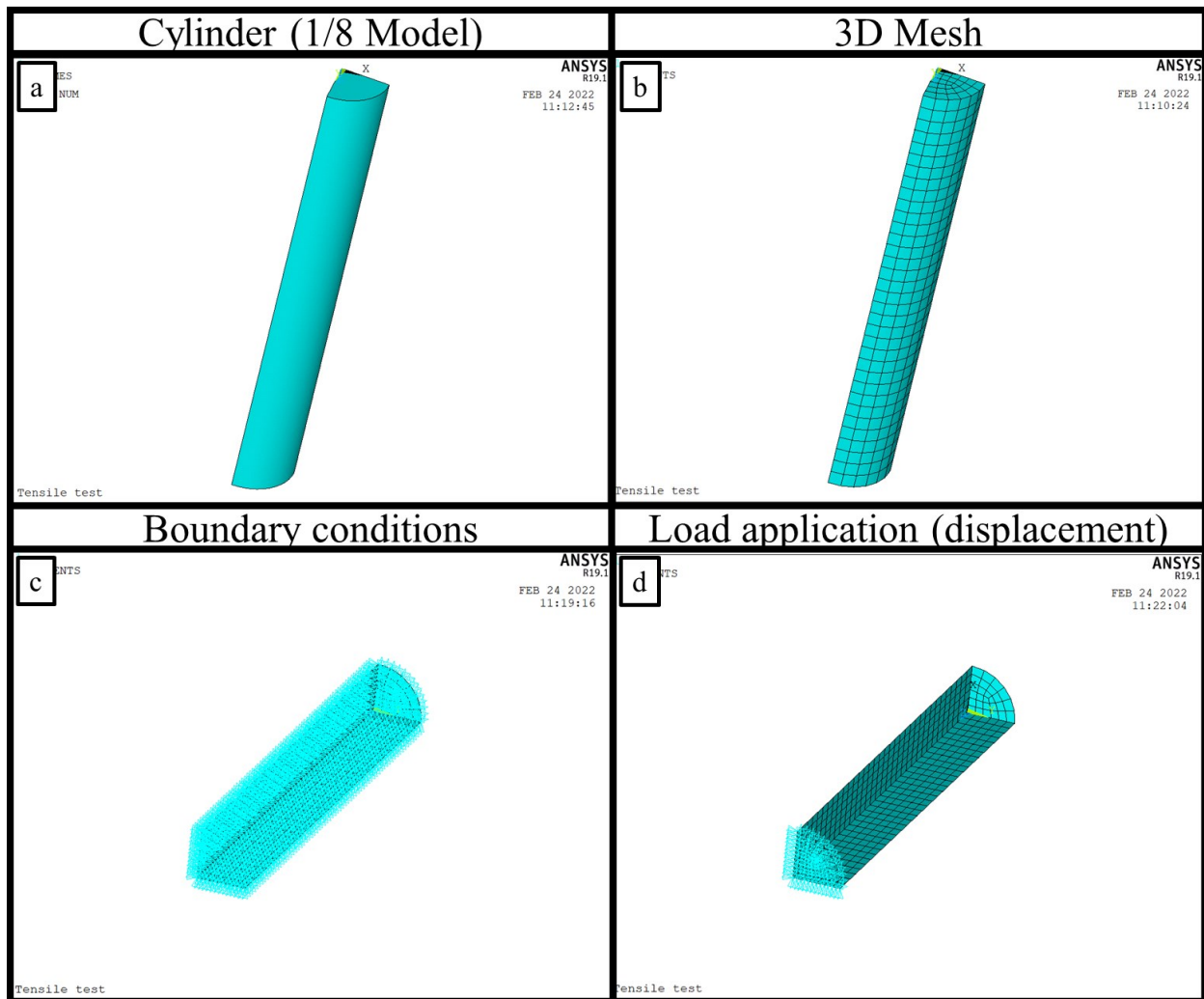
4.1 STRESS-STRAIN CURVE

The iterative algorithm requires a module to calculate the displacements for the applied stresses. Ansys Mechanical APDL was chosen to perform the finite element computations.

The material model employed and the simulation software define whether the engineering or true stress-strain curve is fed. It was not clear in the Mechanical APDL documentation whether the true curve or the engineering curve should be provided.

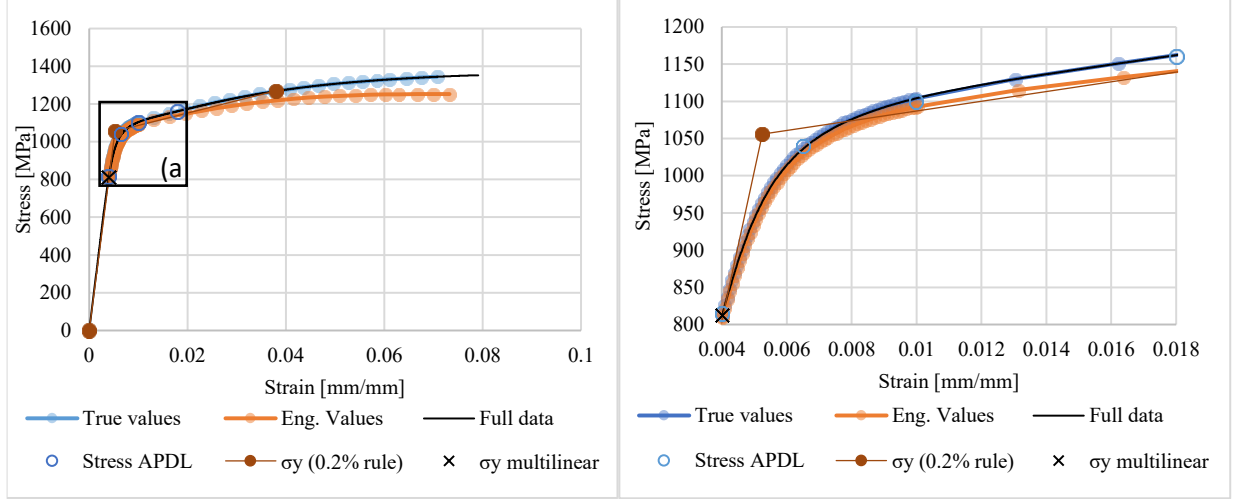
To verify the precision of the elastoplastic model, a tensile test was performed on a simplified specimen (cylinder). A model of 1/8 of a cylinder was employed. Appropriate symmetry conditions were applied at the x, y, and z planes, as illustrated in Figure 4-1c. A point at the origin was locked in all directions to avoid body motion. A specific strain value was induced in the workpiece. A displacement necessary to induce the strain was computed. The displacement was applied at the extreme end opposite to the surface that received the symmetry condition in the z-direction (Figure 4-1d).

Figure 4-1 – (a) 1/8-cylinder model, (b) discretization, (c) boundary conditions, and (d) application of the displacement.



The material was characterized, and the stress-strain curve was generated. A large number of points were obtained during the tensile test. The engineering strain values were converted to true values, and this curve is represented by “Full data” in Figure 4-2. Less than 100 points remained in the final curve named “True values”, which was added to the FE model (Some of the Ansys ® APDL commands impose a limit of 100 points as input data). The true values match the expectation of deviating from the engineering values as it moves further into the elastoplastic region.

Figure 4-2 - Stress-strain curve (left) and amplification/detail “a” (right).



The curve named $\sigma_{y \text{ multilinear}}$ represents the end of the linear region (elastic region) and the beginning of the yielding (Elasto-plastic region) under the multilinear model assumption. The Ansys Mechanical APDL accepts as input data the $\sigma - \epsilon_{\text{plastic}}$ (stress vs plastic strain curve) starting from the point $\sigma_{y \text{ multilinear}}$ up to the rupture point.

The stress vs plastic strain curve employed as input data was computed considering the assumption of the additive strain decomposition (Eq. (4-1)). The elastic strains are obtained from the true stress values (Eq. (4-2)). In this case, the total elastic value is equivalent to the true elastic value, resulting in (Eq. (4-3)).

$$\epsilon_{\text{total}} = \epsilon_{\text{elastic}} + \epsilon_{\text{plastic}} \quad (4-1)$$

$$\epsilon_{\text{elastic}} = \frac{\sigma_{\text{true}}}{E} \quad (4-2)$$

$$\epsilon_{\text{plastic}} = \epsilon_{\text{true}} - \frac{\sigma_{\text{true}}}{E} \quad (4-3)$$

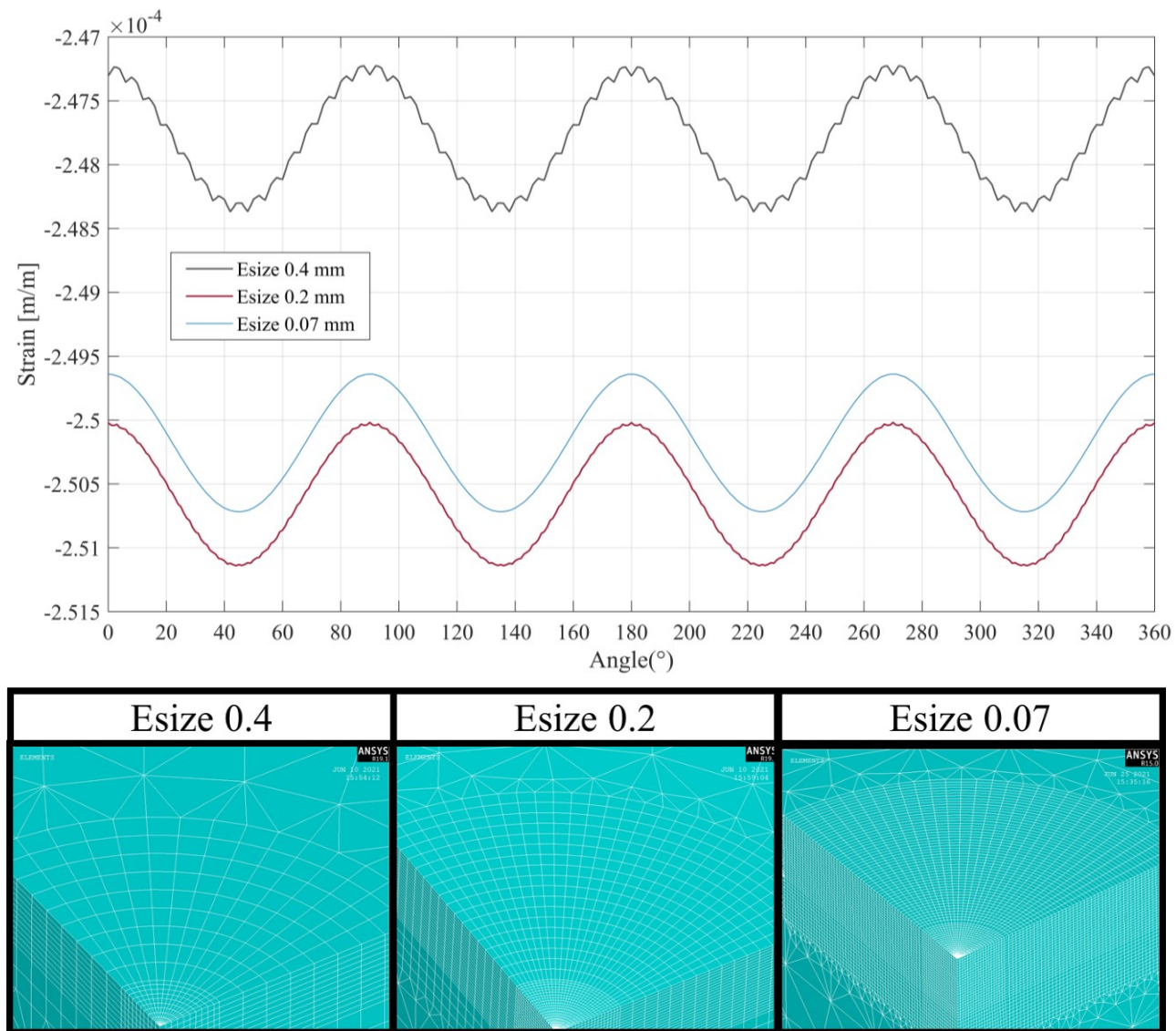
The numerical test was performed for four strain values on this simplified tensile test specimen. It was identified that the multilinear elastoplastic model succeeded in representing the original stress-strain curve, as represented by the curve “stress APDL” in Figure 4-2.

4.2 ANALYSIS OF THE MESH TYPE

In this section, a squared model was employed. Several tests were conducted to validate the model and determine the optimal strategy for load application.

A few different mesh sizes were tested. The final strain profile around the hole is presented in Figure -4-3 for an equibiaxial load – the equibiaxial load was employed to facilitate the visualization. For each mesh size, a different average strain value was obtained, with a total variation of approximately 1% among the strain values. Besides that, a larger noise was generated by the coarser mesh size. It is unclear which portion corresponds to the noise generated by the finite element method and the noise generated by the interpolation tools.

Figure -4-3 – Strain profile around the hole for different element sizes. The profile was obtained by rotating a single strain gauge around the hole.



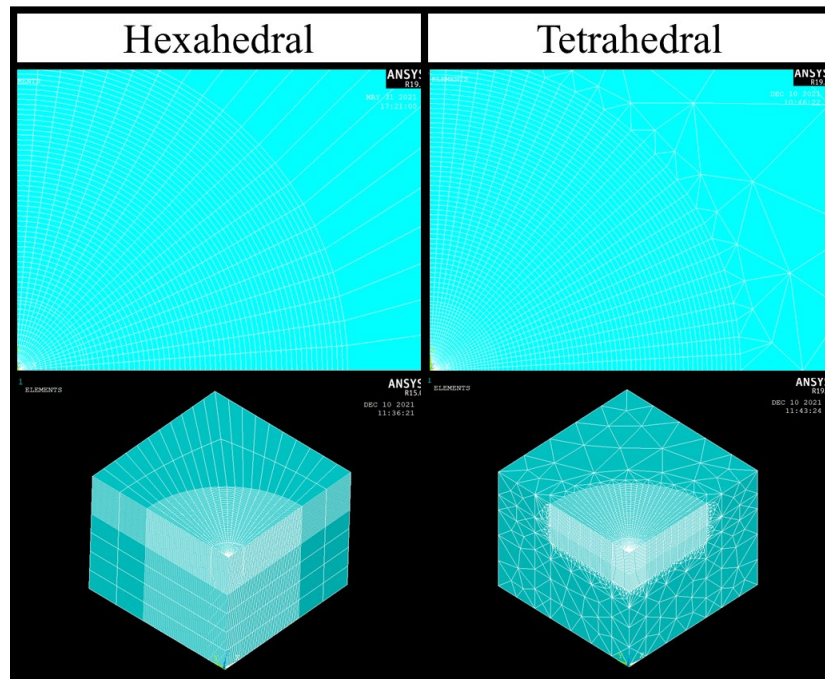
Hole drilling method researchers usually employ a much finer mesh with 2D harmonic elements. In the next step of this project, a model under a uniaxial load considering plasticity effects must be solved. The only reliable strategy known up to date to simulate this condition requires the use of 3D elements. Therefore, a balance between computing time employing 3D elements and the quality of the results must be considered here.

It is a general recommendation that the gages should be spatially represented by elements or nodes at the surface. In this case, the entire profile of the strain values around the hole is of interest. Creating a parameterized model capable of rotating the nodes representing the gage grid limits around the hole turned out to be too time-consuming. Therefore, interpolation tools were employed to compute the displacement between nodes, facilitating the acquisition of the strain profile around the hole for a specific type of rosette (The gages are rotated around the hole).

4.2.1 Tetra vs hexa mesh

The model may be required to run a few times: run until the convergence point, multiplied by the number of increments. For that reason, it is important to make small improvements to speed up the running time. One strategy was to reduce the number of elements by changing part of the model to a tetrahedral mesh, as shown in Figure 4-4. Hexahedral mesh requires that a layer of elements have the same thickness, even far away from the hole, following the thickness of the hole increments. Working with tetrahedral mesh does not require that the same number of divisions along depth be constant, providing greater freedom to increase or decrease the mesh size. This became more important after the model size had to be increased due to the extension of the plastic strains - this topic will be explored in the next section.

Figure 4-4 – (left) Full hexahedral mesh and (right) hexahedral mesh with tetrahedral mesh near the boundaries.



A small difference in the computed strains was identified by changing the element shape at regions away from the hole, as shown in Table 4-1.

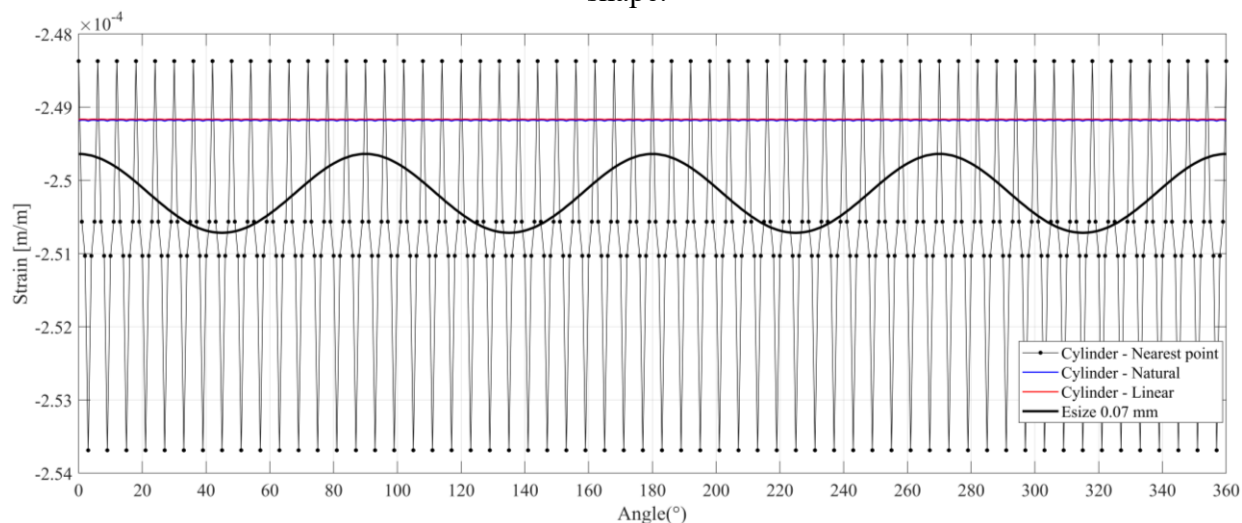
Table 4-1 – Strain variation for hexahedral and tetrahedral mesh.

Equibiaxial							
f		Min Strain (m/m)	Max Strain (m/m)	2x Amplitude (m/m)	Average (m/m)	Deviation 1 (2xAmplitude)	Deviation 2 (Average)
0.9	Tetrahedral	-2.51E-04	-2.50E-04	-1.10E-06	-2.51E-04	0.000%	0.160%
0.9	Hexahedral	-2.51E-04	-2.50E-04	-1.10E-06	-2.50E-04		

An unexpected noise was also identified in the strain profile around the hole. This noise had a sinusoidal shape, indicating that something was affecting the calculation: either there was a problem in the computational procedure or with the tools employed within Matlab® to extract and compute the strain data, such as interpolation tools.

Three interpolation methods available in Matlab® were evaluated: linear, natural, and nearest. The nearest method generated the highest oscillation among the three interpolation methods. A small difference was identified between the other two interpolation methods: linear and natural, as shown in Figure 4-5. However, this difference was small when compared to the sinusoidal signal.

Figure 4-5 – Strain profile obtained around the hole for different interpolating strategies and model shape.



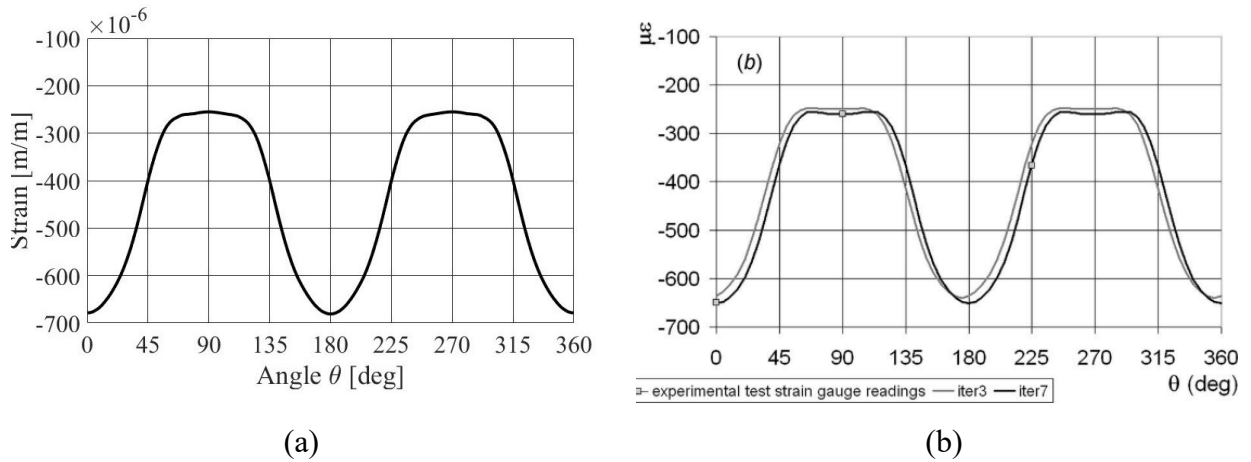
The sinusoidal noise is also present in the displacement data. Tests performed varying the length, width, and gage grid number did not indicate the generating source of this noise. Later, it was identified that the sinusoidal noise was generated by the shape of the FE model. A squared model introduces a small sinusoidal noise in the strain profile measured around the hole. This was verified by creating a cylindrical model and capturing the strains around the hole. The sinusoidal signal disappeared – this sinusoidal signal was more easily seen in the equibiaxial loading case. The sinusoidal signal is shown in Figure 4-5 as “Esize 0.07”. The interpolation tools from Matlab® were tested directly with the cylindrical model.

4.3 VALIDATION OF THE ELASTOPLASTIC MODEL

First, a 2D model was employed to verify whether the results generated with the elastoplastic model agreed with those available in the literature [70, 71]. The results obtained in this work were in close agreement with those reported in the literature. For more details, see APPENDIX A. The next step in the validation process was to verify the strain profile around the hole and compare it with the literature on the field of the hole drilling method.

A numerical analysis was performed to verify whether this plasticity model generates reasonable results. A few works are available in the literature where plastic analyses have been performed. A comparison between the current results and the results obtained by Vangi [5] is presented in Figure 4-6. In this case, a through hole was employed with geometries similar to those presented in the aforementioned article. This comparison indicates that the results obtained with the current FE model are reliable. The results are of the same order of magnitude. The minimum strain values are slightly smaller, and the curve presents a similar shape with a plateau curved downwards.

Figure 4-6 – Comparison of the strain profile around a through hole: a) current model and b) literature.



Source: Vangi and Tellini [5].

The final step of the validation process involves simulating a blind hole and comparing the results. However, the blind hole-drilling process considering plasticity effects requires the evaluation of an additional decision: application of the load. The next task is to decide whether the loads will be applied directly at the hole wall or at the walls located on the extreme boundaries of the model. A thorough investigation regarding this topic is presented in the next section.

4.4 LOADING STRATEGIES: HOLE SURFACE OR EXTREME BOUNDARIES

The loads can be applied either at the hole wall or at the edge in numerical analysis performed within the elastic regime. This is possible considering the superposition principle. In this section, a

detailed investigation of the superposition principle with a specific application in the hole drilling method considering the effect of plasticity will be presented. Three loading ratios were investigated: equibiaxial (Figure 4-7), uniaxial (Figure 4-8), and pure shear (Figure 4-9). For the equibiaxial case, the application of the load is straightforward. However, it is necessary to change the signal because, in this case, the load is applied in the direction of the material, while applying the load to the boundaries of the model requires that it be applied in the opposite direction. A combination of normal load and shear load needs to be applied for the uniaxial and pure shear stress ratios. These values can be obtained with the aid of stress transformation equations.

Figure 4-7 – Stress transformation for the equibiaxial stress state.

Equibiaxial $\Omega = 1$ ($\sigma_x = \sigma$, $\sigma_y = \sigma$)

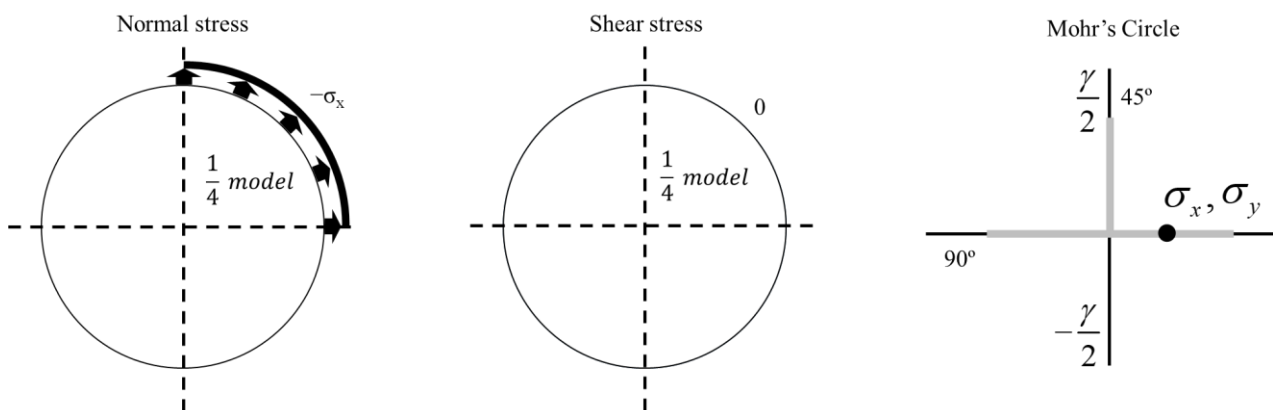


Figure 4-8 – Stress transformation for the uniaxial stress state.

Uniaxial loading $\Omega = 0$ ($\sigma_x = \sigma$, $\sigma_y = 0$)

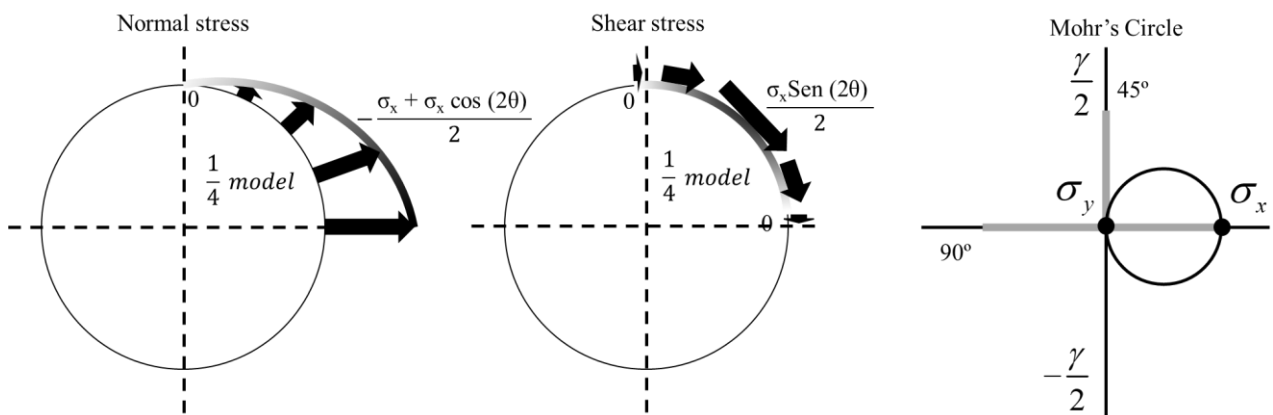
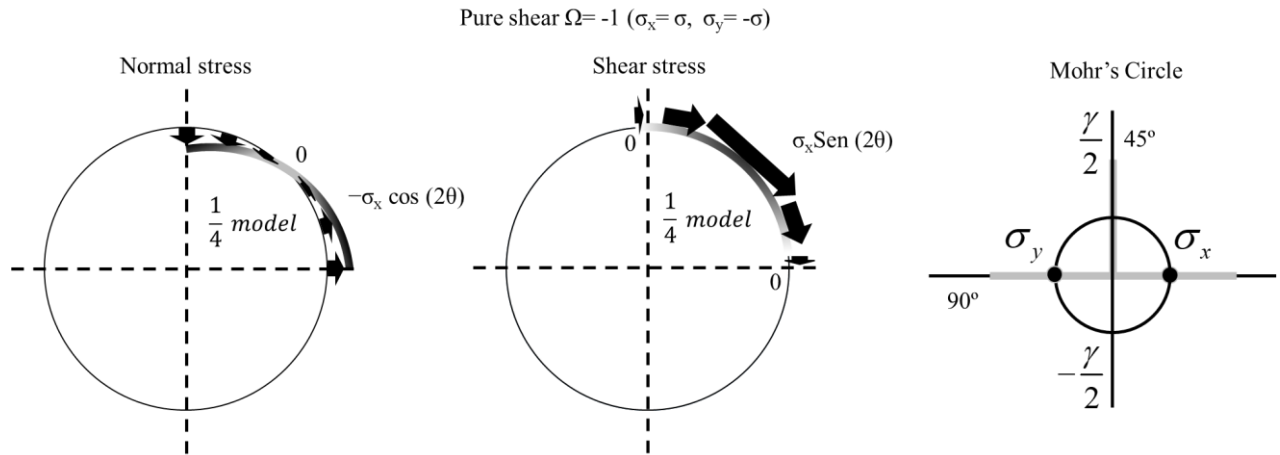


Figure 4-9 – Stress transformation for the pure shear stress state.



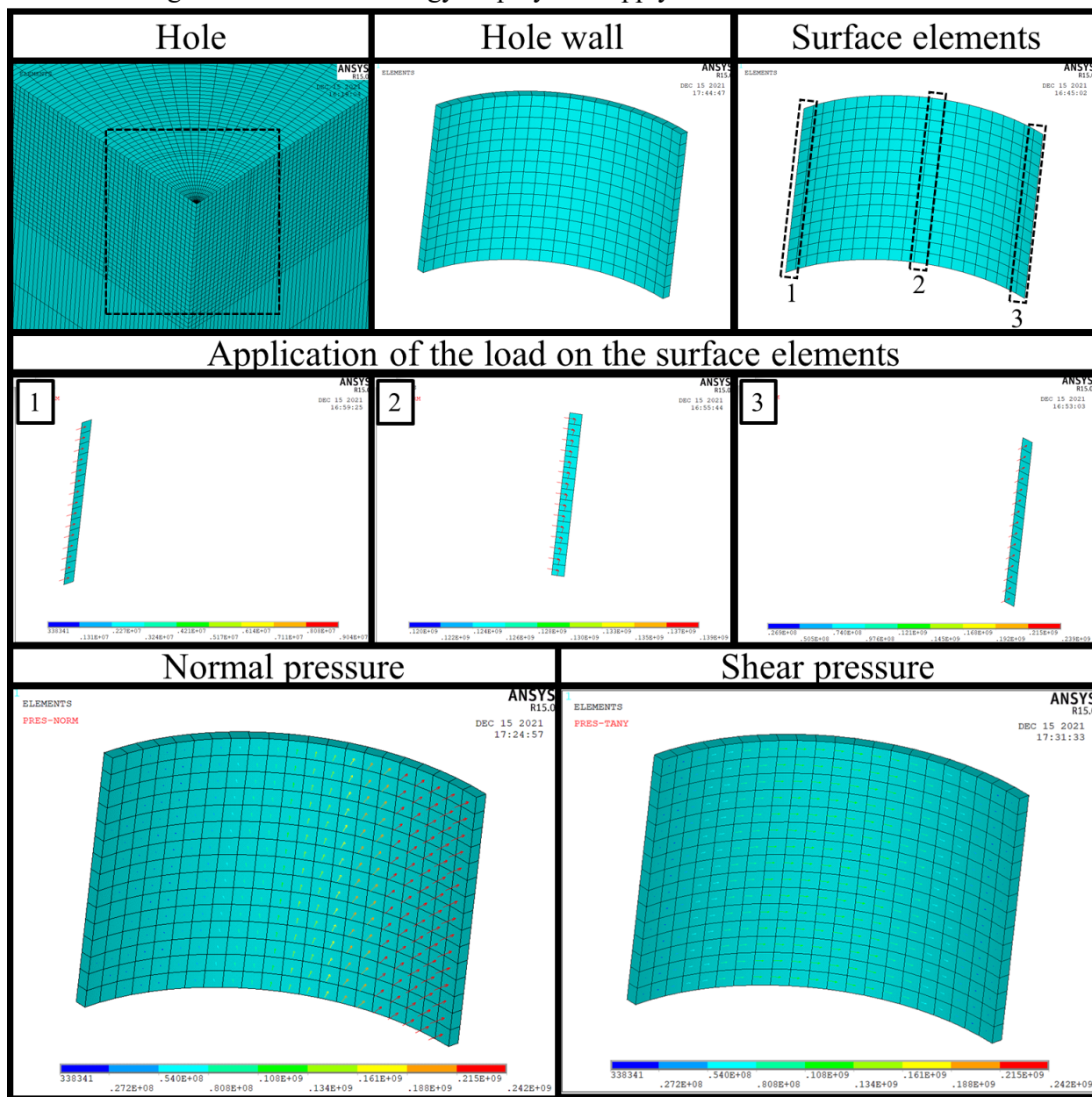
Different finite element software are available to perform this type of simulation. Ansys Mechanical APDL was employed in this work. There are a few peculiarities regarding the implementation of this FE model:

1) Up to date, there is no command available to automatically vary the normal load applied to the hole wall. Therefore, it is necessary to compute the load applied at each three-dimensional element.

2) Typical 3D elements do not accept the application of shear loads. Surface elements with the capability to receive shear loads were introduced in the region of interest, and the loads were applied afterwards.

The hole, hole wall, and surface elements are shown in Figure 4-10. The stress components of the uniform load are computed and applied at each column of surface elements (for a non-uniform load, this computation should be performed individually for each element) as shown in insets 1, 2, and 3 of Figure 4-10. After the application of the load, it is possible to visualize the normal pressure and shear pressure, as shown at the bottom of Figure 4-10.

Figure 4-10 – Methodology employed to apply the load on the hole wall.



A discrete stress value was computed considering the stress at the center of each element. Indeed, the highest precision would be achieved by computing a stress average along the area of the element. However, this simplification generated fairly good results for the elastic case. The case where the load is applied at the extreme ends of the model can be employed as a reference to verify whether the entire procedure described above is correct.

4.4.1 Evaluation of the load application strategies

First, simulations were performed for a full hole with a depth of 1 mm. Then, the strain profile around the hole was obtained for three specific loading cases: uniaxial, equibiaxial, and pure shear stress ratios, as shown in Table 4-2. In the actual strain profile for the uniaxial case, the load is applied

at the edge (boundaries) of the model, regardless of the loading factor being located within the elastic or elastoplastic region.

It is possible to check the precision of the load application by comparing the strains obtained with the two strategies for a plasticity factor within the elastic regime. A load was applied to both models, and the difference between the two strategies for the equibiaxial case within the elastic region was 0.01%, while it was around 0.09% for the uniaxial and pure shear cases (the values are presented in Table 4-2 under $f=-0.2$, which represents the elastic case). It was found that the difference between the two strategies is not large for the uniaxial and pure shear cases, so the investigation continued with the current model.

Table 4-2 - Comparison between two strategies of load application for the uniaxial, equibiaxial, and pure shear cases ($D_0=1.8\text{mm}$, $h=1\text{mm}$).

Equibiaxial			
f	Hole wall - Strain (m/m)	Boundary - Strain (m/m)	Deviation
-0.2	8.83E-05	8.84E-05	0.01%
0.3	1.44E-04	1.47E-04	1.66%
0.6	1.81E-04	1.93E-04	6.29%
0.8	2.11E-04	2.31E-04	9.32%
0.9	2.29E-04	2.50E-04	9.35%
0.95	2.39E-04	2.65E-04	11.03%

Uniaxial (Twice the amplitude)			
f	Hole wall - Strain (m/m)	Boundary - Strain (m/m)	Deviation
-0.2	0.00007406	0.00007412	0.081%
0.3	-	-	-
0.6	-	-	-
0.8	-	-	-
0.9	-	-	-
0.95	0.000363	0.0004561	25.65%

Pure shear (Twice the amplitude)			
f	Hole wall - Strain (m/m)	Boundary - Strain (m/m)	Deviation
-0.2	0.00013627	0.0001364	0.088%
0.3	-	-	-
0.6	-	-	-
0.8	0.0003804	0.0004068	6.94%
0.9	-	Lack of converge	-
0.95	-	Lack of converge	-

A significant deviation was generated by considering the superposition principle, especially for high loads. The deviation is presented in Table 4-2. Taking the loading factor $f=0.95$ into consideration, the uniaxial loading at the hole wall generated a deviation of approximately 25.65%, equibiaxial (11.03%), and the pure shear case did not converge for this loading factor. The strain

profile for the uniaxial (Figure 4-11), equibiaxial (Figure 4-12) and pure shear (Figure 4-13) did not generate a plateau, as opposed to the results shown in Figure 4-6. The validation presented in Figure 4-6 represents the strain profile for a through hole under a high load, resulting in a strain profile that resembles a plateau – representing a very extreme condition.

Figure 4-11 – Strain profile around the hole for two load application strategies considering the uniaxial case.

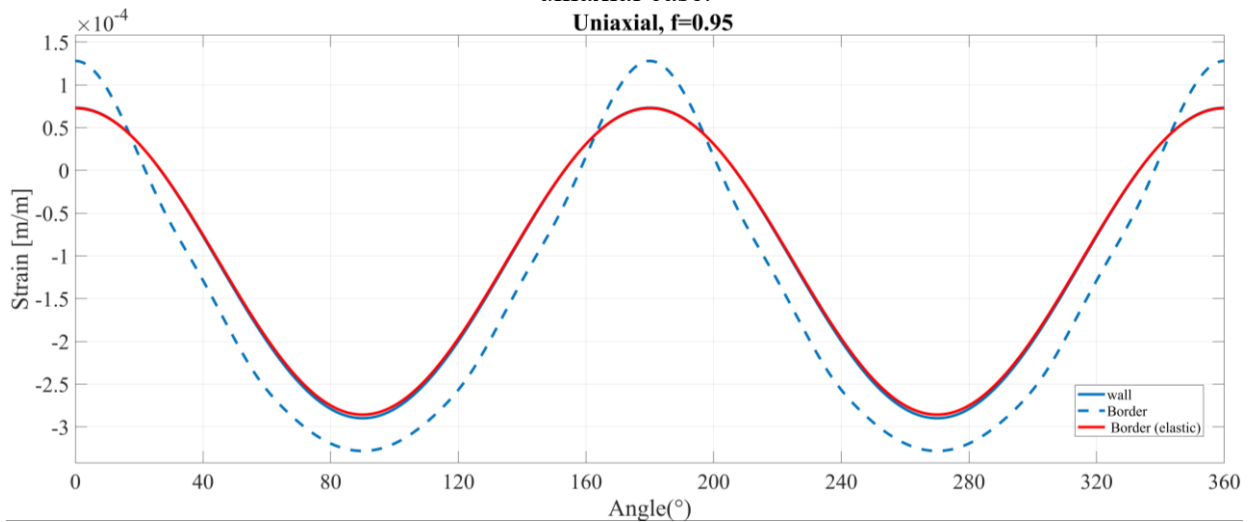


Figure 4-12 – Strain profile around the hole for two load application strategies considering the equibiaxial case.

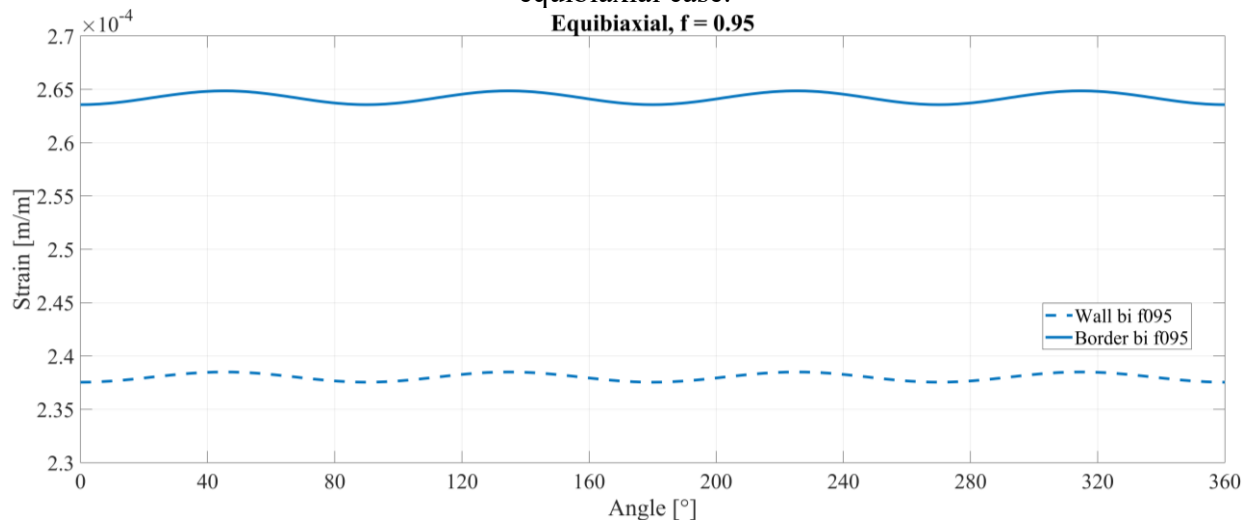
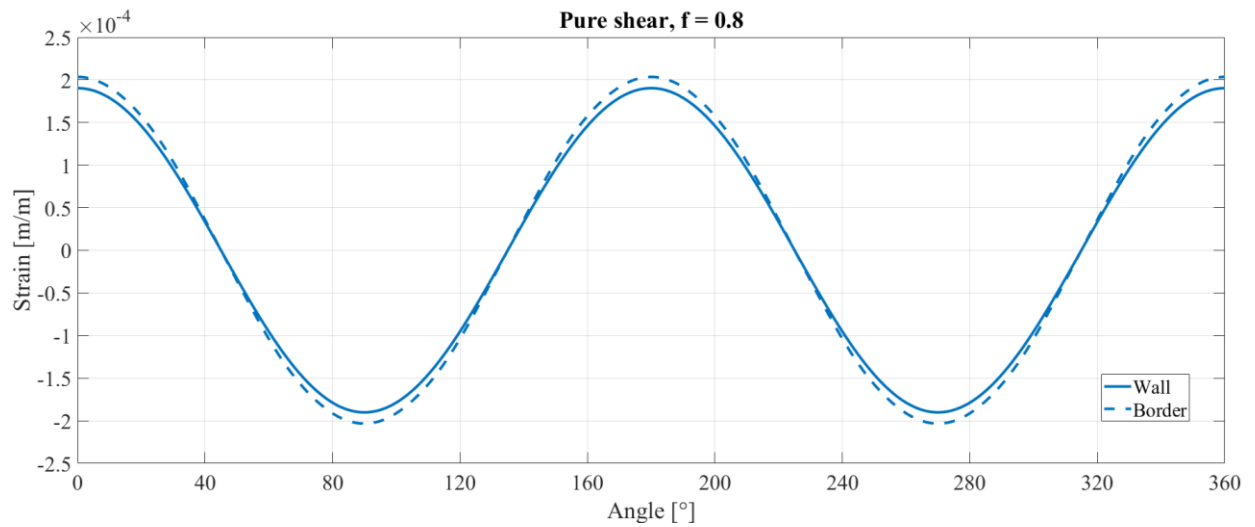
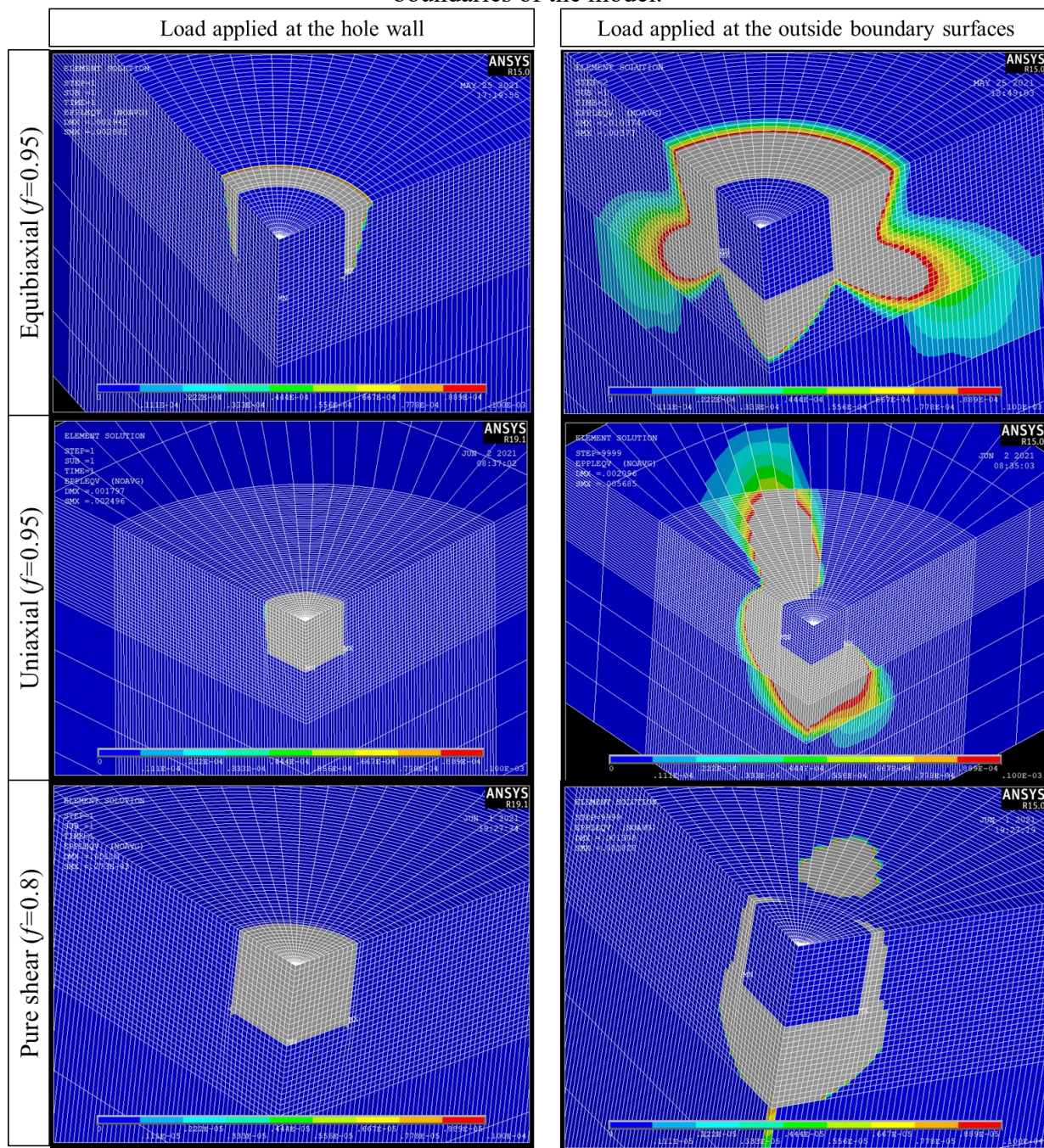


Figure 4-13 – Strain profile around the hole for two load application strategies considering the pure shear case.



The deviation in the strain values for the two application load strategies can be related to the extension of the plastic strains. It is possible to observe in Figure 4-14 that the plastic strains reach a much larger region in the uniaxial case than it does in the other two cases.

Figure 4-14 – Extension of the plastic strains around the hole for two different cases: (left) application of the load directly at the hole wall and (right) application of the load at the extreme boundaries of the model.



Loads were applied directly at the hole wall in the article from Chupakhin [34], focusing on equibiaxial loads introduced by laser shock peening. The same strategy was not adopted in this work because the workpiece under investigation presents a strongly uniaxial stress ratio, which would generate even higher deviations if the loads were applied directly at the hole wall assuming the superposition principle to be satisfied or nearly satisfied. The strains measured around the hole for both the uniaxial and equibiaxial stress ratios are shown in Figure 4-11 and Figure 4-12, respectively. The elastic case is presented together with the two load application strategies for the uniaxial stress case (Figure 4-11). It is possible to observe that the load applied at the wall generates a strain profile

close to the elastic one for the uniaxial case. That means it generates a large deviation from the expected result.

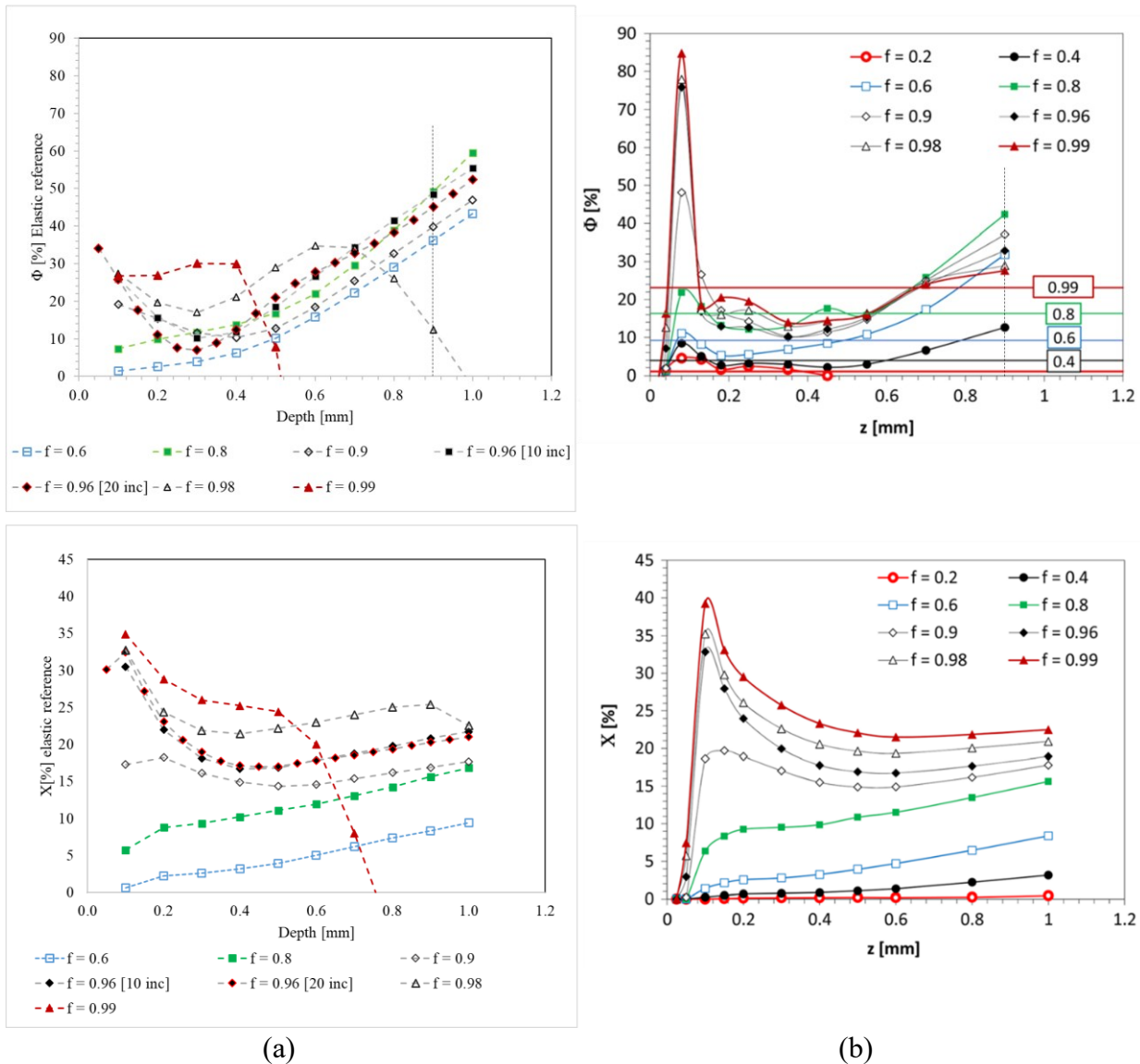
Two additional strategies were tested in the initial phase of this work: the introduction of pre-stress as well as the use of temperature gradients. The details are presented in Appendix B and Appendix C. Substantial development was achieved using these strategies to introduce a non-uniform residual stress profile. However, their use no longer met the requirements of this work.

To sum up, a few validations were performed in this chapter to ensure a reliable elastoplastic model would be employed as the basis for the subsequent development of the proposed methodology. The following validations were performed: validation of the material model, mesh type model, geometry type model (squared/rounded model), elastoplastic model, and, most importantly, the load application strategies. The assumption of the superposition principle yields significant deviation for high levels of uniaxial loads – a strain variation of 25.65% was identified for the full hole and a loading of $f=0.95$. For this reason, the chosen strategy is the traditional application of the load at the boundaries.

5 REFINING THE FINITE ELEMENT MODEL

The results obtained in this work for the elastoplastic model were also compared with a research work from Nobre [33]. HDM parameters were employed in this evaluation considering simple linearized kinematics. An initial assessment revealed that, for high residual stress values, the numerically measured residual stress values deviated from the literature. The strain variation, as well as the nonuniform stress variation, are plotted in Figure 5-1. The strain error X was defined in the literature and represents the variation between the strains obtained using both the pure elastic finite element model and the elastoplastic model. The nonuniform stress profile variation Φ follows the same calculation strategy.

Figure 5-1 –Strain profile variation X and nonuniform stress profile variation Φ for a) the current model and b) results obtained from the literature.



Source: Nobre *et al.*[33].

A few points can be obtained from this comparison:

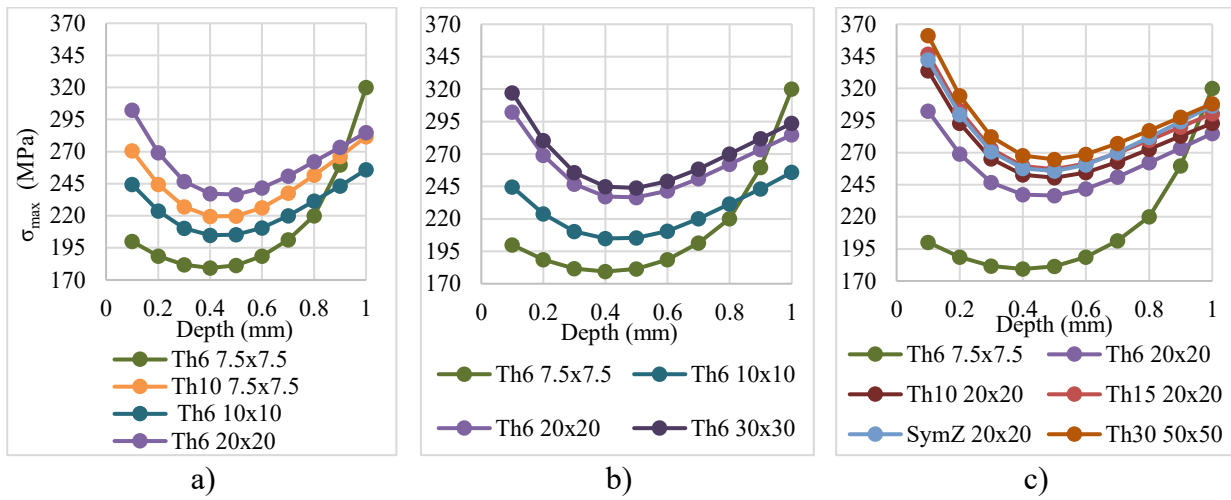
- The first peak is not completely captured by the current model because of the increment size. Increments of 0.1 mm were employed, whereas the results found in the literature were obtained employing increment sizes that reached values of 0.02 mm near the surface.
- A test was performed with increments of 0.05 mm for the plasticity factor of $f = 0.96$. It indicated that the stress peak near the surface was now being represented by the current model.
- A considerable deviation is identified for the highest loads, starting from $f = 0.96$. This deviation is more easily seen in the nonuniform stress variation curve.

After checking the plastically deformed regions, it was identified that these regions extended to the boundaries of the model. For this reason, the size of the model had to be increased. However, working with 3D elements has some computational limitations (hardware and memory) due to the increase in the size of the model. Therefore, it was necessary to optimize the geometry effects on the strains and stress values. Other research work employed harmonic elements, which are much computationally lighter than 3D elements; it was certainly not necessary to worry about the size of the model.

5.1 EVALUATION OF THE GEOMETRY EFFECTS CONSIDERING PLASTICITY

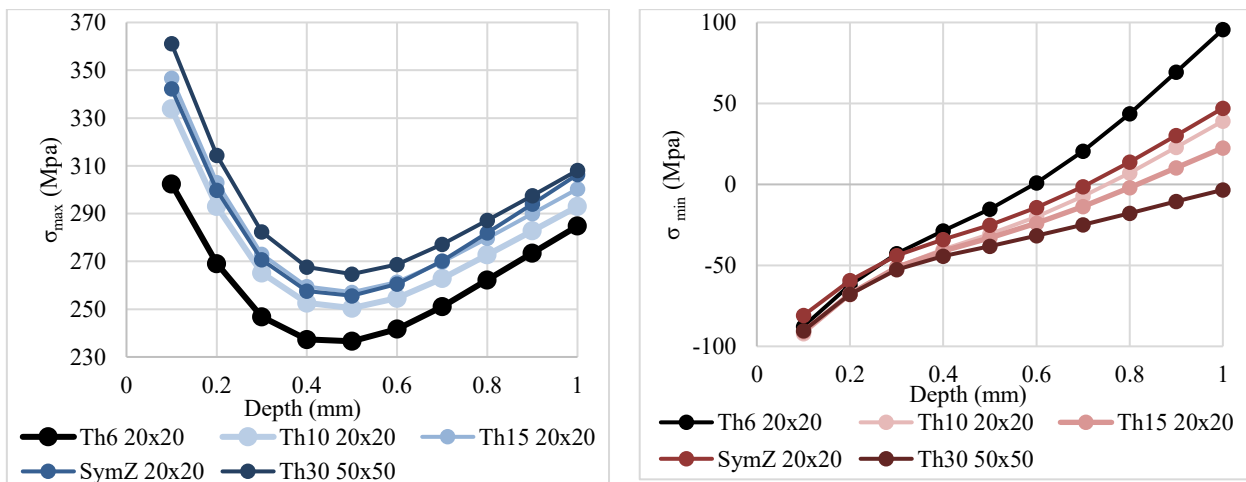
First, a few evaluations were performed varying the thickness and the width of the model. The uniaxial load was kept constant. This first analysis indicated that both variables influenced the residual stress values, as shown in Figure 5-2a. After that, the influence of the size was evaluated for sizes of 7.5 mm, 10 mm, 20 mm, and 30 mm (Figure 5-2b). It is possible to consider that the difference is relatively small between sizes of 20 mm and 30 mm. Lastly, the influence of the thickness was assessed. It is shown in Figure 5-2c that thicknesses of up to 15 mm influence a lot on the residual stress values. For higher thicknesses, including the symmetry analysis performed with the thickness of 15 mm, little variation from the 15 mm case was identified. Therefore, the following dimensions were chosen: (a) width of 20 mm and (b) thickness of 15 mm.

Figure 5-2 – Influence of the model geometry on the final residual stress values. Where, for example, "Th6" means thickness of 6 mm and "10x10" stands for FEM model size of 10 mm x 10 mm. SymZ stands for the model with a symmetric boundary condition applied at Z=0.



Once the geometry was chosen, it is possible to evaluate the variations of the maximum and minimum stresses for a slight variation in the thickness. The variation of the minimum and maximum stresses as a function of the thickness is shown in Figure 5-3. The model with a thickness of 30 mm is considered as a reference. The reference model demands a much higher computational cost and does not generate a large difference from the chosen geometry.

Figure 5-3 – Maximum and minimum stresses for five different thicknesses ($f=1$). Where, for example, "Th6" means thickness of 6 mm and "20x20" stands for FEM model size of 20 mm x 20 mm. SymZ stands for the model with a symmetric boundary condition applied at Z=0.

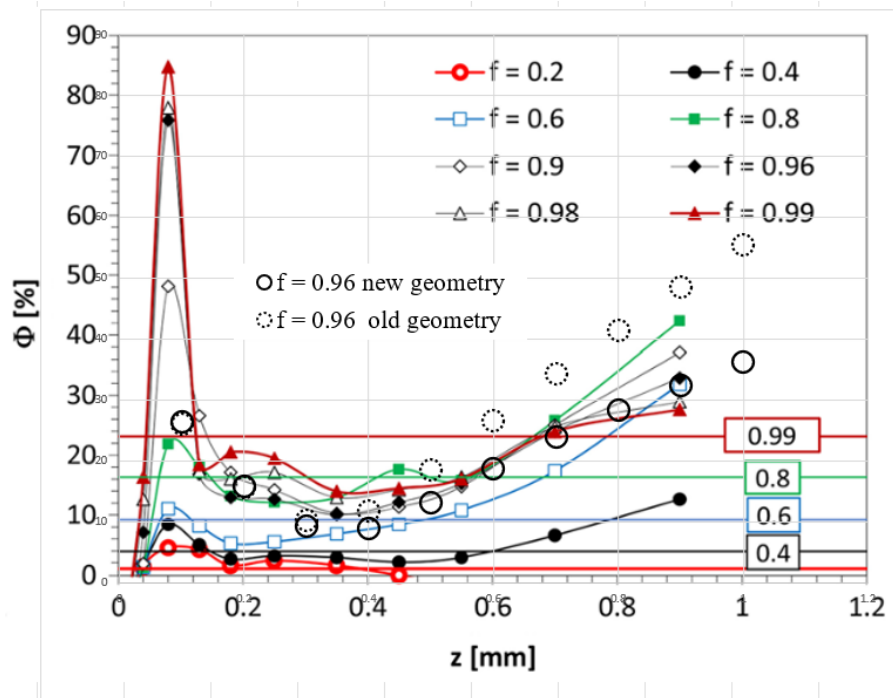


The evaluation was performed once again after the definition of the workpiece dimensions. The non-uniform residual stresses obtained for the new dimensions are presented next.

5.1.1 Validation of the non-uniform profile

The loading factor of $f = 0.96$ was chosen to verify the behavior of the model with the chosen dimensions. One additional simulation was performed for the model with the chosen geometry. The nonuniform stress profile variation Φ was generated. Both curves were put together with the data obtained in the literature, as shown in Figure 5-4. For the chosen geometry, the stresses are closer to the values found in the literature. This time, the non-uniform residual stresses agreed with the expected stress profile.

Figure 5-4 –Nonuniform stress profile variation Φ for the current model ($f = 0.96$) together with the results obtained in the literature.



Source: Nobre *et al.*[33].

For the first increments, the value changed only slightly. This can be explained by the fact that the plastically deformed region induced in the first increments does not reach the edge of the model in either case. In addition, the stress concentration factor and the extent of the plastically deformed region are smaller for shallower holes. In reality, the stress concentration factor for shallower (first increments) holes is smaller in comparison to the stress concentration factor for deeper holes (last increments). The difference is more relevant to higher hole depths. After increment 3, the deviation identified in the residual stress values became higher than 2.5% between the initial model and the refined model, reaching 20% in the last increment.

The dimensions of the model influence the stress profile – it was identified that a deviation of approximately 20% in the stress (profile) of the last increment was generated between the first and the refined model. Once a reliable model was obtained (refined model), it is possible to proceed to the next step: correlate the numerical strain values with the experimental values. A modified algorithm will be presented in the next chapter. Modifications were implemented to enable the

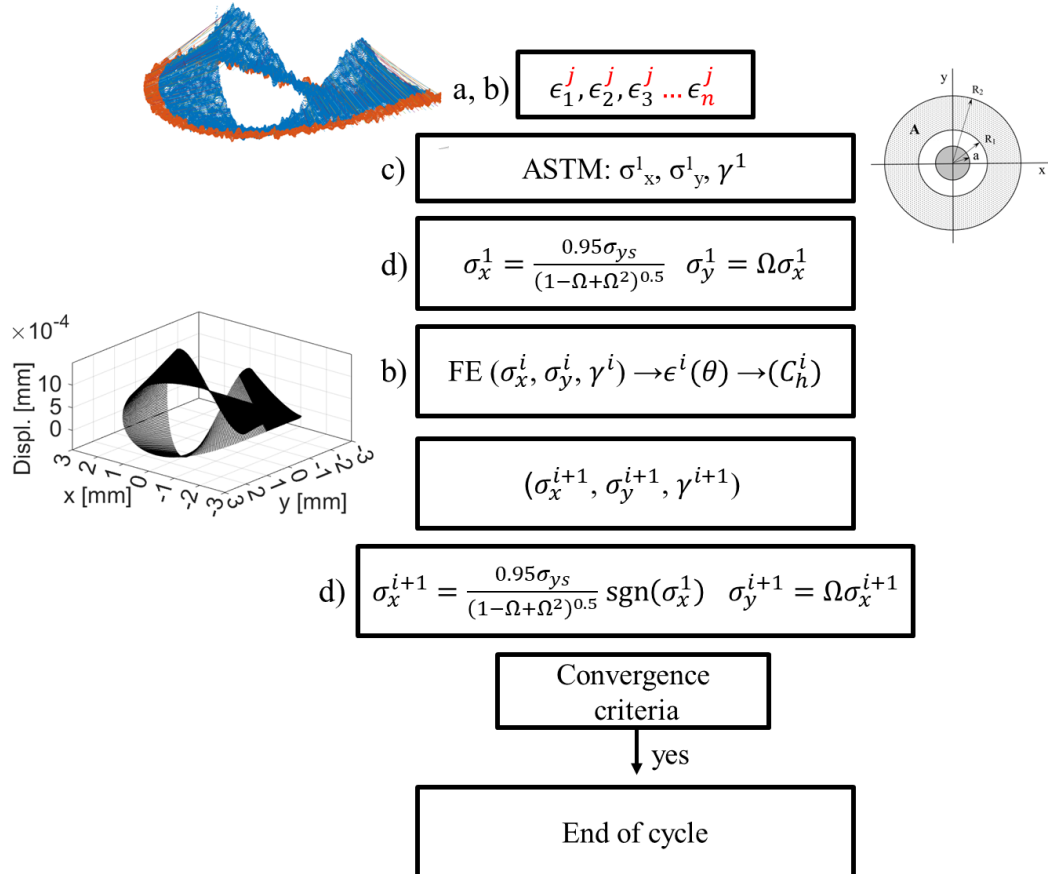
iterative algorithm to receive the full field data obtained from experimental measurements with the aid of an optical device.

6 ITERATIVE APPROACH

The iterative algorithm will be detailed in this section. The adaptation to receive the full field data, as well as the computation of the initial guess value for the next iteration of the iterative approach, will be presented.

The topics covered in this chapter are represented in Figure 6-1: a) use of optical devices to measure the displacement values, b) computation of the inclination of the displacements, c) elastic evaluation of the residual stresses following the procedures presented by Schajer [72], d) convergence issue regarding the signal oscillation of the stress values for the next iteration. Two different strategies will be employed to compute the inclination of the displacements: first, the inclination of the displacements is computed for the numerical data. After that, a strategy will be presented for computing the inclination of the displacements from the experimental data.

Figure 6-1 - Modifications to the iterative algorithm.



6.1 MODIFIED ITERATIVE ALGORITHM

The main objective of the iterative algorithm proposed by Vangi [5] is to match the experimentally measured strain values with the strain profile computed with the numerical model.

This process enables the correction of plasticity effects and the identification of stresses and the direction angle. The iterative process begins with an elastic evaluation of the residual stresses, using the strain in accordance with the standard ASTM procedures. Depending on the stress level obtained in the elastic evaluation, the value can be corrected to bring it to the elastic range. Then, the numerical model obtains the strain profile for this residual stress state. The strain profile is modeled with five-term Fourier coefficients, as shown in Eq. (6-1). The Fourier coefficients C_s were associated with coefficients A and B to link the strain values to the principal stresses (Eq. (6-2)). The superscript ‘ l ’ represents the values for the elastic evaluation.

$$\epsilon^l(\theta) = A^l(\sigma_x^l + \sigma_y^l) + B^l(\sigma_x^l - \sigma_y^l) \sum_{h=1}^m \left[\frac{C_h^l}{C_1^l} \cos(2h(\theta - \beta^l)) \right] \quad (6-1)$$

$$A^l = \frac{C_0^l}{\sigma_x^l + \sigma_y^l}; \quad B^l = \frac{C_1^l}{\sigma_x^l - \sigma_y^l} \quad (6-2)$$

This equation was then forced to pass through the experimental strain gauge readings, resulting in a set of 3 equations and 3 unknown values of the main stresses and direction. Three modifications to the existing iterative algorithm were proposed in this thesis and are detailed below.

The first modification involved substituting the elastic evaluation procedure. The iterative process starts with a residual stress evaluation according to the ASTM standard procedures. To employ the full field data, the traditional strain gauge experimental technique was replaced by an elastic evaluation following the strain gauge formalism for optical techniques [73].

The second modification addresses a convergence issue. The Von Mises equivalent stress is computed according to the relationship in Eq. (6-3).

$$\sigma_{eq} = \sqrt{(\sigma_x^l)^2 + (\sigma_y^l)^2 - \sigma_x^l \sigma_y^l} \quad (6-3)$$

Then, the principal stresses are lowered to 90% of the yield stresses according to Eq. (6-4) and (6-5).

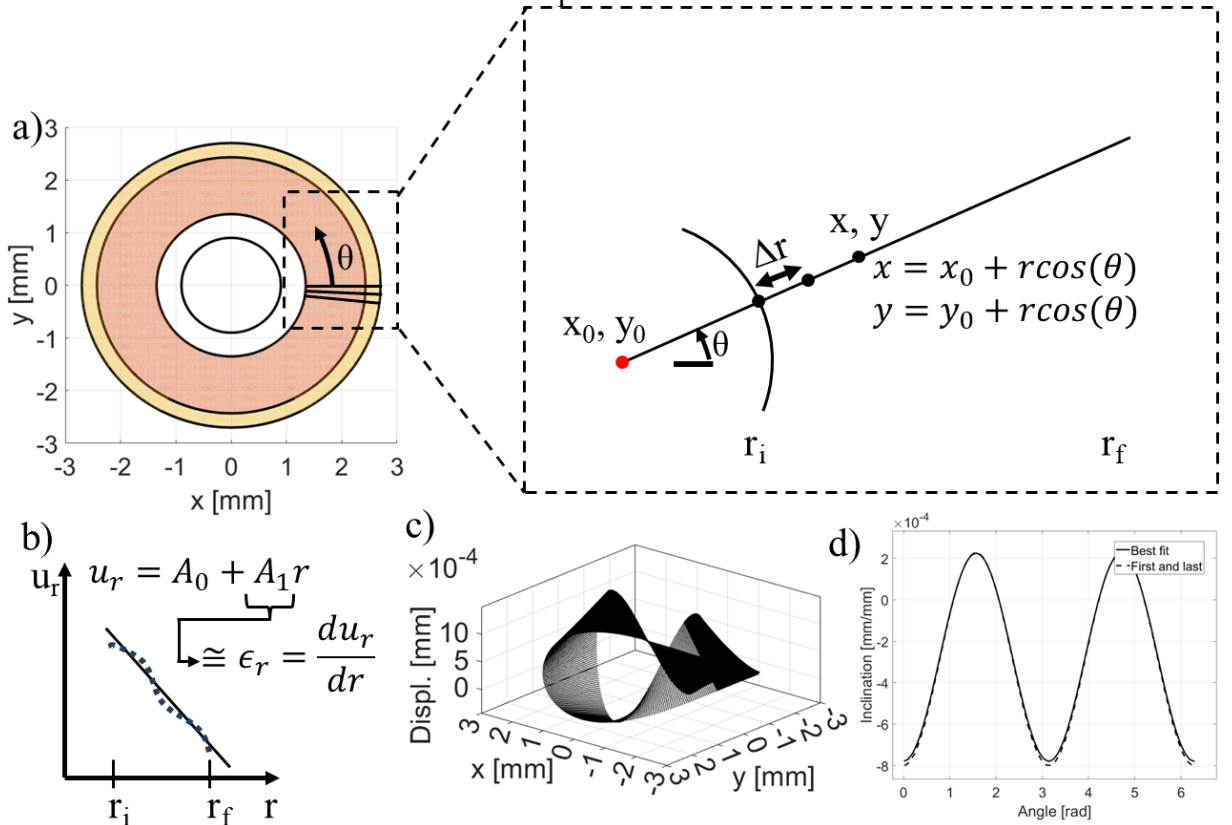
$$\sigma_x^l = \frac{0.90\sigma_{ys}}{\sqrt{1 - \Omega^l + (\Omega^l)^2}} \quad (6-4)$$

$$\sigma_y^l = \Omega^l \sigma_x^l \quad (6-5)$$

For some loads, it was observed that the algorithm oscillated between positive and negative signals for σ_x . The relationship works well for all cases, but it does not keep the negative signal when σ_x is negative. For this reason, the sign of σ_x was linked to the signal of the σ_x elastically evaluated to reduce oscillations and speed up the convergence process, avoiding unnecessary iterations.

The third modification is related to the data collected after each increment. The data obtained with the aid of the optical device can substitute the strain gauge data points. It is necessary to relate the traditional strain gauge measurements to a physical quantity somehow related to it. The partial derivative of the radial displacement with respect to the radius as a function of theta is possible to be computed from the data (displacements) obtained with optical techniques. To compute the partial derivative of the radial displacement with respect to the radius as a function of theta, the annular area was divided into small sectors by a few straight radial lines for each theta (every $\Delta\theta=1^\circ$), as shown in Figure 6-2 for $\theta=1^\circ, 2^\circ$, and 3° .

Figure 6-2 (a) Representation of the annular area divided into sectors evenly distributed around the hole, a few points were captured along the radius with an interval of $\Delta r=0.05$ mm, (b) partial derivative of the radial displacement with respect to the radius (inclination) for a single value of θ and (c) lines adjusted representing the radial displacement for each sector producing the complete profile.



The minimum number of lines is limited by the number of pixels near the inner radius of the annulus region of experimental data. A straight line was adjusted to fit the displacement points on each radial line Figure 6-2(b). Figure 6-2c illustrates the 360 equally distributed lines around the hole adjusted to fit the displacement points for a uniaxial load within the elastic region. A test was performed comparing the fitting of a line connecting the first and last points, as well as a linear least-squares fitting method that considered every point on the line. The latest presented a smoother angular profile for experimental data, while for numerical data, both cases presented a smooth profile with

slight differences (Figure 6-2 (d)). The minimum and maximum radii values were obtained from the recommendations given in the literature [72]. The inclination of the displacement is not constant with the radius, but its variation was considered small within the annuli adopted in this work.

The entire profile around the hole of either the displacements or the inclination of the displacements can be obtained from the experimental data. The inclination of the displacements was chosen, which is closely related to the strain values measured with the traditional strain gauges. This quantity is obtained from both the measurement data and the output of the numerical model. The iterative process compares the inclination of the displacements from the measurement data and the output of the numerical model, trying to approximate them as closely as possible.

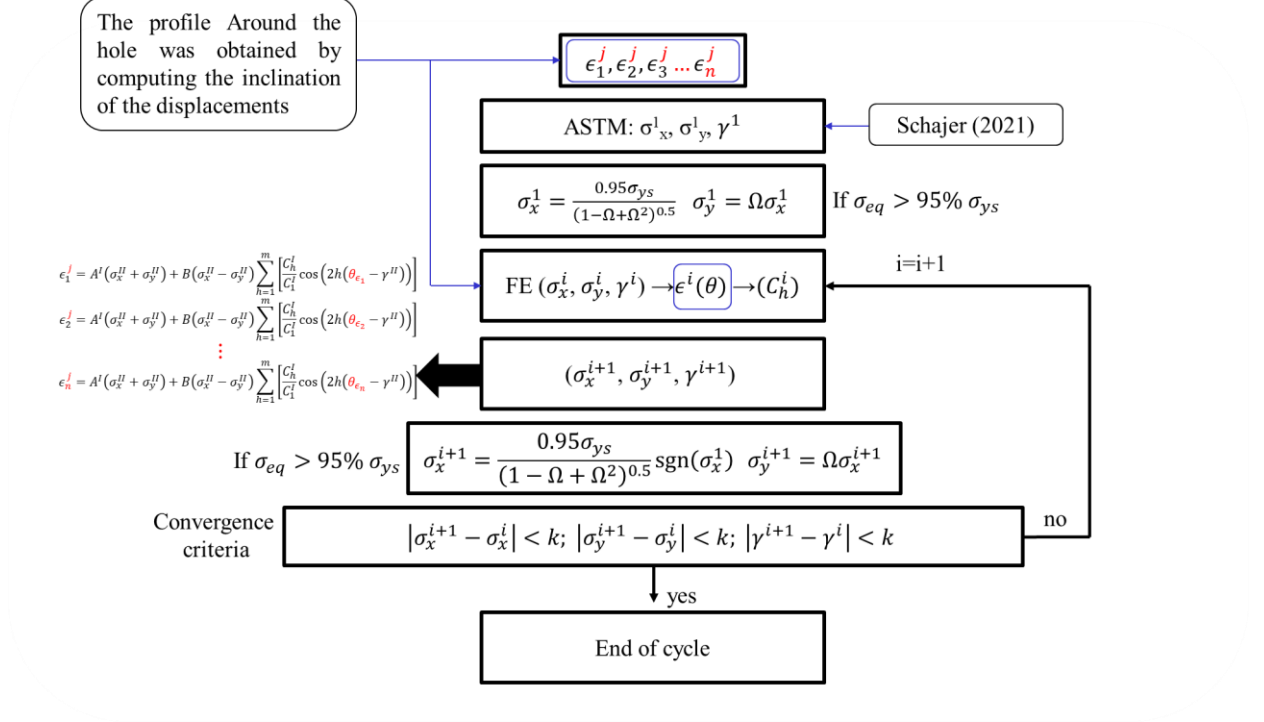
The three strain points typically obtained when strain gauges are used are then replaced by the entire profile of the inclination of the displacements. The discrete data acquisition performed every 1° enables the use of 360 points, as represented in Eq. (6-6). Where A and B represent coefficients associated with the calibration constants, and C represents the Fourier coefficients. The solution contains the values of the principal stresses σ and the direction β .

$$\begin{aligned}
 \epsilon_1^j &= A^I(\sigma_x^{II} + \sigma_y^{II}) + B^I(\sigma_x^{II} - \sigma_y^{II}) \sum_{h=1}^m \left[\frac{C_h^I}{C_1^I} \cos(2h(\theta_{\epsilon_1} - \beta^{II})) \right] \\
 \epsilon_2^j &= A^I(\sigma_x^{II} + \sigma_y^{II}) + B^I(\sigma_x^{II} - \sigma_y^{II}) \sum_{h=1}^m \left[\frac{C_h^I}{C_1^I} \cos(2h(\theta_{\epsilon_2} - \beta^{II})) \right] \\
 &\vdots \\
 \epsilon_{360}^j &= A^I(\sigma_x^{II} + \sigma_y^{II}) + B^I(\sigma_x^{II} - \sigma_y^{II}) \sum_{h=1}^m \left[\frac{C_h^I}{C_1^I} \cos(2h(\theta_{\epsilon_{360}} - \beta^{II})) \right]
 \end{aligned} \tag{6-6}$$

Then, the solution is found for the chosen number of equations, considering the residue of each equation. Each residue represents the difference between the evaluated measured inclination of the displacement and the five-term Fourier equation representing the inclination of the displacements obtained numerically. One point is obtained for every one degree in Eq (6-6). The Fourier coefficients are obtained with the aid of the Matlab command ‘fit’ set to model type ‘Fourier’ and the n number of equations can be solved with the aid of the command ‘fsolve’.

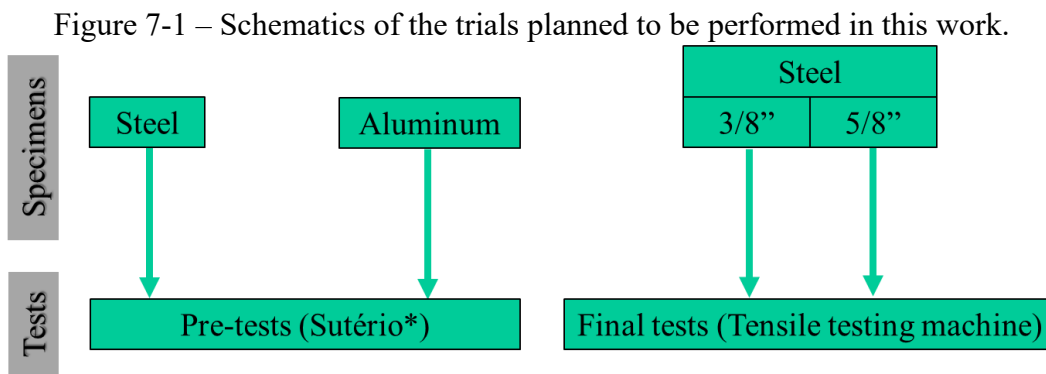
The complete modified iterative algorithm together with the modifications is illustrated in Figure 6-3.

Figure 6-3 – Diagram of the modified iterative algorithm.



7 EXPERIMENTAL PLANNING

The tests were divided into two stages: pre-tests and final tests, as shown in Figure 7-1. The pre-tests were designed to understand the experimental setting and identify the potential of the technique. The manual loading device was designed by a previous Ph.D student called Sutério [74]. The pre-tests were primarily created due to the time constraints associated with the availability of the tensile testing machine. Then, the tests were performed on a tensile testing machine employing specimens with a more suitable geometry.

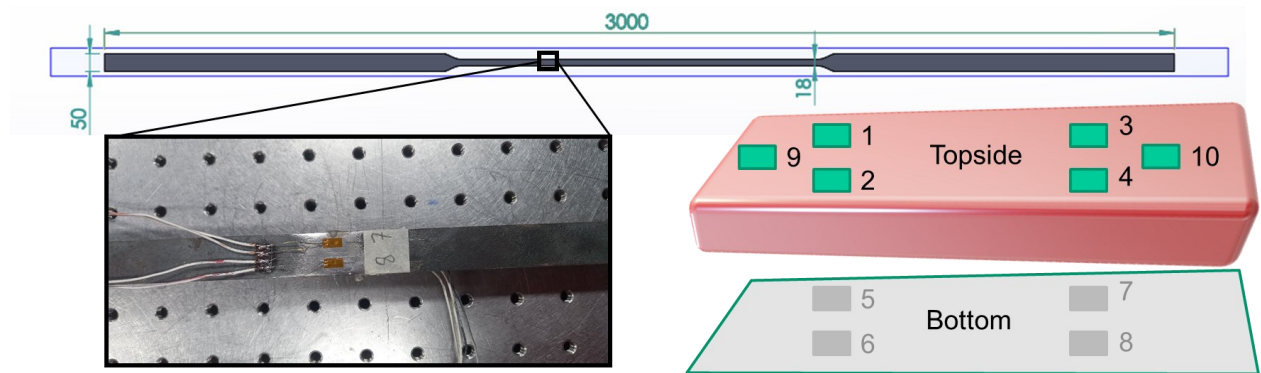


The details regarding the equipment, specimen material, and geometries are presented in this chapter.

7.1 Pre-test

A manual loading device available at LABMETRO was employed to induce a stress state on steel and aluminum specimens[74]. This equipment has a loading capacity of approximately 20000 N and a total length of roughly 3 m. Dog-bone-shaped specimens were prepared for the steel workpiece shown in Figure 7-2. The reduction in cross-sectional area was necessary to induce high levels of equivalent residual stresses using the available equipment. However, keeping a uniform stress distribution during the loading phase was challenging.

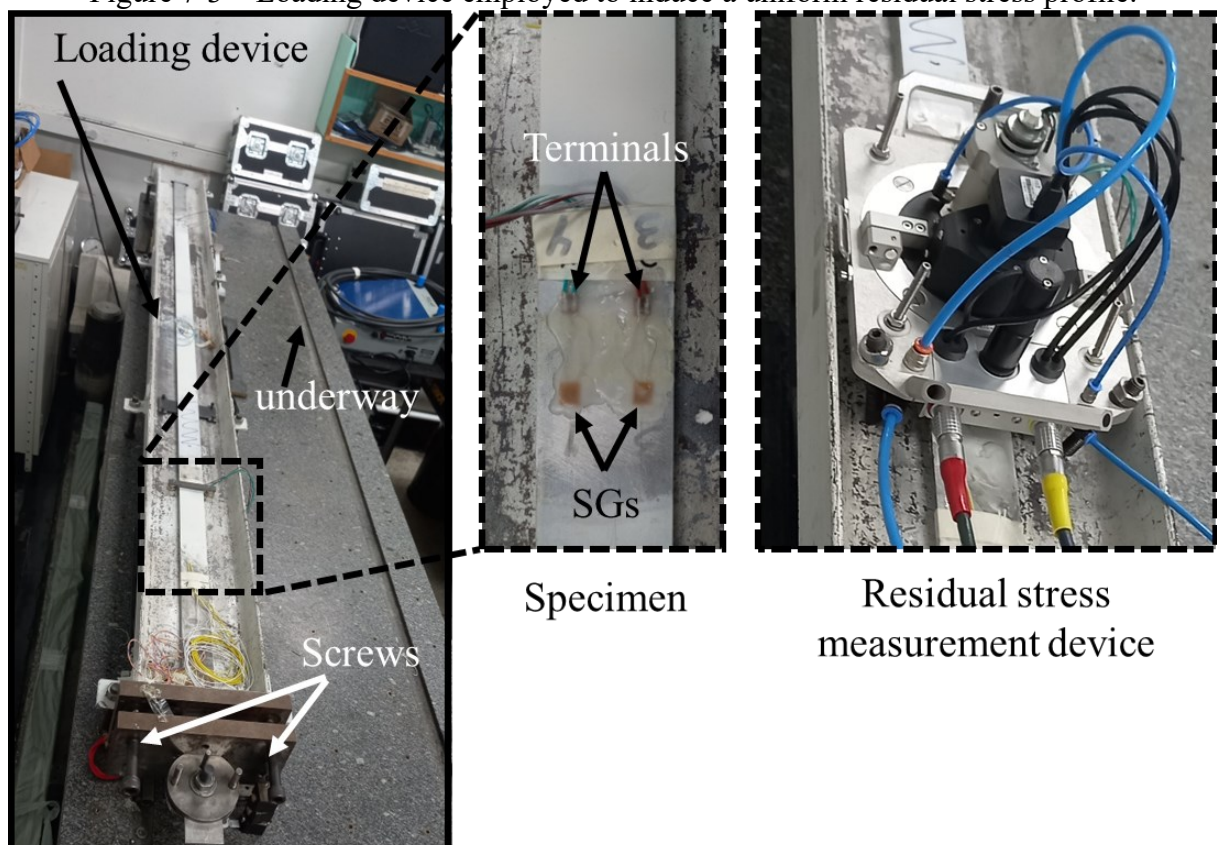
Figure 7-2 – Long dog-bone-shaped steel specimen instrumented with 10 strain gauges.



The loading process involved tightening six screws, three on each extreme end of the beam. High divergence was identified between the 10 strain gauge measurements due to unbalanced loading. The gauges presented a coherent measurement under upward and downward bending, as well as lateral bending. However, for both uniaxial traction and compression, the gauges did not present coherent results. Details are given in Appendix D. Therefore, a second workpiece made of a less stiff material was employed.

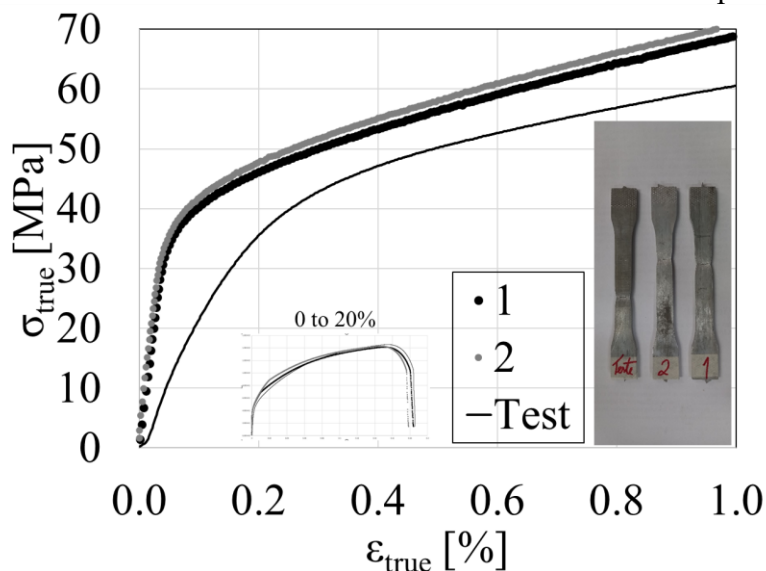
Aluminum was chosen to substitute the steel bar. In this case, it was not necessary to create the bone dog shape, keeping the width of 50 mm and thickness of 5 mm as shown in Figure 7-3. Strain gauges were attached to the beam to control the loading process.

Figure 7-3 – Loading device employed to induce a uniform residual stress profile.



A few specimens were prepared to perform tensile tests on this material. The tensile test was performed to obtain the stress-strain profile, with the primary goal of determining the yield stress (48.65 MPa) and the tangent modulus. The stress-strain curve obtained is presented in Figure 7-4, as well as the specimens after the tensile test.

Figure 7-4 - True stress vs strain curve for three aluminum specimens.



Specimen “Test” was employed to determine the approximate profile of the stress-strain curve, allowing the technician to evaluate the optimal timing to remove the clip gauge and avoid introducing any damage to it. Some problems associated with this first experiment made its results less reliable than the others; therefore, it was not considered when computing the yield stress and the tangent modulus.

The average yield stress value obtained was 48.65 MPa. The tensile machine employed in this work is not suitable for estimating the Young modulus. For this reason, the Young modulus was obtained from the literature.

The workpiece was instrumented to measure strains, allowing the consistent application of a uniform load. The applied stress was computed based on the measured strains. Six strain gauges were attached longitudinally to the specimen, two on each side of the top side of the specimen. Omega strain gauges specially developed for use in aluminum with 120Ω were connected to an HBM Canhead CB1014-120 system configured to work with 120Ω strain gauges with a $\frac{1}{4}$ bridge. The orientation of the strain gauges is shown in Figure 7-3.

Before starting tractioning the specimen, it was necessary to set the strain measurement to zero. The zeroing process was performed by pressing the specimen against a straight guide to be rectified, and only then were the strain gauges zeroed. This process ensured that the beam was nearly straight before zeroing.

The specimen was tractioned to a strain of $677.5 \mu\text{m/m}$. This is the average strain measured by a total of six gages at the end of the loading process – this equipment does not perform force

measurement. The minimum value was 652 $\mu\text{m/m}$ and the maximum value was 702 $\mu\text{m/m}$. The maximum uncertainty obtained for this device is 8% for measurements with the 100 $\mu\text{m/m}$ to 1000 $\mu\text{m/m}$ range [74]. The average stress value can be obtained by $\sigma = \epsilon E$, yielding an average stress of $\sigma = 46.7 \text{ MPa}$ ($f = 0.9399$). One single drilling test was conducted to evaluate the applicability of the method. No repetition was performed.

For better performance, the hole diameter was acquired after the hole was completed. The acquired full-field data can be employed to feed three different regions: near, medium or intermediate, and far. As specified by Schajer [72], each region presents its annulus diameter: near ($R1 = 1.5 \cdot RF$; $R2 = 3 \cdot RF$; $Rx = 2.7 \cdot RF$), medium ($R1 = 2 \cdot RF$; $R2 = 4 \cdot RF$; $Rx = 3.6 \cdot RF$), and far ($R1 = 2.5 \cdot RF$; $R2 = 5 \cdot RF$; $Rx = 4.5 \cdot RF$).

7.2 FINAL TESTS

A material testing machine was used for the final tests. The machine must be robust enough to apply a very powerful tensile force. Another requirement of the equipment is that it must keep the force stable in the same level during the experiment. Tractioning a robust workpiece to the yield stress requires a powerful machine. An MTS C45.305 with a force capacity of 200 kN was employed. The maximum force was limited by the LPS.105 load cell, which has a capacity of 100 kN and meets the calibration requirements of ISO 376 Class 0.5 for the following loading range: 1 to 100% of the load cell capacity. Therefore, the machine has a maximum capacity of 100 kN, but its use is restricted to a range of 20% to 80% of its maximum capacity.

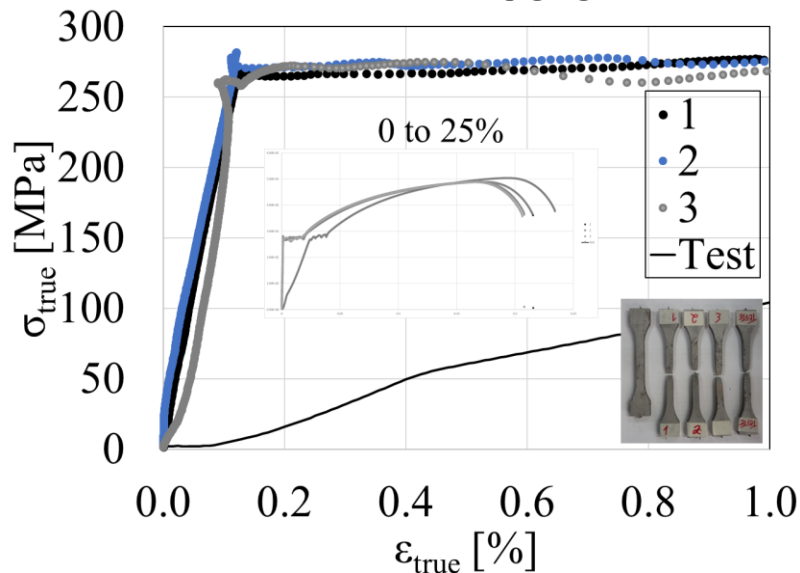
First, the specimen was loaded to the required tensile force, and then the machine was shut down. However, it did not hold the force. The force decreased abruptly and continued to decrease.

A second option was to employ a hold command. Shortly before the machine entered the hold command, the force dropped approximately 5% and then stabilized at a force lower than the target value. One strategy that mitigated this problem was to reduce the speed shortly before the hold command. It decreased the initial drop, achieving values closer to the target. Once the drop magnitude was identified, it was summed to the target value to stabilize as close as possible to the target value.

The specimens were made of steel ABNT 1020 and underwent a stress relief heat treatment. Their shape follows the pattern of tensile test specimens with shoulders compatible with the wedge grips and a test cross-section designed according to the maximum load available. A few auxiliary specimens were prepared to perform tensile tests on this material. The main and the auxiliary specimens underwent the heat treatment together. The tensile test was performed to obtain the stress-strain profile, with the main goal of determining the yield stress (269.69 MPa) and the tangent modulus. The stress-strain curve obtained is presented in Figure 7-5, as well as the specimens after

the tensile test. As mentioned earlier, the specimen “Test” was employed only to identify the approximate profile of the stress-strain curve.

Figure 7-5 - True stress vs strain curve for four 3/8” steel specimens. Curve “Test” was obtained without the use of a clip gauge.

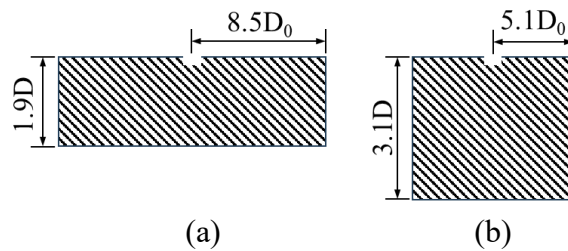


The raw material was acquired in accordance with the geometrical constraints imposed by the wedge grips (width) and the available bar dimensions. Flat bars with two different thicknesses were chosen: 9.5 mm (3/8 in) and 15.9 mm (5/8 in). The main specimens were designed to introduce high levels of stresses with a plasticity factor near unity by using 80% of the maximum capacity of the tensile machine – the operator indicated a range of 20% to 80% of the machine capacity. This was the main design parameter to specify the width of the specimen, together with the specification of the minimum thickness defined in the standard ASTM [11]. The final geometry was obtained with water jet cutting. Besides the main specimen, smaller specimens were prepared to characterize the material with tensile tests. The small specimens enabled the acquisition of the $\sigma \times \epsilon$ curve for both thicker and thinner specimens.

The specimens underwent a stress relief heat treatment on a FORTELAB horizontal tube furnace. They were centered in the middle of the chamber – there is better temperature control in this region. First, a vacuum was applied three times to the chamber to remove air from inside the chamber. The temperature was kept at 900°C for one hour with a warming rate of 8°C/min and slow cooling inside the furnace. Using a gas mixture with a concentration of 95% Argonium and 5% Hydrogen. The system indicated the temperature of 900°C, while the pyrometer indicated a value slightly higher than 925°C at the one-hour stage.

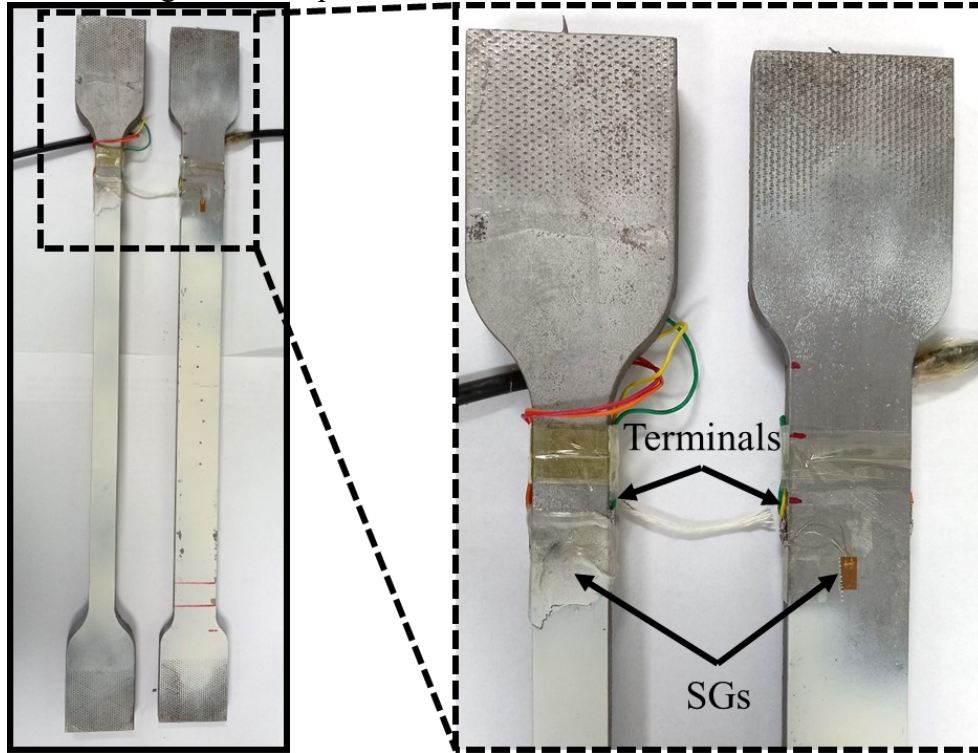
A few relevant aspects need to be taken into account when dimensioning the specimen. First, the dog-bone shape is employed to use the largest area available in the wedge grip, reducing the chances of slipping. Second, the cross-section was defined based on two parameters: the stress required in the component and the maximum loading capacity of the machine. Induced stresses near the yield stress are required in this work. The standard presents recommendations for the minimum thickness of the workpiece: ASTM 20a [11] ($W/D \geq 0.6$) and ASTM 13a [75] ($W/D \geq 1$). In addition, Schajer [73] presents the minimum required distance from the edge is approximately equal to $3D_0$. Plasticity effects enlarge these spacing values. For this reason, two specimens were prepared: a) three times the minimum thickness and three times the minimum distance to the edge, b) five times the minimum thickness and 2 times the minimum distance to the edge. The cross-sectional area remained constant for both cases, but the bending stiffness is four times higher for the thicker specimen.

Figure 7-6 – Specimen cross-section for thicknesses of (a) 9.5 mm (3/8 in) and (b) 15.9 mm (5/8 in).



The workpiece was designed to be robust with the maximum possible cross-sectional area according to the loading capacity of the tensile machine. The width of the optical device also needs to be taken into account when defining the length of the specimen due to the large size of the wedges. Therefore, the maximum allowable specimen length for this machine was employed. The specimens were instrumented with two strain gauges, one located on the frontal area and the other on the back. Terminals were glued on the right and left sides, as illustrated in Figure 7-7.

Figure 7-7 – Specimens for residual stress measurement.



First, one side of the specimen was fixed with the aid of the wedge. Then, the other side of the specimen was fixed. A small load was applied by the grips due to the vertical displacement. Finally, the full load was applied to the specimen.

A portable residual stress device was employed to perform the stress measurements. It was necessary to fix a support base on the specimen to firmly position the device and avoid rigid body motion during the experiments, as shown in Figure 7-8 and Figure 7-9.

Figure 7-8 – Simulation of the experimental setting within the frame of the tensile machine.

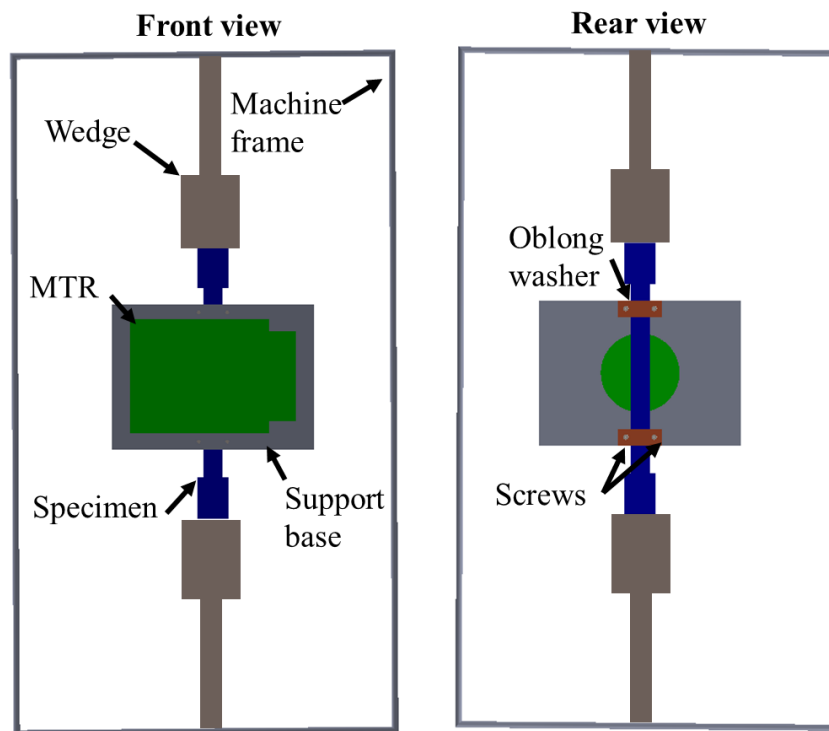
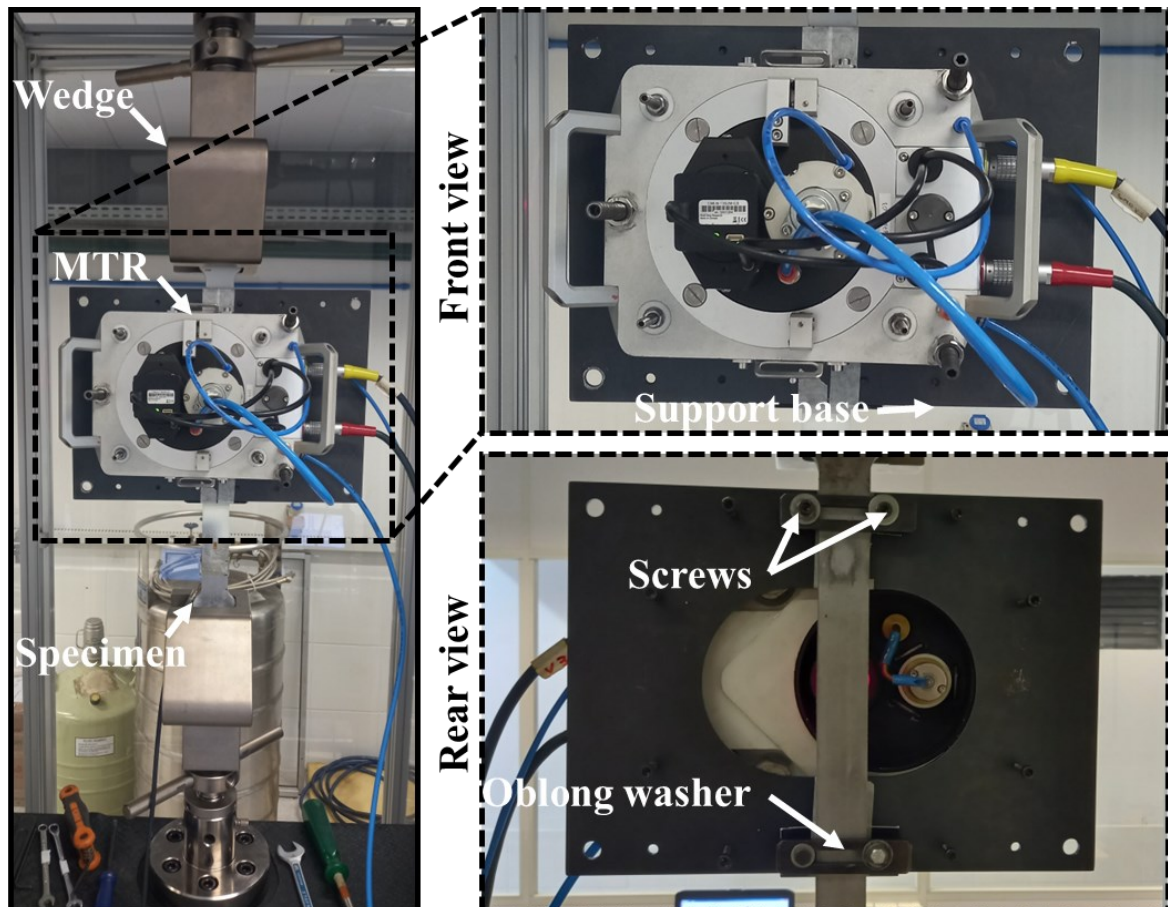


Figure 7-9 – Experimental setting within the tensile machine.



The machining (drilling) parameters employed were the feed rate $v_f=0.20$ mm/min and the rotational speed of approximately 220000 rpm. The tool employed was a carbide milling cutter inverted cone (Carbide Komet h2.314.016) with a diameter of 1.6 mm. Each tool was employed to drill one single hole.

Once the machine applied the load, three residual stress measurements were performed: T1, T2, and T3. The load employed was 71 kN, which is equivalent to a stress value of $\sigma=248.4$ MPa ($f=0.8816$) for the specimen with thickness of 9.5 mm (3/8 in).

8 NUMERICAL RESULTS AND DISCUSSIONS

This section provides a brief overview of the computation of the partial derivative of the radial displacement with respect to the radius as a function of θ , along with an analysis of the grid spacing. Then, a thorough numerical validation of the iterative algorithm is presented, investigating three different annuli: a smaller radius (near), an intermediate radius (medium), and a large radius (far).

8.1 COMPUTATION OF THE PARTIAL DERIVATIVE

The displacement data generated by the numerical model presents an irregular spacing. It is possible to see in Figure 8-1 that for the near case, the spacing is very irregular. For instance, the outer annular region contains only a single row of nodes. The outer annular region of the intermediate case comprises two rows of nodes that do not have the same angular spacing, as shown in Figure 8-2.

Figure 8-1 – Nodes located within the inner and outer annular area (near case) together with the marginal nodes located near the inner and outer limits. Annulus diameter: near ($R1=1.5*RF$; $R2=3*RF$; $Rx=2.7*RF$).

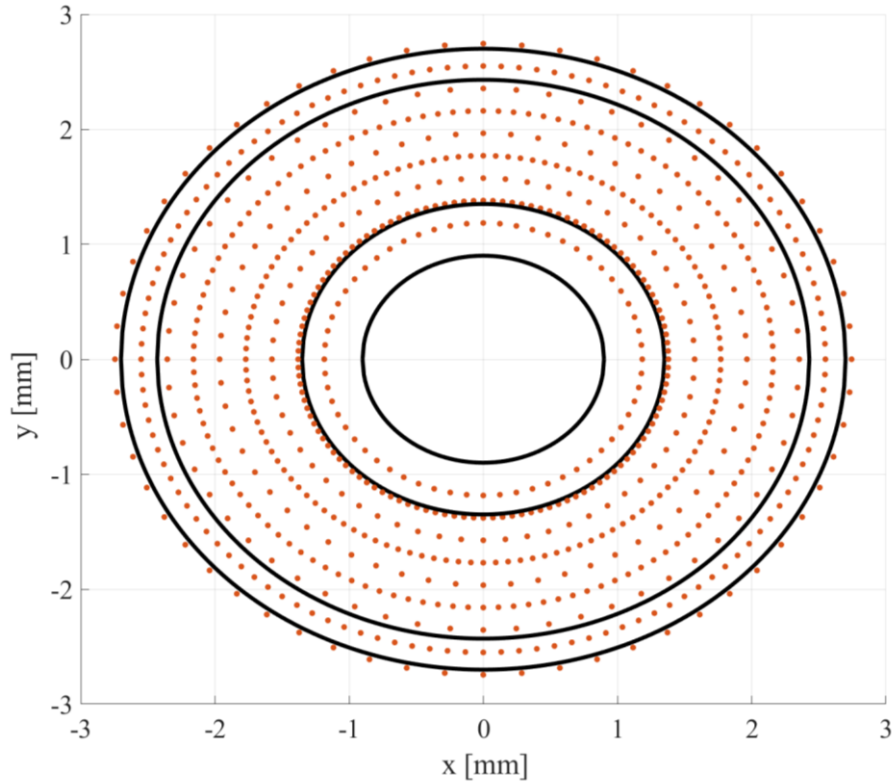
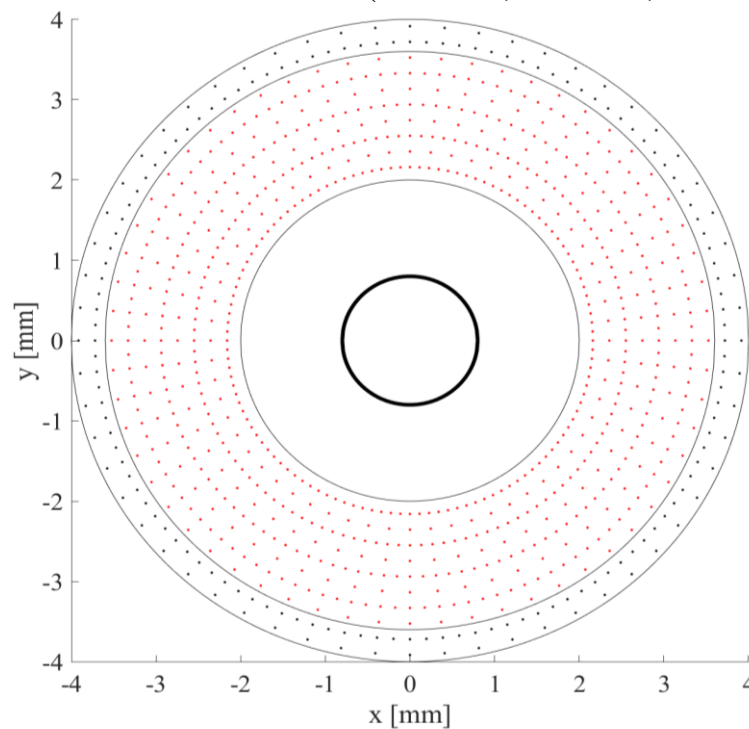
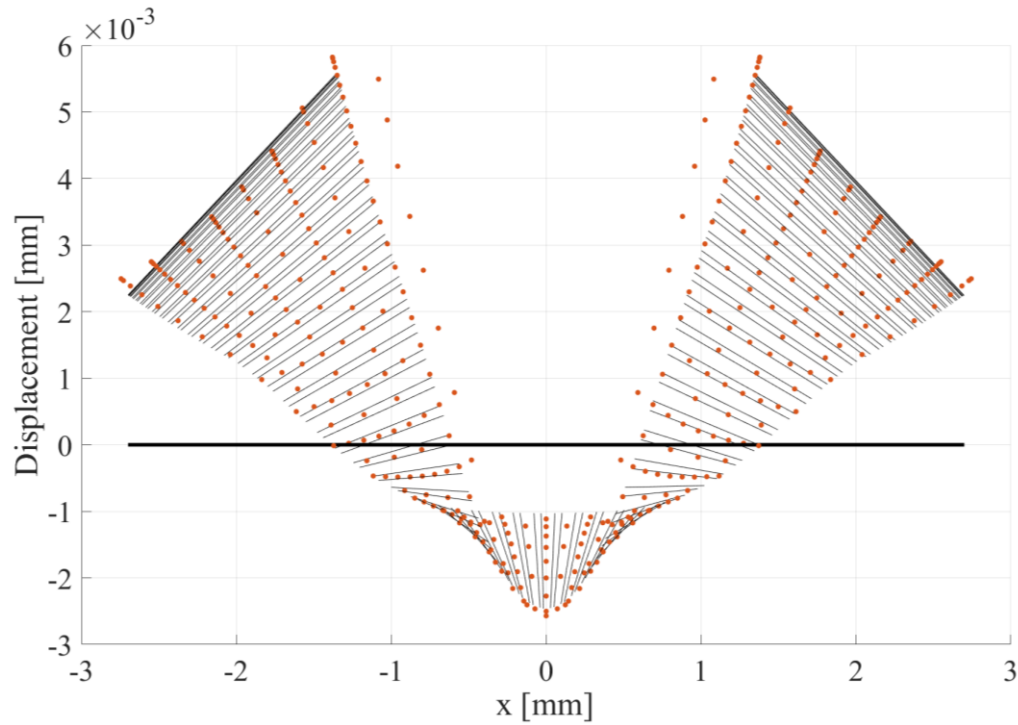


Figure 8-2 – Nodes located within the inner (red dots) and outer (black dots) annular area (intermediate). The circle with the smallest diameter illustrates the approximate dimensions of the hole. Annulus diameter: medium ($R1=2*RF$; $R2=4*RF$; $Rx=3.6*RF$).



A bidimensional interpolation map was employed to generate straight curves every $\theta = 1^\circ$ representing the displacement, as shown in Figure 8-3. The profile around the hole was obtained by computing the inclination for every 1° , generating the partial derivative of the radial displacement with respect to the radius as a function of the angle theta.

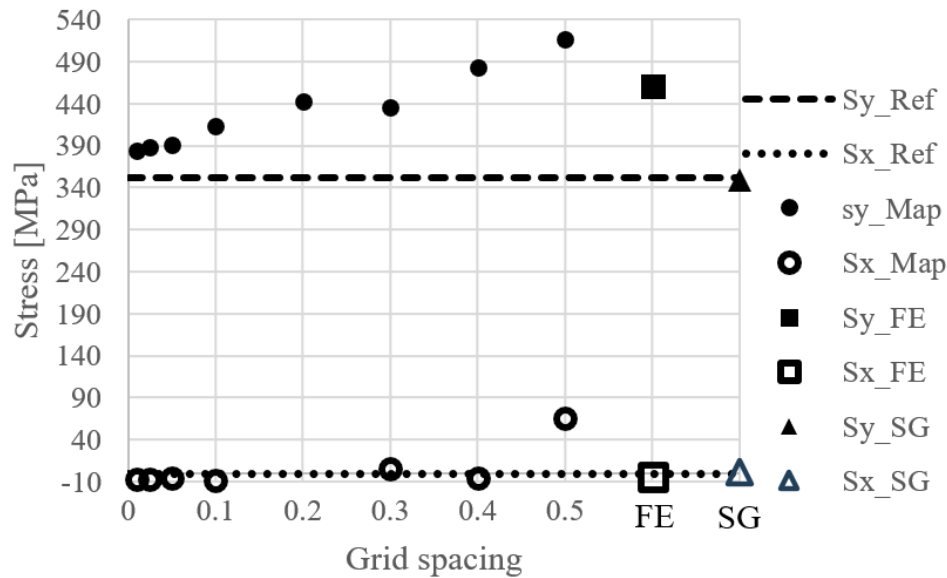
Figure 8-3 – Nodes located within and nearby the inner and outer annular area (near case) employed in calculating the partial derivative. The black lines represent the approximate displacement profile for the annular region.



The mesh refinement in FE models is generated considering the sensitivity analysis and the cost-benefit between the sizing and computational cost. Numerical data obtained from nodes have a much smaller density point when compared to the density of pixels in experimental data. Besides that, the node density on a numerical model usually changes with the distance from the hole.

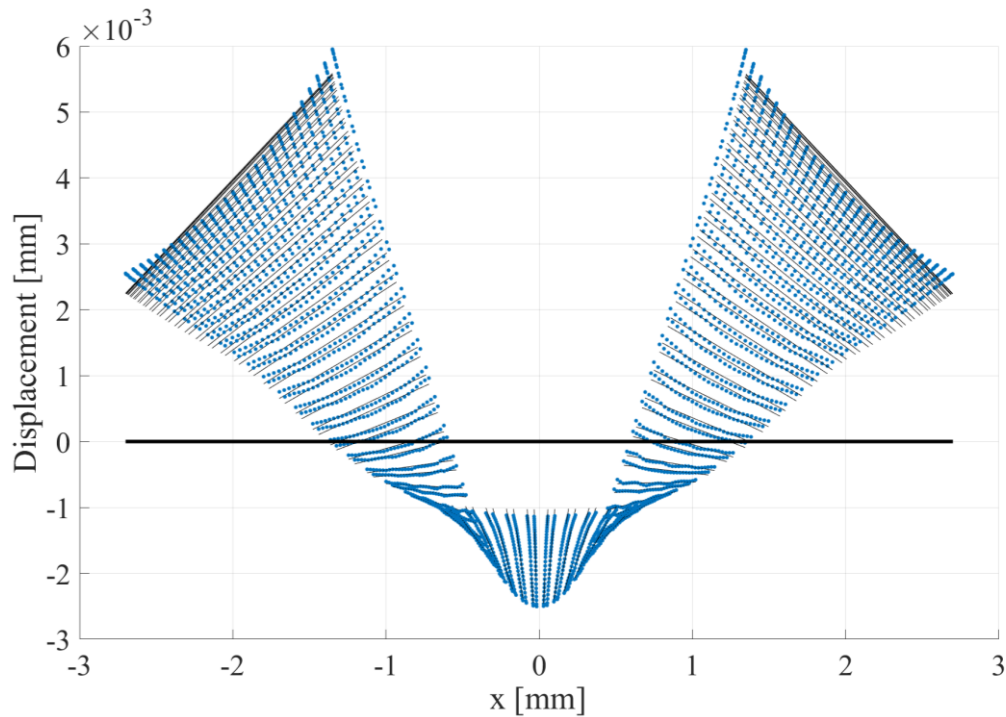
This issue must be addressed to prevent it from affecting the computed residual stress values. It only affects the residual stress evaluation of a numerical model following the optical technique by using the strain gauge formalism. A few values were tested for the grid spacing, varying from 0.5 mm to 0.01 mm. The latter is close to the mm/pixel ratio value usually employed in experiments by our group (0.01064 mm/pixel). This analysis was performed with a small load within the elastic region. It is possible to verify that the grid spacing influences the computed residual stress value. The computation of residual stress values from non-equally spaced data obtained directly from the FE analysis results in a deviation from the reference values. Residual stress values approach the reference values for smaller grid spacing, as shown in Figure 8-4. A variation of approximately 1.5% in the computed stresses was identified for the range 0.05 and 0.01 mm. Creating the grid and interpolating values for a grid spacing of 0.01 mm requires a considerable computational effort. For this reason, a reasonably fine and uniformly spaced grid was modeled with a spacing of 0.05 mm, and displacement values were interpolated and attributed to each point of the grid.

Figure 8-4 Residual stress computation for different grid (node) spacing following the strain gauge formalism for the optical technique. The term ‘Ref’ represents the reference stress values, ‘Map’ the stresses computed considering the mapped grid, ‘FE’ the original mesh from the finite element model, and ‘SG’ the stress computation according to the ASTM standard procedures (Type B).



The mesh grid employed to compute the initial guess was also brought to the straight lines (Figure 8-4) employed to compute the inclination of the partial derivative of the radial displacement with respect to the radius as a function of theta. This procedure was necessary to standardize the computation process for both the pure numerical procedure employed in the validation step and the experimental data.

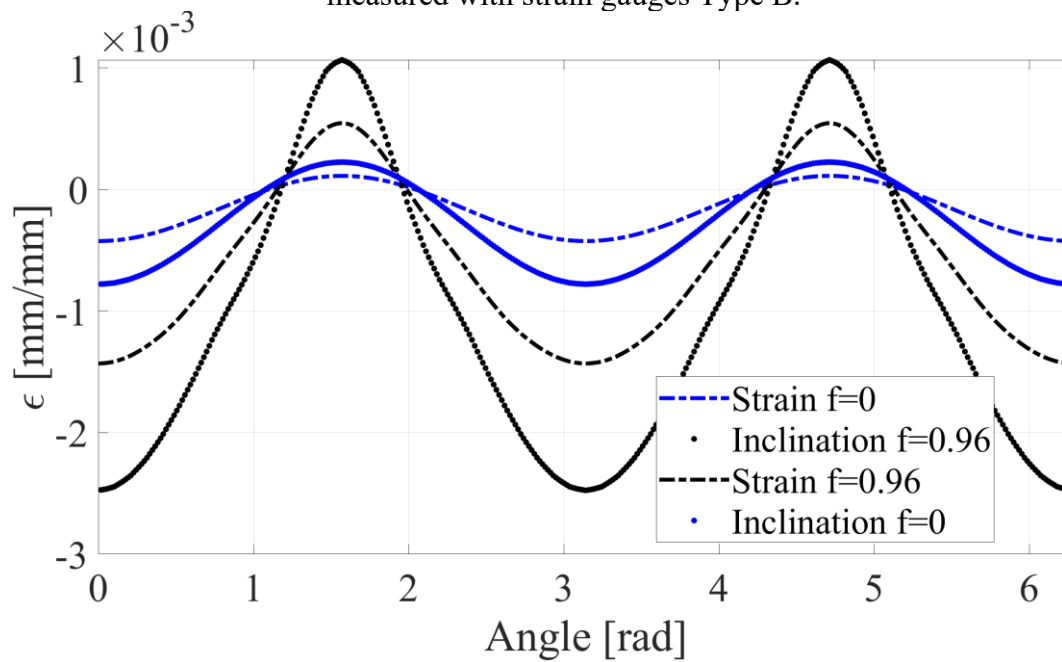
Figure 8-5 – Nodes located within and nearby the inner and outer annular area (near case) employed in the calculation of the partial derivative. Straight black lines represent the approximate displacement profile for the annular region.



The inclination of the straight lines presented in Figure 8-5 represents the partial derivative of the radial displacement with respect to the radius. By computing the inclination around the hole, it is possible to obtain the partial derivative of the radial displacement with respect to the radius as a function of theta (Figure 8-5). The strains measured with traditional strain gauges are much lower than the partial derivative, as shown in Figure 8-5.

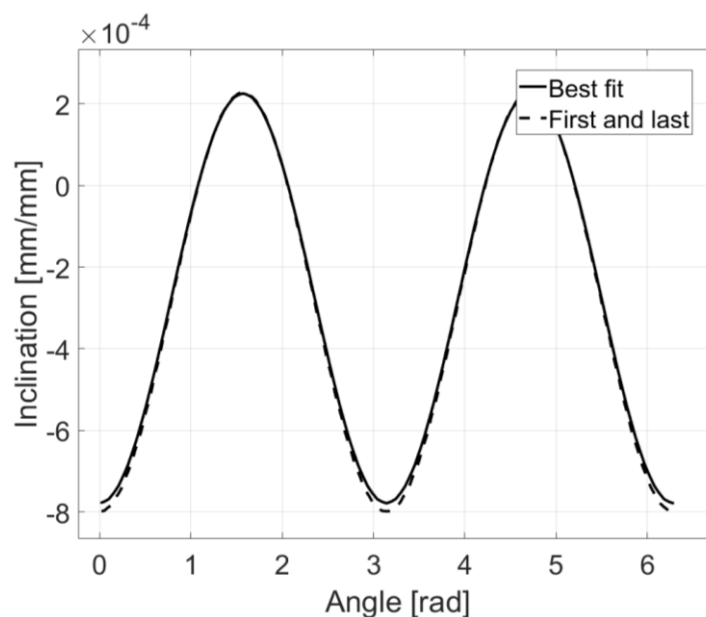
The strain gauge begins approximately in the middle of the annular region (Figure 8-2) and extends further beyond the outer limits of the annular region. Strains are higher near the hole. Additionally, the strain gauge width value represents the average of its neighboring points. For these reasons, the inclination of the radial displacements is higher for the optical strategy. Higher-order behavior indicates that plastic strains are higher near the hole and, therefore, measurements capable of correcting plasticity effects can better measure the plasticity effects.

Figure 8-6 – Comparison between the inclination of the radial displacements (Near case) or partial derivative of the radial displacement with respect to the radius as a function of theta and the strains measured with strain gauges Type B.



A test was performed to compare the fitting of a line connecting the first and last points, as well as a linear least-squares fitting method that considered every point on the line. The latest presented a smoother angular profile for experimental data, while for numerical data both cases presented a smooth profile but slightly different.

Figure 8-7 - Partial derivative of the radial displacement with respect to the radius as a function of theta for two different fitting strategies: connecting the first and last points and linear least-square fitting method considering every point on the line.



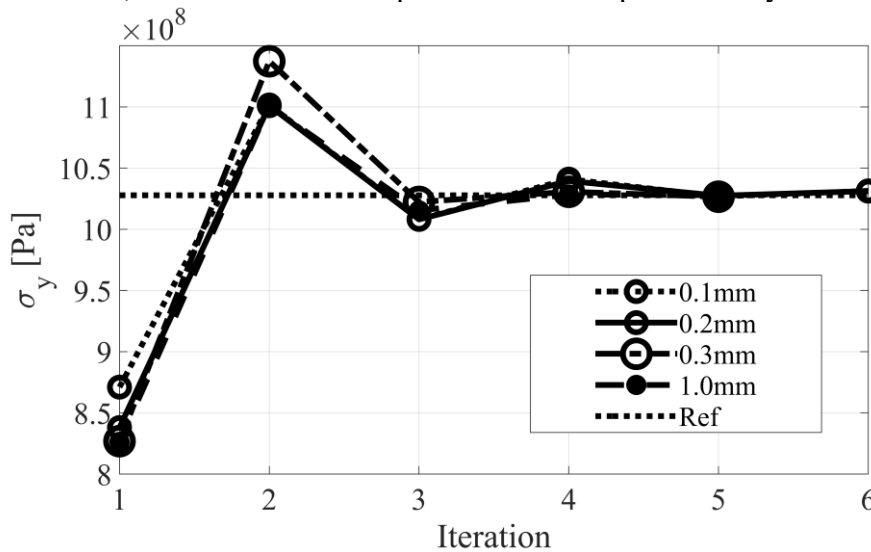
In summary, a reasonably fine and uniformly spaced grid with a spacing of 0.05 mm was employed to compute the partial derivative. The linear least-squares fitting method was chosen to perform the fitting of the straight lines considering every point on the line.

8.2 NUMERICAL VALIDATION OF THE ITERATIVE ALGORITHM

Increasing the load reaching considerable plasticity factors requires attention to deviations in the residual stress computation. High loads near the yield stress require the use of the iterative algorithm to correct the plasticity effects. In this case, the first guess values were evaluated elastically and allowed the beginning of the iterative procedure, which obtains in each iteration the full relieved curve for the inclination of the displacements and compares it with the same curve obtained for the numerical input representing simulated experimental data. A load near the yield stress ($f=0.96$) was employed to assess the correction ability. The excessive distortion in the strain profile for high loads is captured by the algorithm through the processing of data collected around the hole.

The first and second depths for both the intermediate and far cases were not able to converge and did not get any close to the expected value. They presented convergence problems and the results obtained by the iterative algorithm considerably diverged from the correct value. Only the near case converged for the first few increments. In addition, for lower depths smaller than 0.3 mm, a higher number of iterations (6 iterations) is required for the near case, as shown in Figure 8-8. As already mentioned by Vangi, the ability of the iterative algorithm to obtain the corrected stress values depends on the loading factor, stress ratio, and the tangential ratio r [5]. In addition, it was possible to verify that the h/D ratio also affects the convergence process.

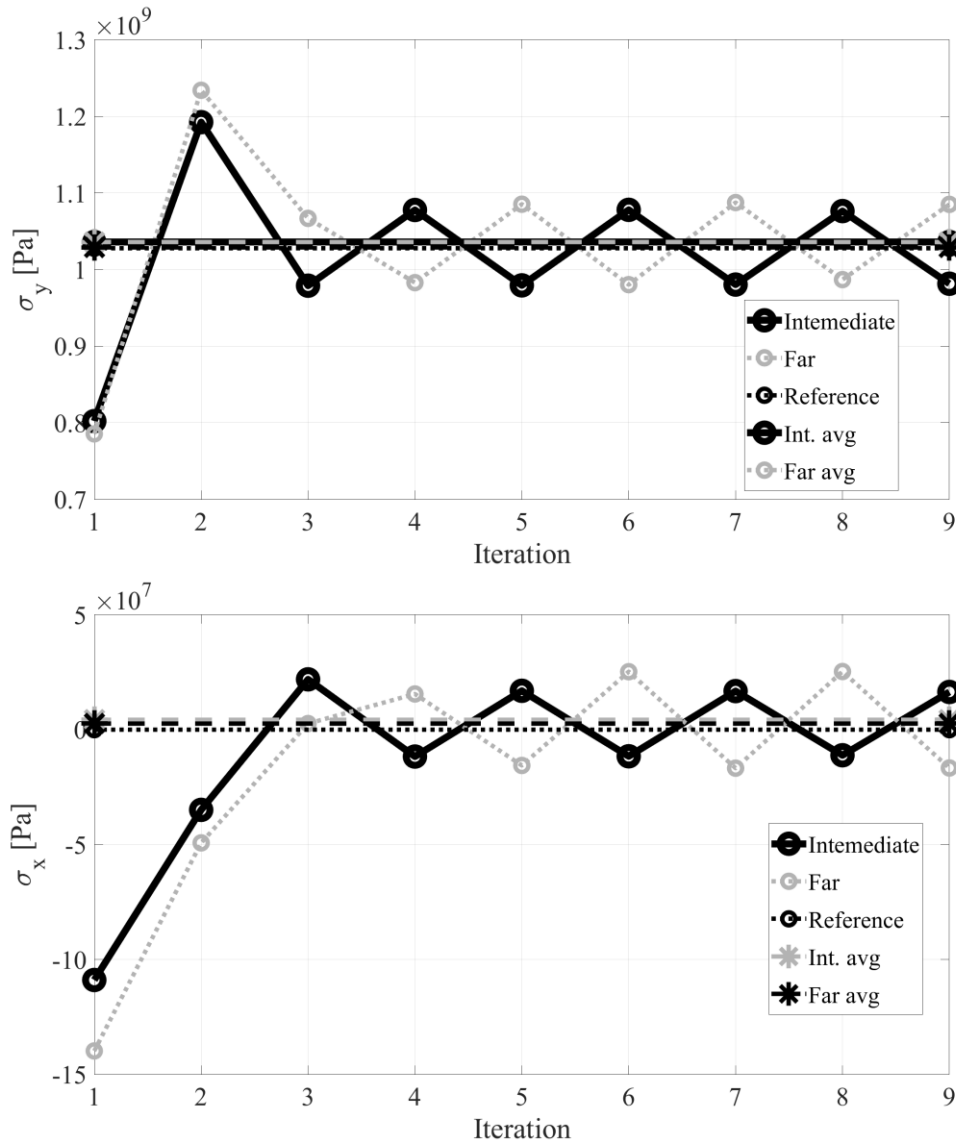
Figure 8-8 - Iterations required by the near case to achieve full convergence for depths of 0.1, 0.2, 0.3, and 1.0 mm. The expected value is represented by 'Ref'.



Both the intermediate and far cases oscillate around the correct stress value for the increment 0.3 mm. A variation of approximately 100 MPa would allow the iteration to converge, possibly generating a considerable deviation from the reference values. It shows that the diameter of the annulus also influences the ability of the iterative algorithm to obtain the corrected stress values. However, the average of the last two values given by Eq. (8-1) is close to the reference, as shown in Figure 8-9. The average value is taken as the final residual stress in the first few increments where the iterative algorithm does not converge to the reference value but alternates between slightly higher and slightly smaller values. This minimizes the deviation in the final results.

$$\begin{aligned}\sigma_x^{avg} &= \frac{1}{2}(\sigma_x^I + \sigma_x^{II}) \\ \sigma_y^{avg} &= \frac{1}{2}(\sigma_y^I + \sigma_y^{II})\end{aligned}\quad (8-1)$$

Figure 8-9 - Stresses in the x and y direction for depth 0.3 mm, indicating convergence issue and their respective average for the last two points for the intermediate (Int. avg) and far (Far avg) cases, together with the expected (reference) values for the simulation.

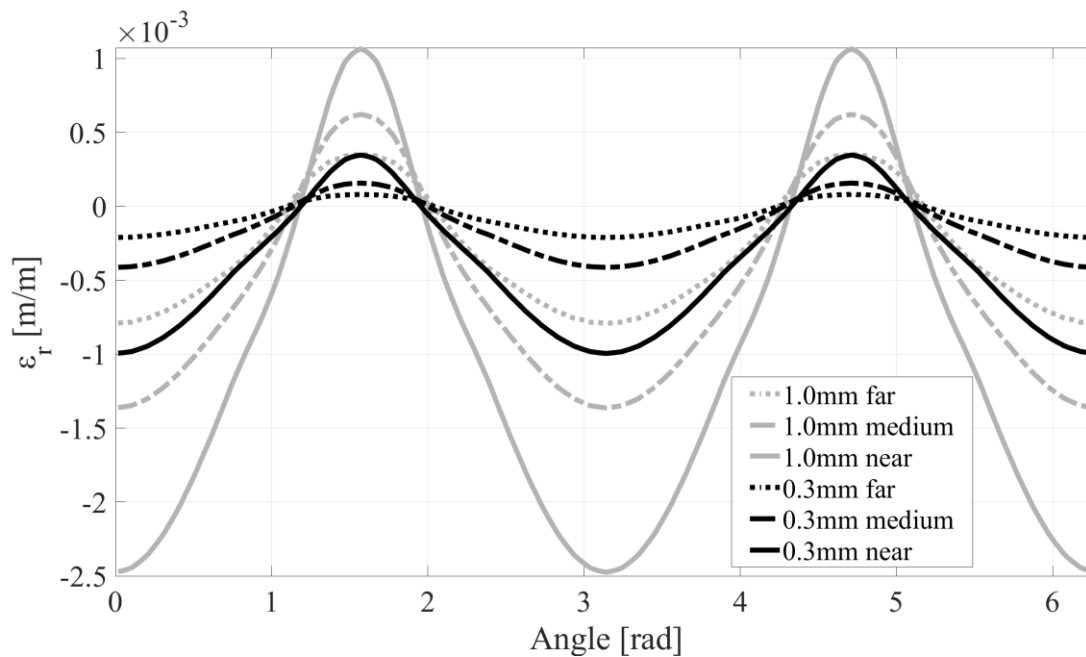


The average of the previous two iterations can also be employed in the next iteration's initial guess vector. In this case, the zeroth iteration elastically evaluated should not be considered. Two tests were performed considering the moving average (MA) of the last 2 iterations and also of the last 3 iterations. The moving average was computed only after the third iteration. The results indicated that the initial estimate vector did not influence the final result for the initial estimates analyzed, and that the final result obtained is similar to the result obtained with the initial estimate vector considering the result of the previous iteration (technique adopted throughout this thesis). Details of this trial are shown in Appendix E. It was not possible to obtain convergence, as the oscillation persists, but it is necessary to reinforce that the average of the last two points is very close to the reference value: the moving average (or average of the last two points) corresponds to the value of approximately 1028.9 MPa from the fifth iteration, a deviation of 0.1% from the reference value (1027.8 MPa).

Better convergence ability is expected for higher depths. The full hole with 1 mm depth was analyzed and, for this depth, all the three annuli employed in the analysis converged well to a very precise residual stress value.

There is a large difference in the strain profile between the first few depths and the last depth. The signal acquired in each annular region is also different. It presents a higher signal near the hole where the displacements are greater, while farther from the hole the signal is smaller. Smaller depths also obtain less signal. Figure 8-10 presents the signal obtained for depths 0.3 mm and 1 mm for three annuli. The reduction of the signal by reducing the depth or increasing the annulus radius increases the difficulty of the algorithm to converge. It tries to converge but oscillates around the target value. The signal becomes too small for the intermediate and far cases with depths of 0.3 mm or smaller. It is possible to identify some higher-degree oscillations in the inclination of the strain profiles for both medium and near cases. The plasticity effects are higher closer to the hole and generate higher-degree oscillations if compared to the elastic regime.

Figure 8-10 - Partial derivative of the radial displacement with respect to the radius as a function of the angle theta for the data collected within the annular areas far, medium, and near, as well as two different hole depths of 0.3 mm and 1.0 mm. Annulus diameter: near ($R_1=1.5*RF$; $R_2=3*RF$; $R_x=2.7*RF$), medium ($R_1=2*RF$; $R_2=4*RF$; $R_x=3.6*RF$), and far ($R_1=2.5*RF$; $R_2=5*RF$; $R_x=4.5*RF$).



The higher signal obtained for higher depths contributes to the convergence ability. The algorithm required only four iterations to converge for the depth of 1 mm. The results for each iteration obtained for the full depth are shown in Figure 8-11. The reference value and the iterative algorithm output (4th iteration for all cases) had a good agreement for the three different annuli. The shape of the curve is different for each of the three cases. The last iteration strain peaks and valleys match with the reference values, and the shape of the curve also agrees with the Fourier higher number of terms profile which can be visually identified for the near case. The residual stress values match with the reference values in the last two iterations, as presented in Table 8-1. The near case obtained the smallest deviations for both the stresses in the x and y directions.

Figure 8-11 - Partial derivative of the radial displacement with respect to the radius as a function of the angle theta obtained with the numerical model for each iteration performed by the iterative algorithm for the a) near, b) intermediate, and c) far cases.

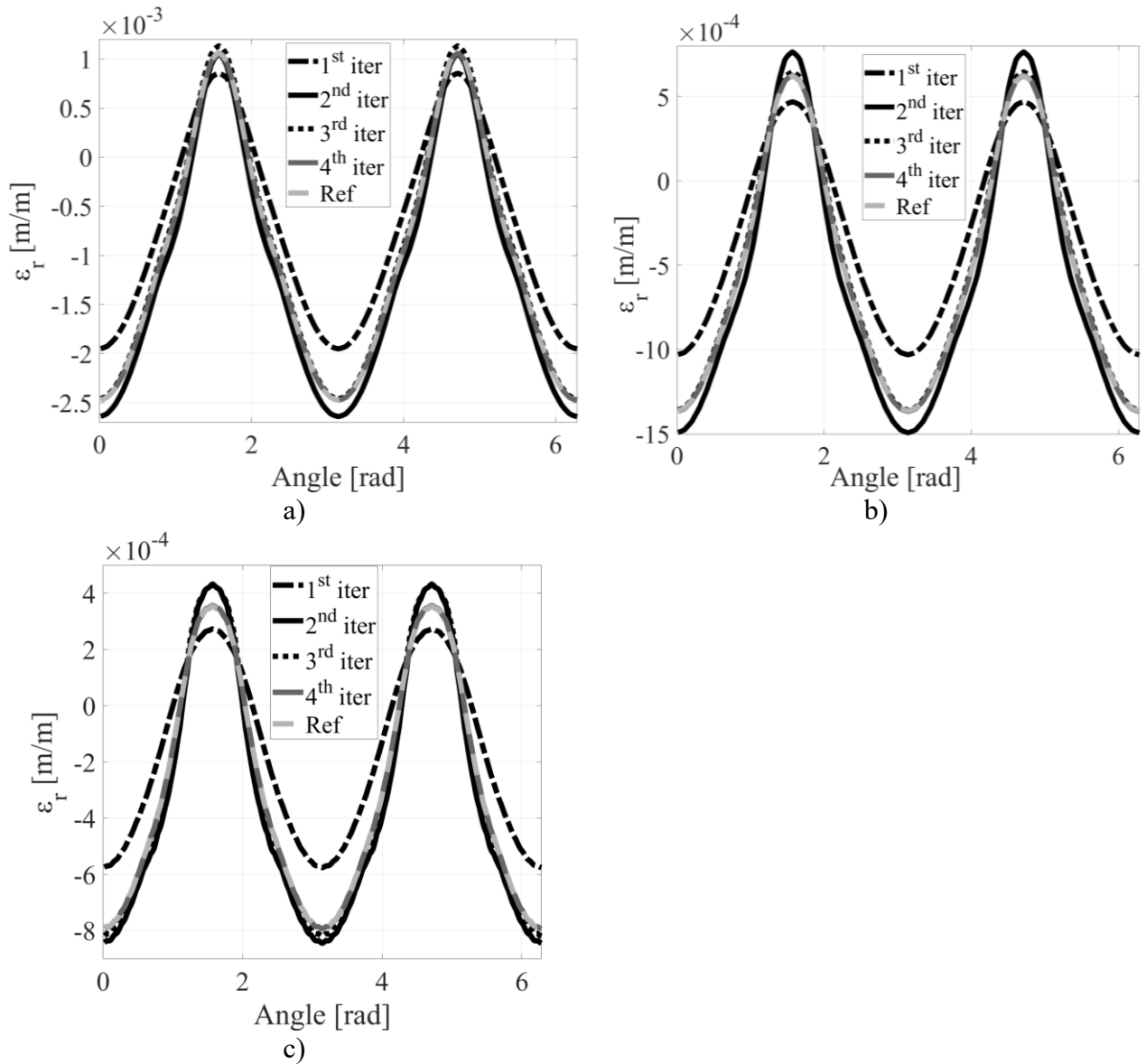


Table 8-1 Iterative algorithm stress values output for the depth of 1 mm. Reference values: $\sigma_x=1027.8$ MPa $\sigma_y=0$ MPa.

Iteration:		Elastic	1	2	3	4
near [MPa]	σ_x	-61.6	32.7	-26.6	2.1	0.0
	σ_y	826.3	1100.9	1015.4	1028.4	1027.9
medium [MPa]	σ_x	-93.4	-0.7	-25.2	0.0	-0.3
	σ_y	810.2	1150.5	1018.6	1030.3	1026.9
far [MPa]	σ_x	-125.0	-18.9	-27.3	-1.7	2.1
	σ_y	793.5	1196.4	1086.3	1029.6	1030.2

The full simulation for 10 increments takes about 8 minutes to find the nonlinear FE solution, while the first two simulations (loading + increment) required for the first increment take about 1.6 minutes. A standard computer from 2019 was employed with an Intel Core i7-8700 @3.20GHz and 16 GB of RAM. A larger number of iterations are required for the first few increments in the iterative algorithm, but at the same time, these increments obtain the nonlinear FE solution much quicker. The graphical processing and elastic residual stress computation that precedes the iterative algorithm requires 18 seconds, while the graphical processing and the solution of the 360 equations and the computation of the next iteration requires 1.5 minutes. The material stress-strain curve was accurately represented near the yield point. The iterative algorithm typically requires 2 to 6 iterations to converge. The zig-zag oscillation mentioned above runs more than 60 iterations and continues on if allowed to do so. It is recommended to set a limitation of about 6 to 10 iterations to avoid problems with infinite loops, especially for both the intermediate and far cases for the first few increments.

The numerical approach employed to validate the proposed procedure is not affected by uncertainties present in the experimental procedure. There are a few factors that can impact experimental measurement, such as center misalignment, effectiveness of the stress-relieving heat treatment, and hole diameter variation, among other factors.

A few conclusions can be taken from this chapter:

- It was necessary to obtain a full profile around the hole with the mechanical signature of the total displacements influenced by both the elastic and plasticity effects (containing the elastoplastic strains). The profile around the hole for the inclination of the displacements was generated by processing the full-field data.
- Three different annuli were investigated: smaller radius (near), intermediate radius (medium), and large radius (far). A residual stress state was simulated to validate the method numerically, and the inclination of the displacements was computed. This entire profile around the hole was used instead of three to four points of strain gauge experimental data as input for the iterative algorithm. The iterative process yielded results close to the reference values. Care must be taken when computing the displacements for two different density regions, for instance, simulated data and experimental data. The initial increments were more computationally expensive, requiring more iterations within the iterative algorithm. Both medium and far strategies also tend to decrease the convergence ability. Higher level curves were obtained and are more visible in the near case, where the data is captured closer to the hole edge. The higher levels of plastic strains near the hole certainly contribute to the modification of the strain profiles in the near case.

- The ratio h/D_0 influences the convergence ability as well as the annuli diameters. The higher signal presented by the near case tends to reduce the divergence of the corrected residual stress values from the target value.

9 EXPERIMENTAL RESULTS AND DISCUSSIONS

The experimental results are presented in this section. It is divided into two sub-sections: pre-test and tests. The pre-test presents the preliminary analysis performed on a manual loading device, while the “tests” section summarizes the results and discussions based on more robust residual stress measurements. In this last case, they were performed with the aid of a tensile machine to induce high levels of stress.

9.1 PRE-TEST

A residual stress measurement was performed. The full-field acquisition of the displacements allowed the computation of the partial derivative of the radial displacement with respect to the radius. The displacements measured for each annulus are shown in Figure 9-1, Figure 9-2, and Figure 9-3 for the near, medium, and far cases, respectively.

Figure 9-1 – Near case: radial displacement around the hole together with the partial derivative of the radial displacement with respect to the radius.

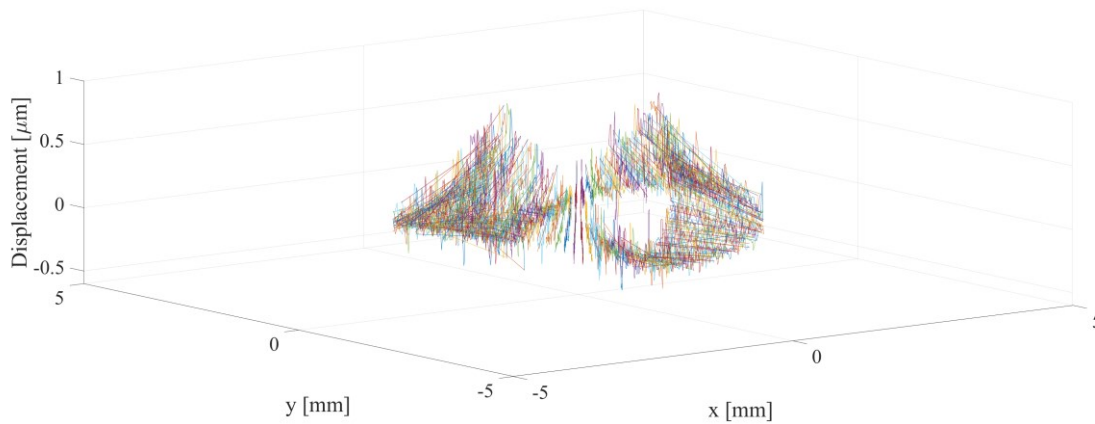


Figure 9-2 – Intermediate case: radial displacement around the hole together with the partial derivative of the radial displacement with respect to the radius.

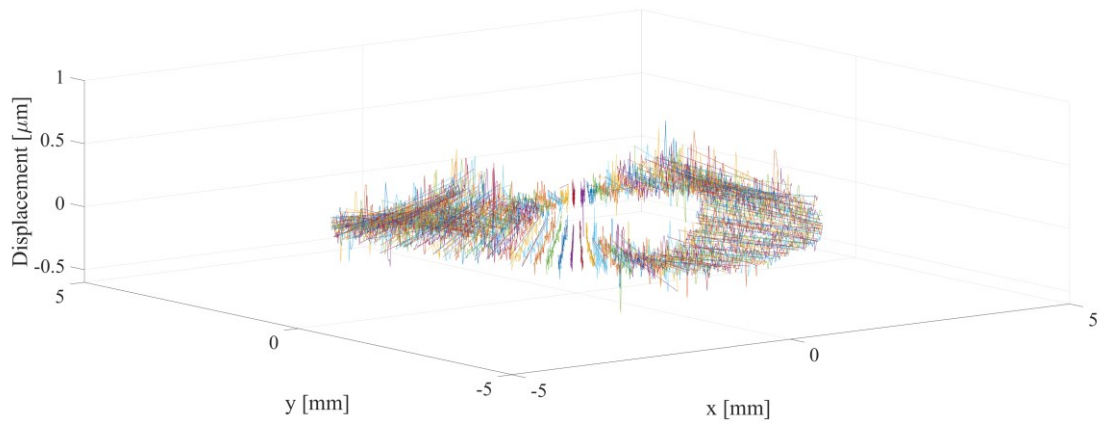
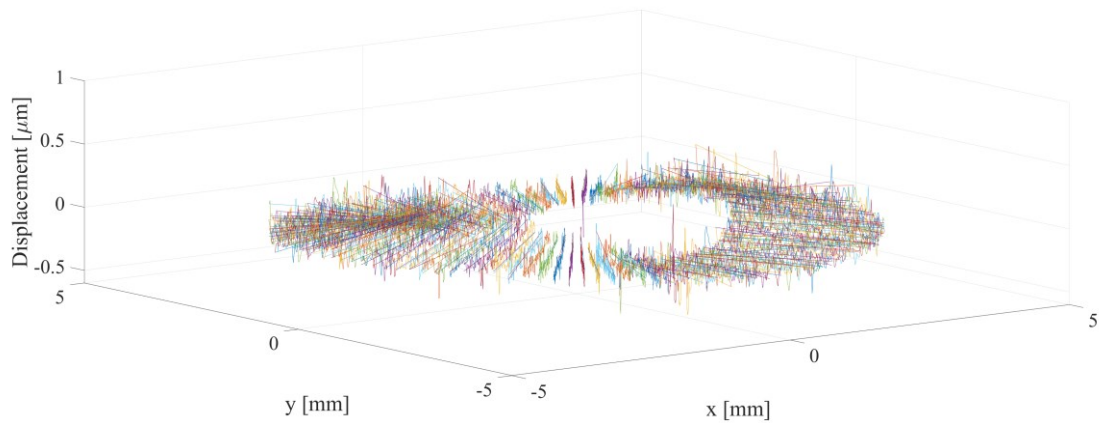
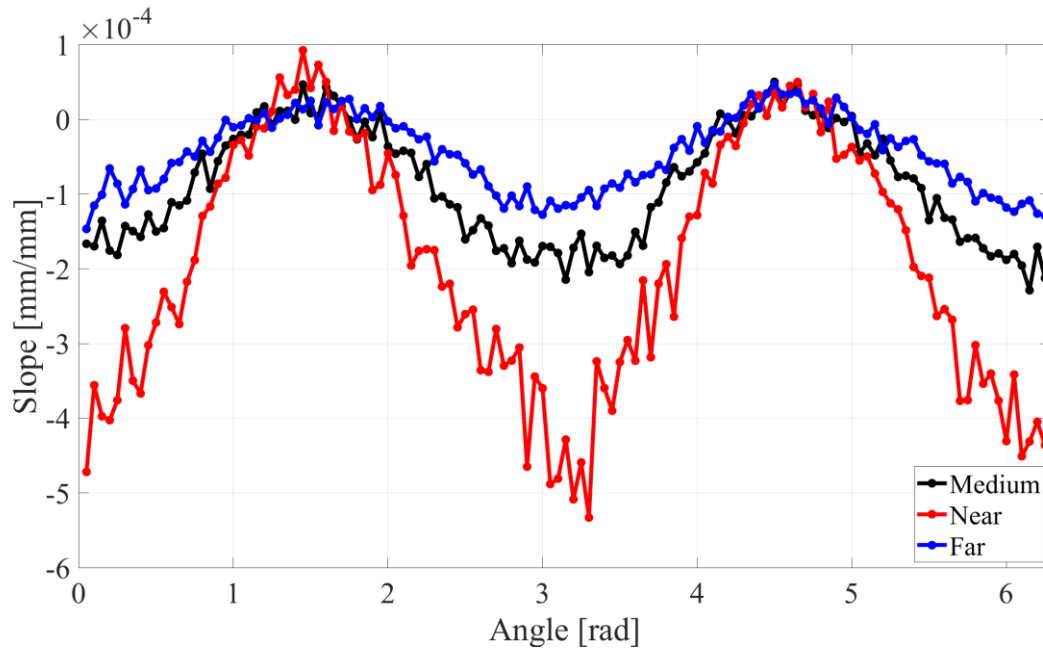


Figure 9-3 – Far case: radial displacement around the hole together with the partial derivative of the radial displacement with respect to the radius.



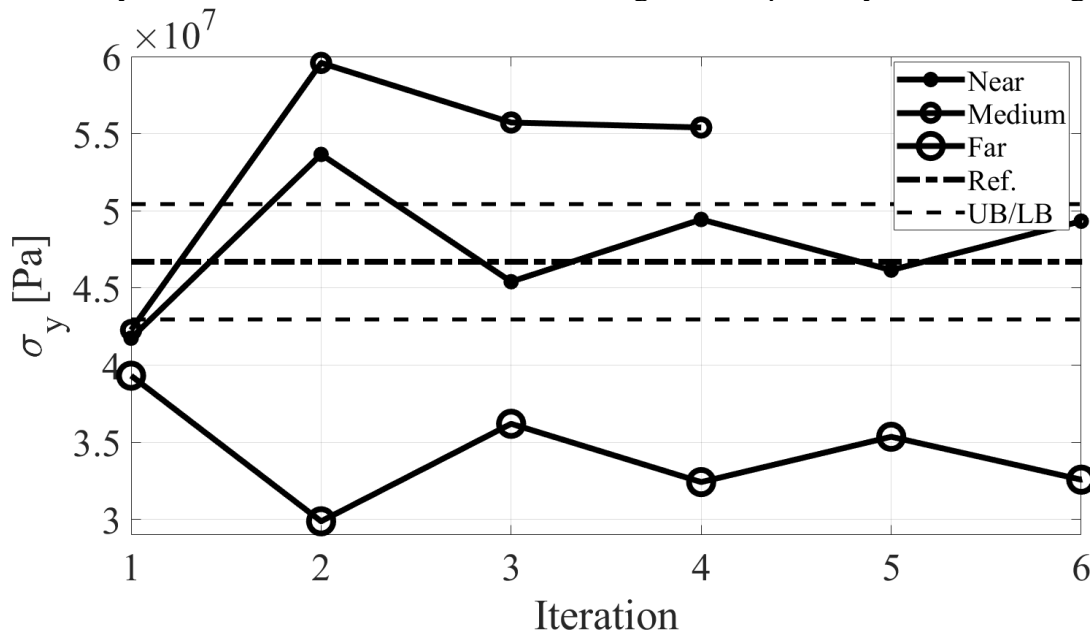
From the partial derivative of the radial displacement with respect to the radius, it is possible to obtain the full profile around the hole as a function of theta for each annulus, as shown in Figure 9-4. Medium and far cases present signals with good behavior, although the signal has a smaller magnitude.

Figure 9-4 - Partial derivative of the radial displacement with respect to the radius as a function of theta.



These slope profiles were plugged into the iterative algorithm. The solution obtained for each annulus region is shown in Figure 9-5. Only the medium case obtained values compatible with the expected stresses. The far case obtained values much lower than the induced stresses.

Figure 9-5 – Convergence process of the iterative algorithm for the near, intermediate, and far cases (depth of 1 mm). UB stands for Upper Bound and LB for Lower Bound of the reference value uncertainty. The first iteration indicates the initial guess computed by the iterative algorithm.



Besides iterating the stress values, the algorithm also varies the orientation of the principal stress direction. The variation is visible in iterations 2nd, 3rd, and 4th from Figure 9-6, Figure 9-7, and Figure 9-8.

Figure 9-6 – Case: Medium, Iteration: 2nd. Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.

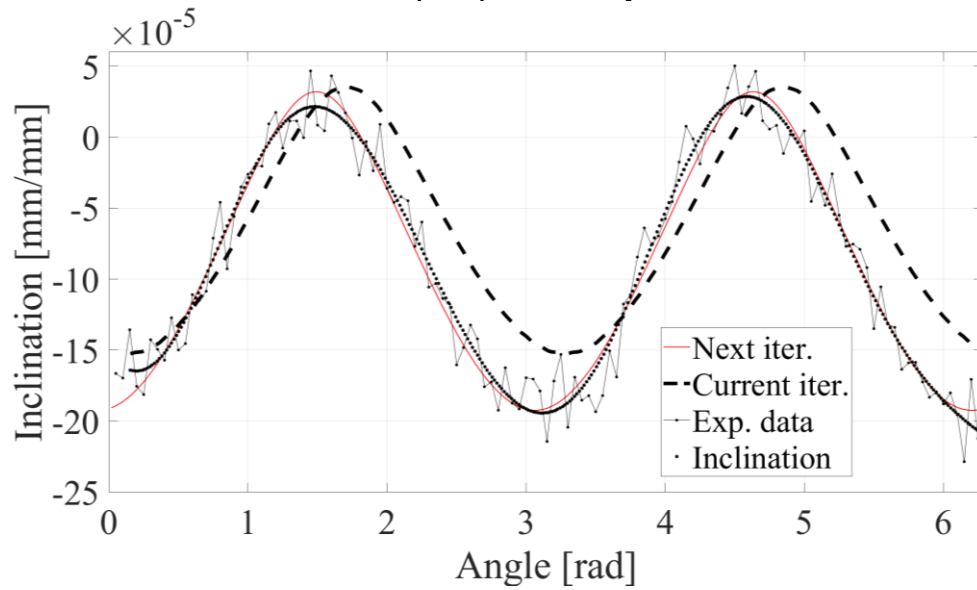


Figure 9-7 – Case: Medium, Iteration: 3rd. Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.

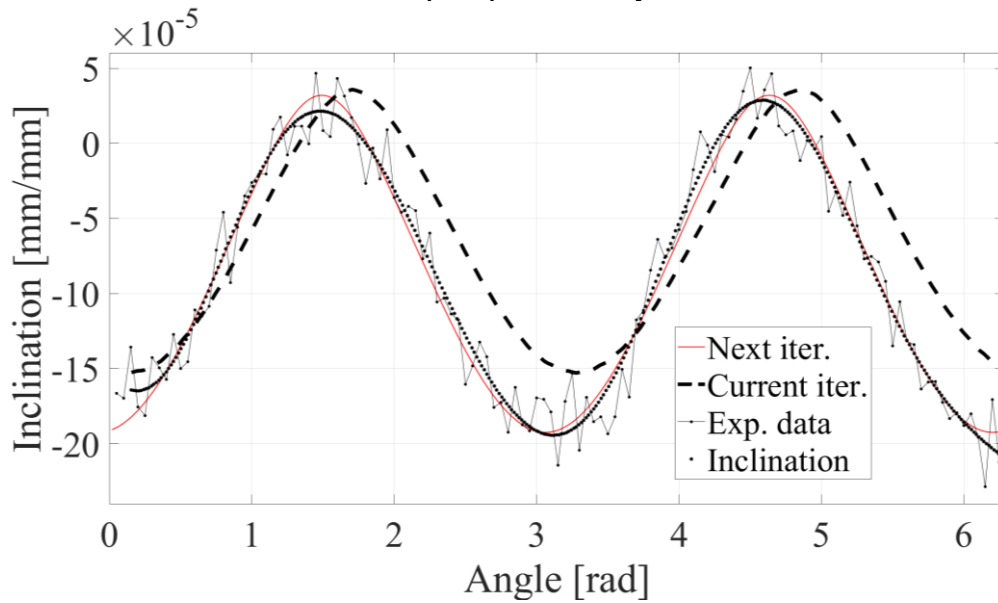
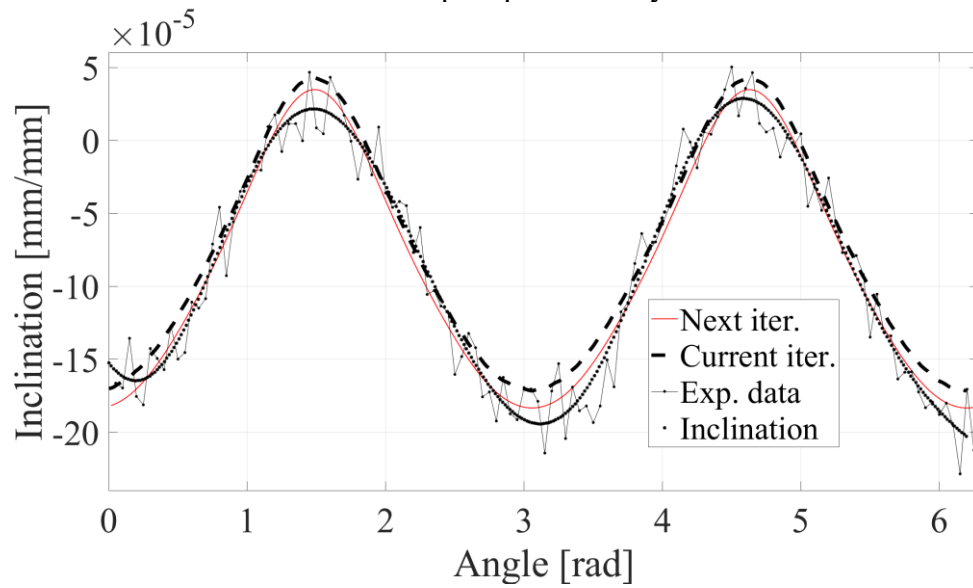


Figure 9-8 – Case: Medium, Iteration: 4th. Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.



The strains do not match the experimental reference values. It is possible to verify in the convergence process shown in Figure 9-5 that the algorithm oscillates around the target values for the last increments; this behavior in the residual stress values is also seen in the strain values for the last few iterations (Figure 9-9 and Figure 9-10).

Figure 9-9 – Case: Medium, Iteration: 5th. Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data and its rounded slope represented by “Inclination”.

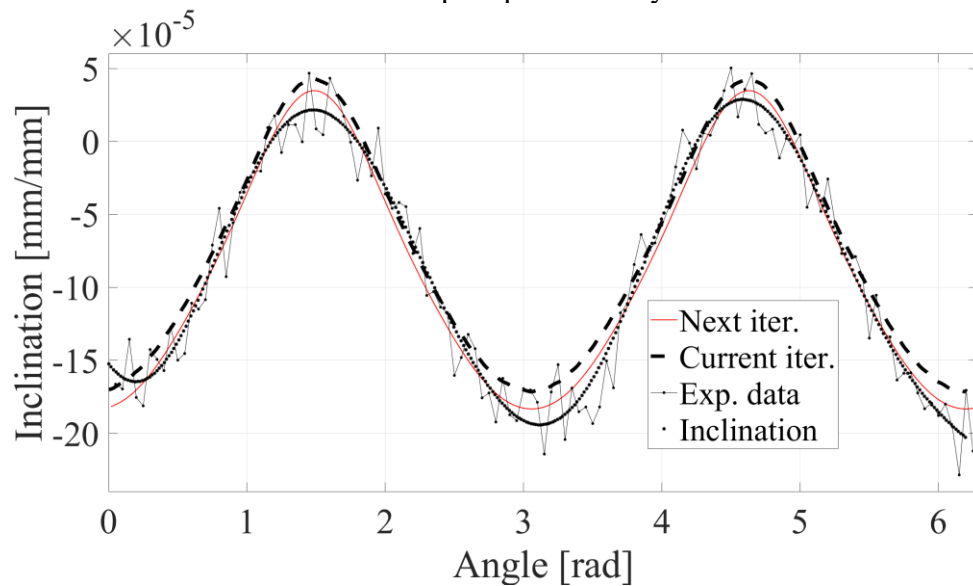
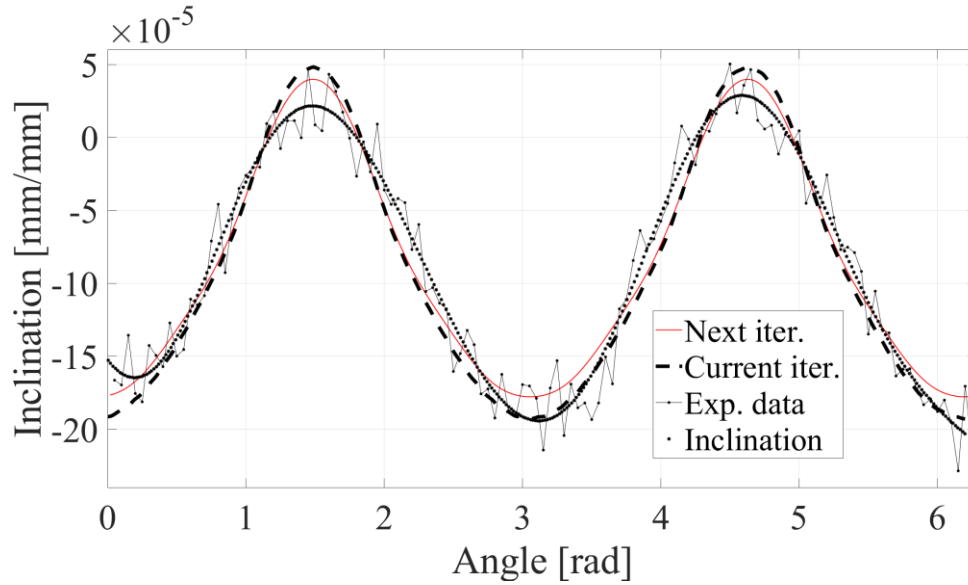


Figure 9-10 – Case: Medium, Iteration: 6th. Partial derivative of the radial displacement with respect to the radius as a function of theta for four items: Next iteration, current iteration, experimental data, and its rounded slope “Inclination”.



Lastly, a test was performed by changing the geometry of the finite element model. The width and thickness were set equal to the real specimen. Both stress curves of the iterative process for the far case presented the same behavior, and the results in the simulated stresses differed by no more than 2% for each iteration. The initial guess is not affected by variations in geometry. It shows that the geometric difference between the current model and the real workpiece dimensions does not significantly influence the result.

This pre-test on aluminum is not a robust test for residual stress measurements involving high levels of stress. The results indicate that the technique is capable of measuring, but it has some limitations. In spite of that, it was relevant to present the algorithm's behavior for the three cases: near, medium, and far. Although only one out of three evaluation strategies achieved a stress value located near the applied stress (or within the expected range, considering the uncertainties), this validation was an important step in the development process. The inclination profiles obtained for the three cases seem reasonable and present good behavior. Convergence issues as well as minor issues were fixed in this stage of the development process. The evaluation indicated that the strategy can potentially correct high levels of stress. More robust residual stress measurements were performed and will be presented in the following section.

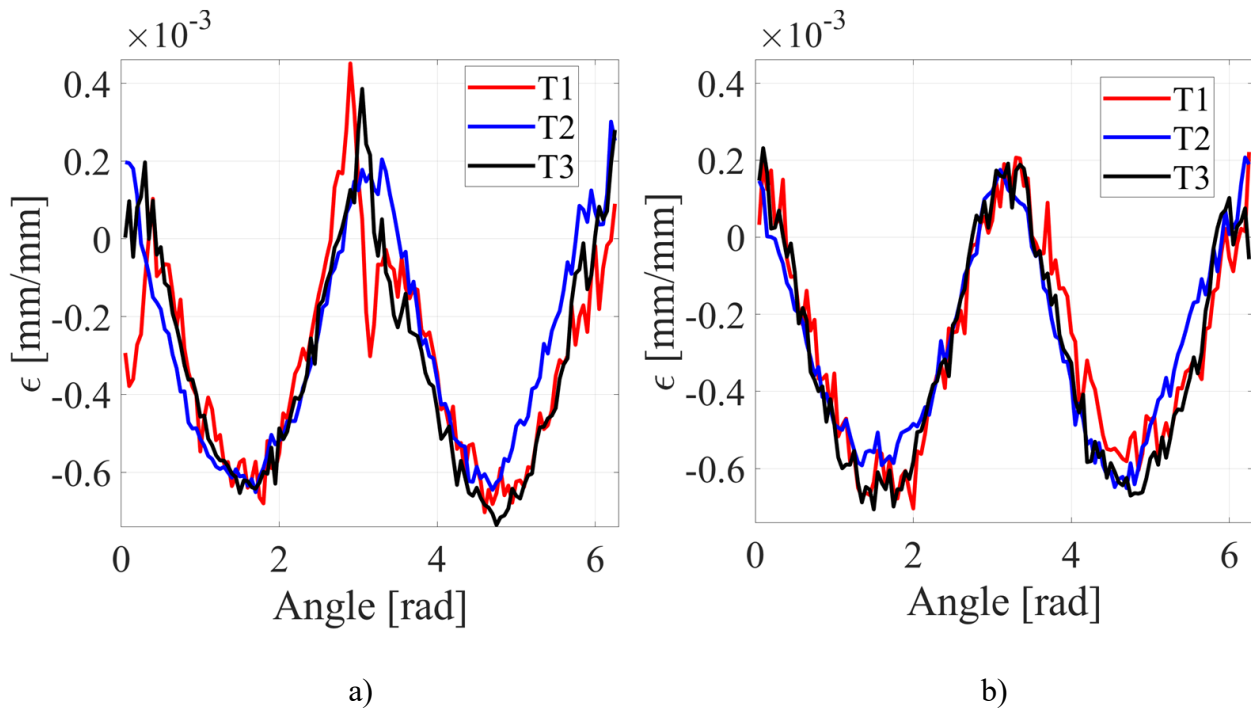
9.2 TESTS

This step was performed on the tensile test machine. As presented earlier, a structure was designed to keep the measurement device in a vertical position. Unfortunately, considerable rigid body motion was identified. The highest and smallest strain values as a function of theta presented a considerable difference for the two peaks and two valleys. The strain profile around the hole was

certainly affected by the rigid body motion. Rigid body motion generates cat's whiskers-like fringes pointing to the center, which does not directly affect the inclination of the radial lines (measured displacements). However, cumulative body motion increases the lack of correlation, introducing noise to the measured strain signal. It posed a difficulty for the algorithm to converge.

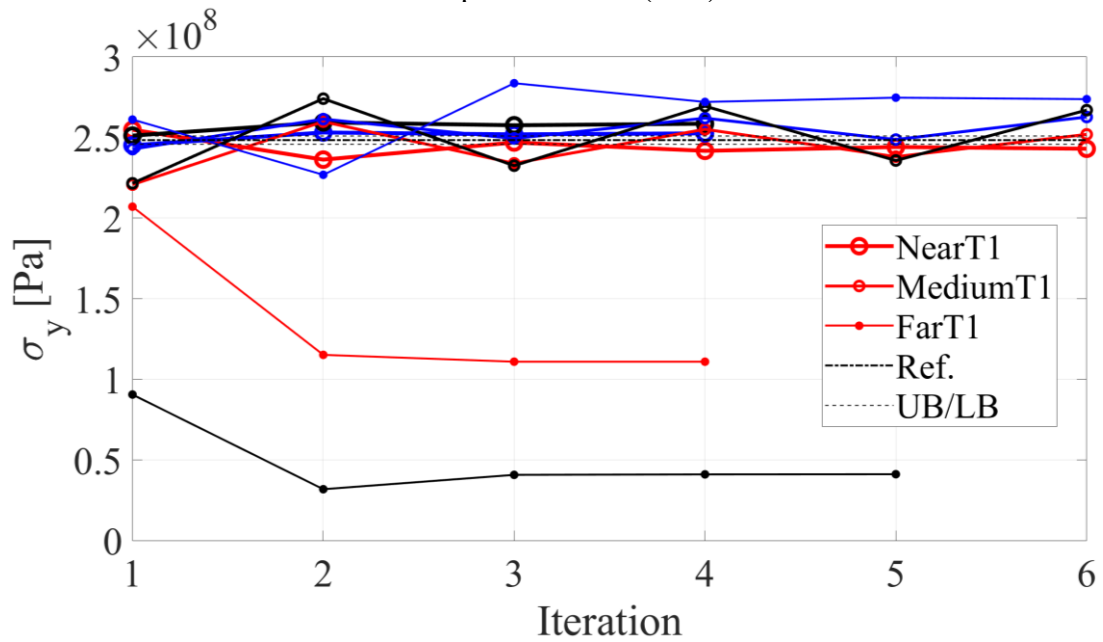
There are few techniques available to reduce the effect of rigid body motion. The experiments carried out in this section were first analyzed considering the reference image was taken before the hole drilling procedure started. This analysis can also be performed incrementally. The incremental method employs the previously acquired image as the reference for obtaining the incremental differences for each increment. After applying a low-pass filter, the incremental displacement values were summed to obtain the accumulated displacements for each increment. Then, the displacement partial derivative with regard to the radius was obtained for the incremental method, as shown in Figure 9-11b.

Figure 9-11 – Case: Near, Depth= 1 mm, T1 stands for first test, T2 for the second test, and T3 for the third test. Partial derivative of the radial displacement with respect to the radius as a function of theta for a) the traditional and b) the incremental method.



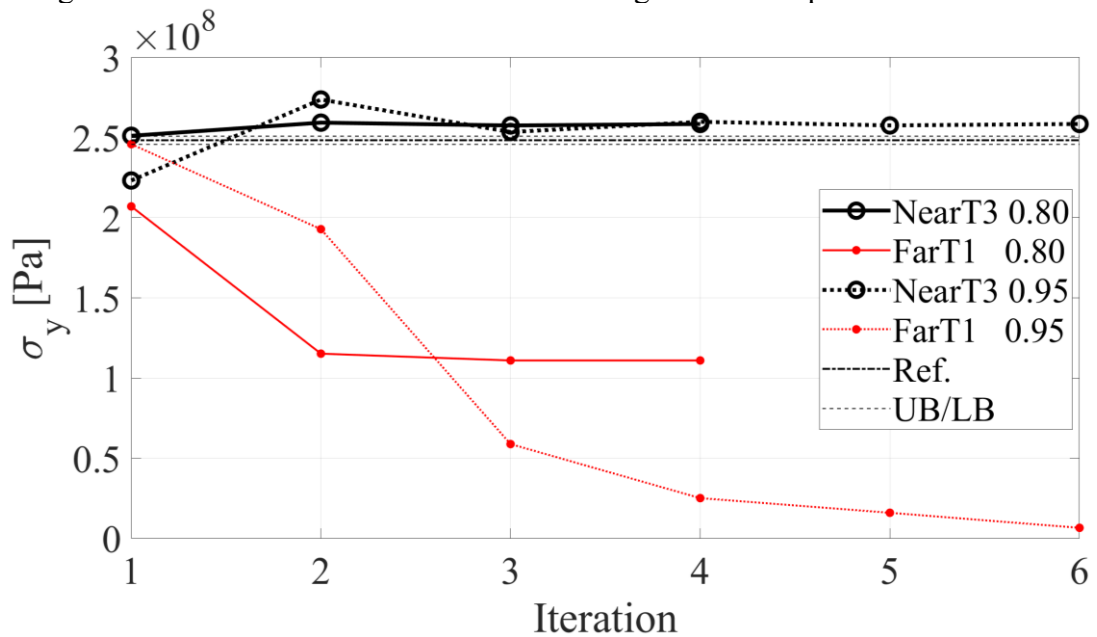
The new profile obtained with the incremental method facilitated the convergence process. This time, the algorithm successfully reached convergence for a few cases, as shown in Figure 9-12. The near and intermediate cases converged to the expected values. Far cases obtained the highest deviation, especially for experiments T1 and T2 that converged (or diverged) to a distant value.

Figure 9-12 – Iterative process obtained for experiments T1 (red), T2 (black), and T3 (blue). Annular regions: near, medium, and far. The dotted lines represent the upper and lower bounds and the expected values (Ref.).



In this case, most initial guesses are located near the reference line. Initial guesses too close to the expected value may mask the ability of the algorithm to converge. However, it is possible to test the convergence ability. The convergence ability can be tested by changing the initial guess. This can be done by changing the initial guess provided to the iterative algorithm, as shown in Figure 9-13.

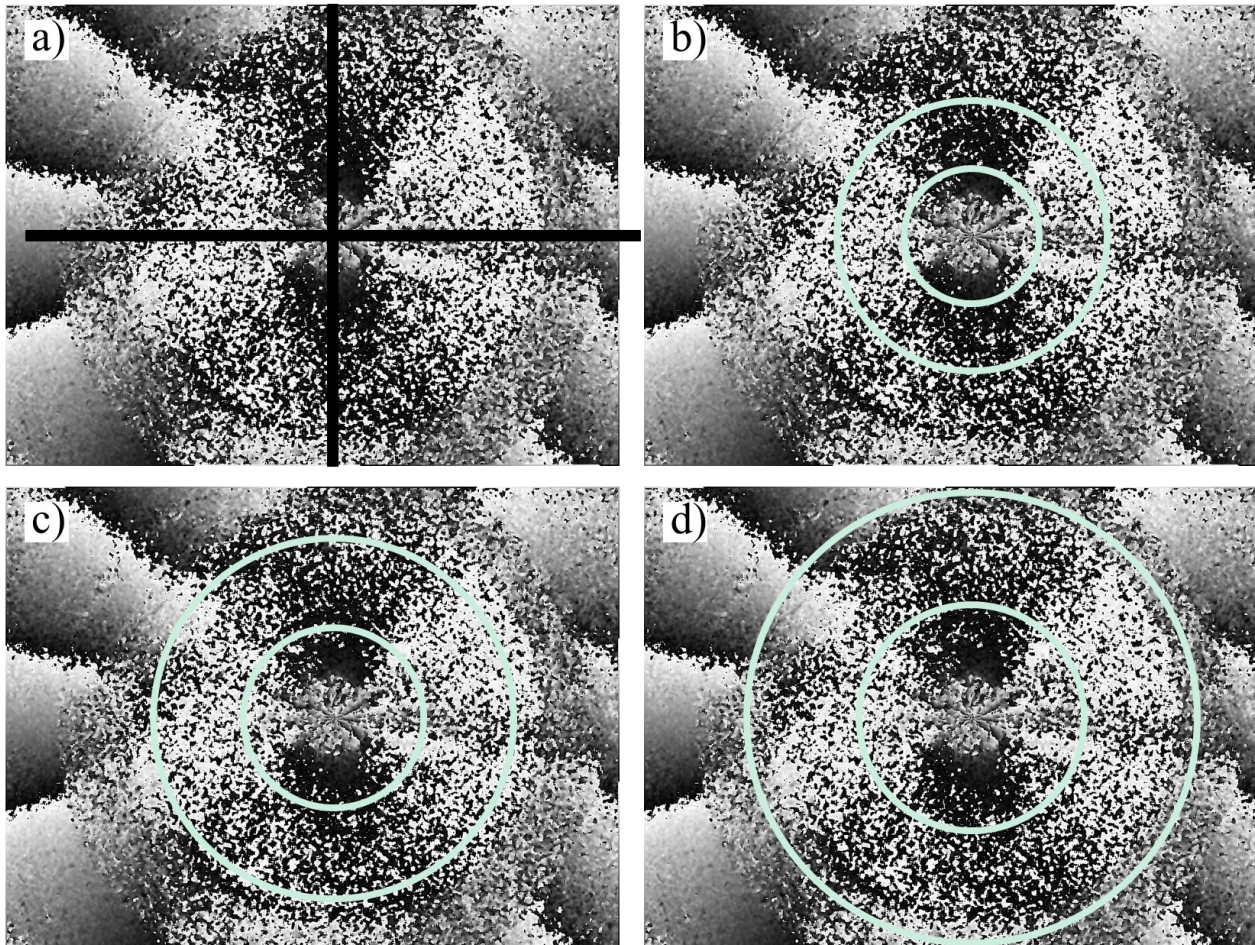
Figure 9-13 – Iteration behavior for two initial guesses multiplier: 0.80 and 0.95.



There are a few factors that may influence the convergence ability of the far case. It can be noted that the optical center of the interferometer does not coincide with the central coordinate of the image, as shown in Figure 9-14a. Although the center of the circle with radial sensitivity is slightly

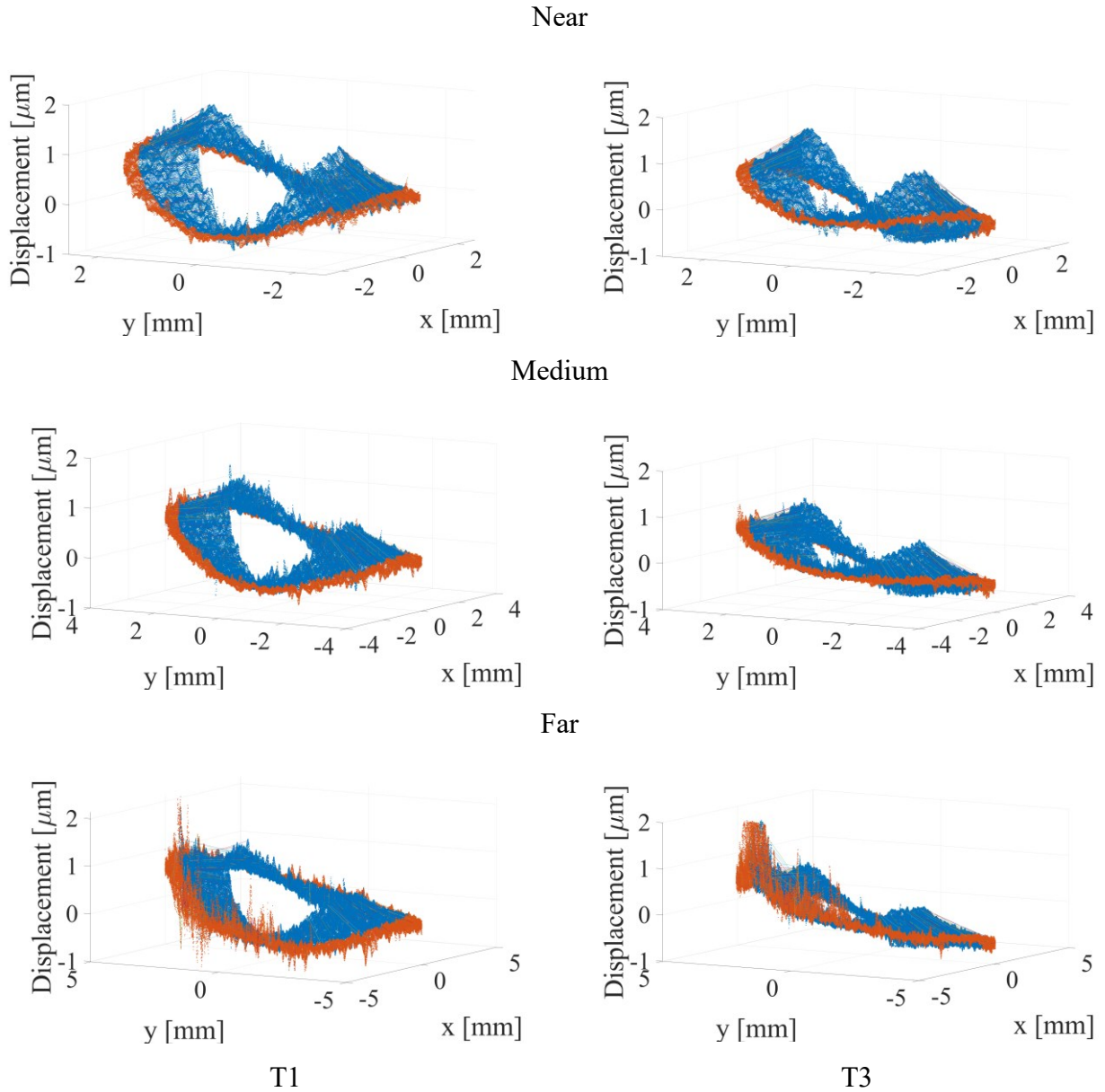
decentered, the largest annular external limit does not reach the edges of the image; however, it reaches regions with low radial sensitivity (correlation) with inaccurate radial displacement values. For instance, T1 processing indicates that the radial displacements in the upper left quadrant go through regions that do not present sufficient correlation (Figure 9-14d).

Figure 9-14 – Phase difference for the first increment of experiment T1 indicating a) the center of the area sensitive to radial displacements, annular regions: b) near, c) medium, and d) far.



The region with missing correlation affects the profile of the partial derivative of the displacements with respect to the radius. The displacements for the three annular regions are presented in Figure 9-15. The modification of the profile at localized regions led to a sudden slope variation.

Figure 9-15 – Radial displacements and partial derivative of the radial displacement with respect to the radius. x- and y-axis are not equal for different annulus radii.



A very localized abrupt change may not interfere with the algorithm. However, modifications that affect a few theta values around the hole change the curve profile. It is more common in the far case, hindering the convergence of the algorithm.

It is possible to conclude that the modified algorithm can correct the plasticity effect for a limited range of annuli: near (good correction capacity), intermediate (reasonable correction capacity), and far (not applicable at this point). This analysis is summarized in Table 9-1.

Table 9-1 – Summary of the algorithm's performance for the pre-test and final tests for each annulus diameter: near ($R1=1.5*RF$; $R2=3*RF$; $Rx=2.7*RF$), medium ($R1=2*RF$; $R2=4*RF$; $Rx=3.6*RF$), and far ($R1=2.5*RF$; $R2=5*RF$; $Rx=4.5*RF$).

	Pre-test	Tests		
	1	T1	T2	T3
Near	✓	✓	✓	✓
Medium	✗	✓	✓	✓
Far	✗	✗	✓	✓

Lab conditions presented in the pre-tests did not require the incremental evaluation of the displacements – the optical device was rigidly positioned horizontally. The incremental evaluation became necessary in the second step; in this case, the optical device was attached to the support, which was, in turn, connected to the specimen. This certainly reduced the rigidity of the system.

10 CONCLUSIONS

A methodology was developed to correct deviations generated by plasticity in residual stress measurements under high levels of stresses. The full-field area around the hole was taken into account. Experimental tests with high levels of stress ($f \approx 0.9$) were performed to validate the methodology.

The capacity of the algorithm was tested for different annuli radii. The region closest to the hole (near case) presented a good correction capacity, while the intermediate region performed well in most tests, and the region furthest from the hole did not perform well and was not capable of correcting the plasticity effects.

Simulations indicated that both intermediate and far cases had the potential to correct plasticity effects. However, replicating it with experimental results presented some challenges. Improvements in the optical device to reduce the loss of correlation could eventually improve the method and extend its applicability to the far case.

The methodology relies on a robust elastoplastic model. The elastoplastic model was developed taking into account the material behavior. Specific evaluations were performed to assess the performance of the model. There are a few variables that interfere with the results. Even the shape of the model, such as square or round, influences the strain/displacement signal generated around the hole. For elastoplastic analysis under high loads (generalized plasticity), the dimensions of the model influence the stress profile – it was identified that a deviation of approximately 20% in the stress (profile) of the last increment was generated between the first and the refined model for a Beghini's factor of $f=0.96$. Besides that, the thickness also has a significant influence on the final stress profile, even for thicknesses a few times larger than the recommended minimum thickness for the pure elastic models. The difference is more relevant to higher hole depths. After increment 3, the deviation identified in the residual stress values became higher than 2.5% between the initial model and the refined model, reaching 20% in the last increment.

It was verified that the application of the load at the boundaries of the model is the most appropriate strategy to apply the load on a hole-drilling model taking into account plasticity effects. The assumption of the superposition principle yields a significant deviation for high levels of uniaxial loads – a strain variation of 25.65% was identified for the full hole and a loading of $f=0.95$. Therefore, this strategy was not adopted in this work. The assumption of the superposition principle should be used very carefully under any other loading ratio. Although the equibiaxial stress state can be considered as “gentle”, ignoring the consideration of the superposition principle under high loads should be carefully evaluated. Loadings tested in this work indicate that wrong premises can lead to considerable deviations.

The numerical iterative algorithm was first tested numerically for a uniform stress profile near the yield stress ($f = 0.96$). The convergence process was evaluated for different depths and annuli radii. The final convergence results obtained were close to the target value. The ratio h/D_0 influences the convergence ability, and so does the annuli diameter. The initial increments were more computationally expensive, requiring more iterations within the iterative algorithm. Both medium and far strategies also tend to decrease the convergence ability. The higher signal presented by the near case tends to reduce the divergence of the corrected residual stress values from the target value.

A few modifications to the original algorithm were implemented to tackle the convergence issue and integrate the entire profile of the inclination as a function of the polar angle θ . A methodology was developed to compute the inclination profile for both the numerical and experimental data. Care must be taken when computing the displacements for two different density regions, for instance, simulated data and experimental data.

The iterative algorithm was adapted to receive the experimental data. The full-field displacement data obtained with the aid of an optical device were converted into the partial derivative (inclination) of the radial displacement with respect to the radius. The profile of the inclination was obtained as a function of the polar angle θ . A similar methodology was employed to compute the inclination from the numerical data generated by the numerical model – it was employed in the acquisition of the inclinations from the numerical model. The iterative algorithm matched the experimental and numerical inclination profiles.

Numerical trials did not reveal any convergence issues in the final version of the iterative algorithm. On the other hand, several issues were raised in the experimental validation. Three different responses were identified in the iterative procedure of the experimental tests: 1) the algorithm converged, 2) the algorithm converged to a value far away from the expected result (diverged), and 3) the algorithm entered an infinite loop of oscillating around the target value.

In general, the responses of the iterative algorithm for the experimental measurements were related to the diameter of the measurement annulus. The straight convergence or infinity oscillation near the target value were observed for the Near case, while the medium cases did not present straight convergence, and far cases presented two divergent results. Besides the smaller signal measured in the medium and far cases, they were also more affected by the lack of correlation in some areas in the outer region of the evaluated area.

11 FUTURE WORK SUGGESTIONS

In this work, it was verified that strain gauges attached to specimens loaded to the yield limit started measuring values much higher than possible and continued going up after 90% of the yield stress. This phenomenon was not completely understood. Strain gauges with 120 Ω ended up with about 125 Ω /126 Ω and even stopped being recognized by the equipment in subsequent tests. Strain gauges presented weird behavior for loads near the yield stress of the specimens. Even though the strain gauges remained properly glued, their value skyrocketed. No special marks were identified on the specimens; more specifically, near the gauge no marks were identified. The applied strains were much lower than the maximum specification of the strain gauge. It seems that no obvious reasons may have affected the experiments. This behavior requires certain work to be understood.

Three extra tests were supposed to be performed on a different specimen. However, loading issues arose with the second specimen. A few trials were performed, but in none of them was it possible to achieve and hold the target load for the thicker specimen. The load was pretty much the same, but for some reason it was not possible to hold the load. This mission was aborted. No obvious reason accountable for this behavior has been identified. Besides that, considerable bending was identified right after tightening up the specimens with the gripping system (before loading). Future work on this topic is also acknowledged.

The most probable result obtained for iterations that entered an infinite loop is to employ the average value. Eliminating this convergence issue could be done by increasing the target maximum difference between increments. A probable solution would be the consideration of the previous values in the estimation of the next iteration, which did not work in the numerical trials.

The reduction in the deviation of the residual stresses computed following the optical technique by changing the mesh grid can also be performed by specifying the size of the area associated with a specific node (weighting the points instead of using a uniform grid). The computed displacements can also be replaced by the displacement profile, avoiding the computation of derivatives that tend to increase noise.

Lastly, the objective of this thesis was initially to correct the measurement of high levels of non-uniform residual stresses. However, in the qualifying exam, the evaluation board identified scientific limitations regarding the tackling of such a task at once. Limitations start with the strategy of load application up to the involved variables related to the hole-drilling method as well as the resolution achieved with optical strategies. Some insights regarding the load application were added to APPENDIX B and APPENDIX C to help guide future research on this topic.

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APPENDIX A - EVALUATION OF A 2D MODEL CONSIDERING PLASTICITY

INTRODUCTION

Materials in general can be subjected to loadings that go beyond their elastic limit, especially at stress concentration regions. The material is not able to recover itself back to the initial state once the load exceeded the elastic limit of the material. There are different possibilities for generating numerical models/material models in APDL taking into account the plasticity effect. Some authors in the past have already thoroughly investigated the stresses and strains present in thin plates with a hole on its center, and loaded intentionally to reach the plastic region of the material. In this appendix, a comparison between the results presented by Cockerham and Eaton (1976) and the numerical results obtained with mechanical APDL will be given, together with a full description of the numerical model.

MATERIAL PROPERTIES

The material was modeled with a bilinear kinematic hardening behavior (BKIN), which fairly represents the properties of the aluminum alloy employed in the experimental tests. Further details regarding the material properties are presented in a previous article, which reports an experimental analysis performed on a plate with similar dimensions. The following data was gathered (THEOCARIS; MARKETOS, 1964): $\sigma_y = 238$ MPa (Initial yield stress), $E = 68$ GPa (Elastic Modulus), and $T_M = 2.2$ GPa (Tangent Modulus E_T).

In order to divide the model into finite elements, PLANE182 was employed with the following input:

Keyopt,1,1,2 (Enhanced strain formulation)

In addition to that, plasticity commands were employed:

tb,bkin,1,0, !TB, Lab=Bilinear kinematic hardening, MAT, NTEMP, NPTS, TBOPT, EOSOPT, FuncName. Activates a data table for material properties or special element input.

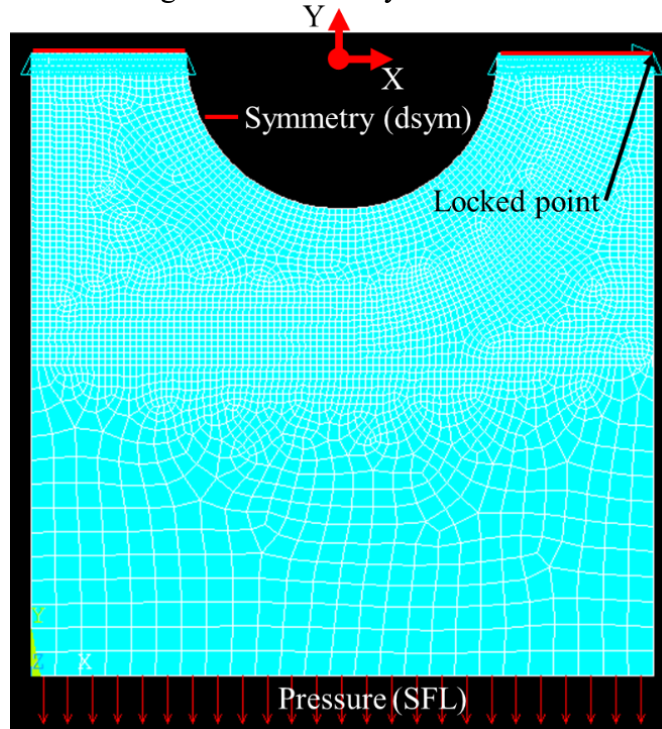
tbtemp,0.0 !Defines a temperature for a material data table.

tbdata,1,SY(σ_y),TM !Defines data for the material data table.

BOUNDARY CONDITIONS AND SOLUTION SETTINGS

The boundary and loading conditions are similar to the conditions presented by Cockerham and Eaton (1976). A 2D model with the following dimensions was employed: 4D (length) and 2D (width), where D represents the diameter of the circle, as shown in Figure 1. Different pressure values were employed: 68 MPa (I), 80 MPa (II), 99 MPa (III), 112 MPa(IV), and 121 MPa (V).

Figure 1 – Boundary conditions.



Finally, it is necessary to set some solve parameters in the /solu processor:

nlgeom,on ! Includes large-deflection effects in a static or full transient analysis.

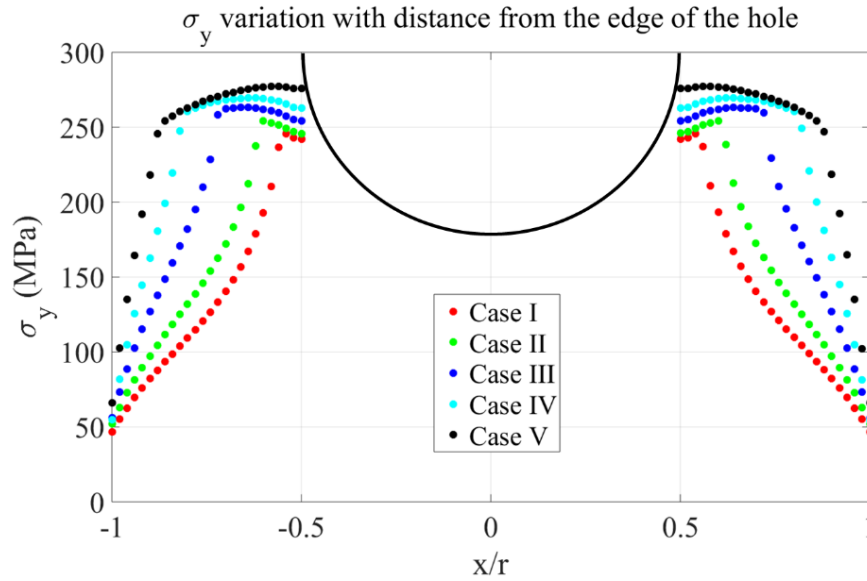
nropt,full ! Specifies the Newton-Raphson options in a static or full transient analysis.

nsubst,40,60,30,on ! Specifies the number of substeps to be taken for each load step.

RESULTS

The data obtained along the symmetry plane for each load (cases I to V) is presented in Figure 2. Although y-stresses in this case are expected to be higher near the edge of the hole for simulations within the elastic regime, it is possible to see that when taking into account plasticity effects the y-stresses at the edge of the hole do not always present the highest value.

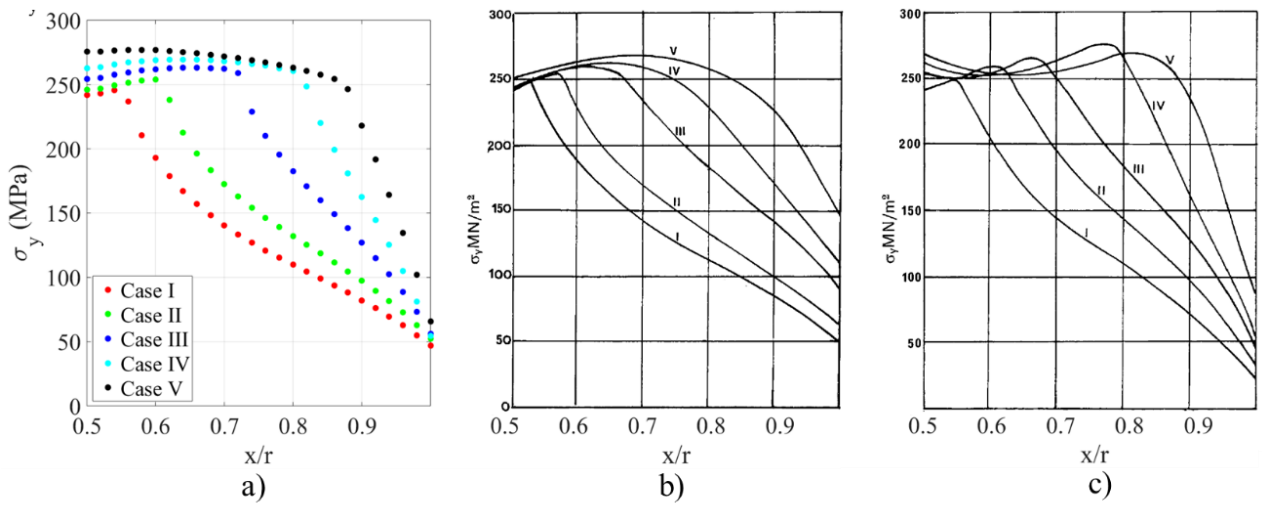
Figure 2 – Y-stresses along the symmetry axis.



To validate the results obtained with Mechanical APDL, two comparisons were performed: y- and x-stresses along the cross-section (at the mid-plane of the hole where the symmetry condition is located), and the plastically deformed regions identified on the plate. Comparing these values with results obtained previously by other researchers confirms that the numerical model is performing well and reflecting the real behavior of such a plate subjected to stresses near the yield point.

The y-stresses obtained along the cross-section are presented in Figure 3, together with the results previously obtained by other authors. Although differences can be identified, generally speaking the results follow the same trend. For the highest load (Case V), the plastically deformed region goes from the edge of the hole ($x/r=0.5$) until approximately $x/r=0.9$ for the three approaches (a, b and c); the only difference is the abrupt change at the transition point for the results obtained with Mechanical APDL, while the other two curves are quite smooth. The stress profile within the plasticized region differs for the three cases; however, the magnitude of the stresses at the edge of the hole varies by less than 10%.

Figure 3 – Y-stresses obtained with Mechanical APDL (a), simulation (b) results and experimental (c) obtained in the literature.

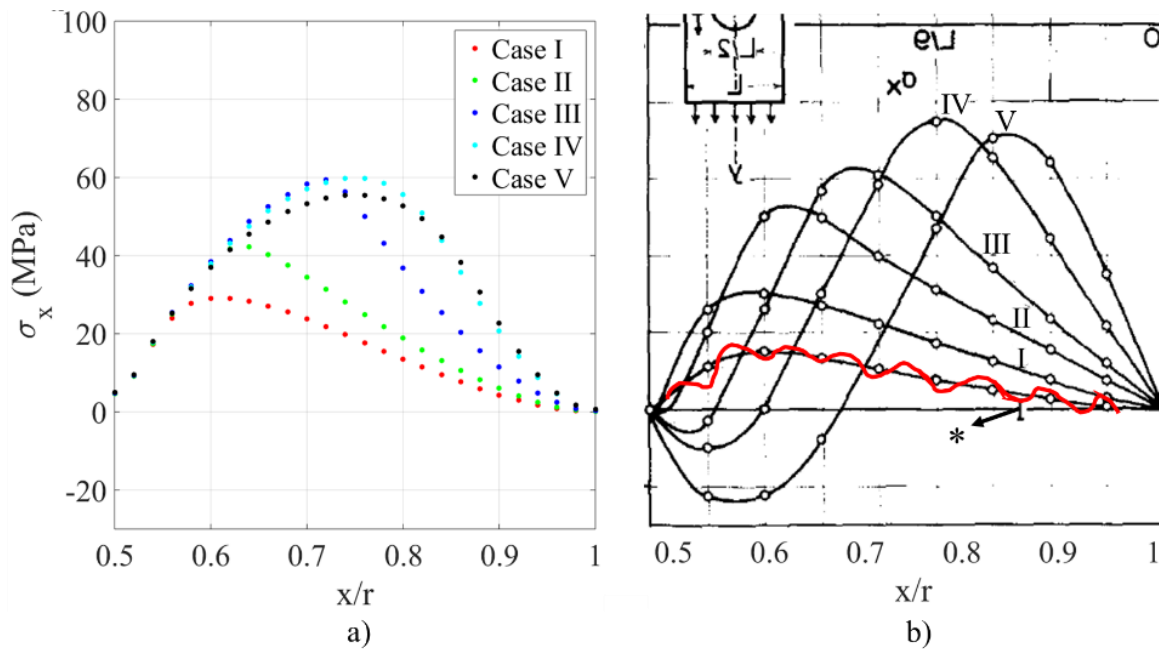


Source: Cockerham and Eaton (1976) (b), and Theocaris and Marketos (1964).

It is important to mention that the simulation results obtained by Cockerham and Eaton (1976) considered a Poisson coefficient of $\nu=0.5$, while for the experimental values the real value was 0.3. The simulation was run twice for these cases and exactly the same values for σ_x and σ_y were obtained for $\nu=0.5$. The only difference found between the two cases ($\nu=0.3$ and 0.5) was the displacements in the x-direction.

The results in the x-direction present a reasonable difference from the experimental values, as shown in Figure 4 (no simulation results were found in the literature for the stresses in the x-direction). It is interesting to point out that the curves follow a trend of shifting to the right as the load increases. However, the magnitude differs by about 20% on the upper side of the curves. Moreover, the simulated values do not go below zero at any time, while for the experimental case stresses down to -20 MPa were identified.

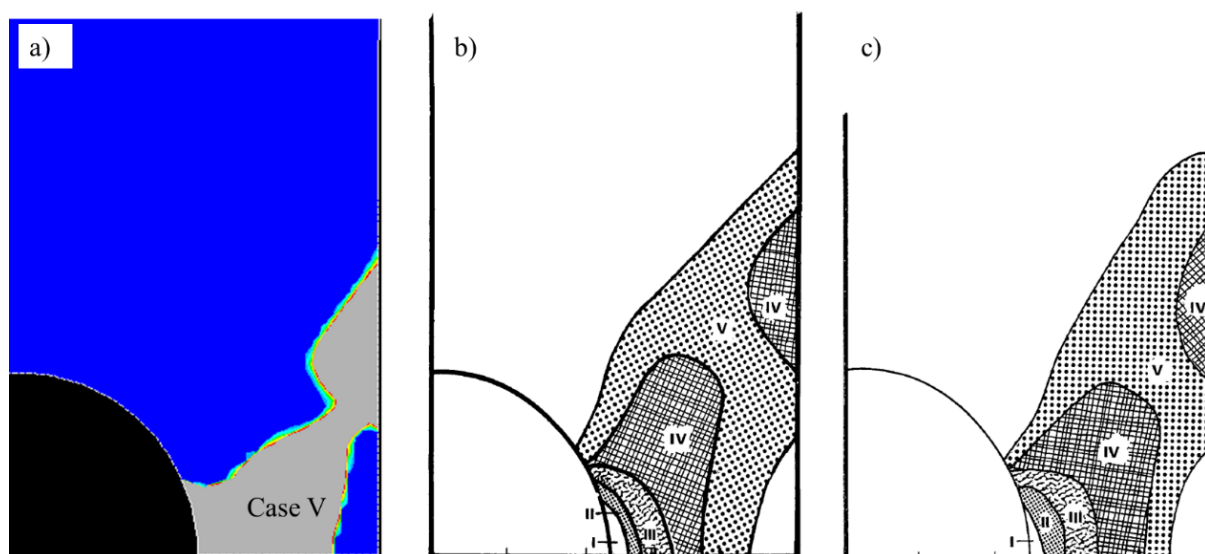
Figure 4 – X-stresses obtained with Mechanical APDL (a), and experimental data obtained by Theocaris and Marketos (1964) (b).



*The first case from Theocaris' article was not considered by Cockerham because it does not achieve the plastic regime (assumption).

It seems strange that the loads near $x/r=0.5$ (hole edge) are different from zero. The actual value is $\sigma_x(x/r=0.5)=5$ MPa. This result was obtained employing a mesh size of 0.5 mm, which is about 0.02 times the hole diameter. The actual σ_x approaches zero as the mesh size is reduced; however, the x-stresses between $x/r=0.5$ and 0.6 keep the same pattern and do not achieve negative values, as obtained experimentally by Theocaris and Marketos (1964). Figure 5 shows the simulated results at the edge of the hole for a coarse and a fine mesh (Case V). It is possible to see that the x-stress (radial) at the edge of the hole is smaller for the finest mesh.

Figure 6 – Von Mises plastic strain obtained with Mechanical APDL (a), simulation results for the plastically deformed region obtained by Cockerham and Eaton (1976) (b), plastic zone obtained experimentally by Theocaris and Marketos (1964) (c).



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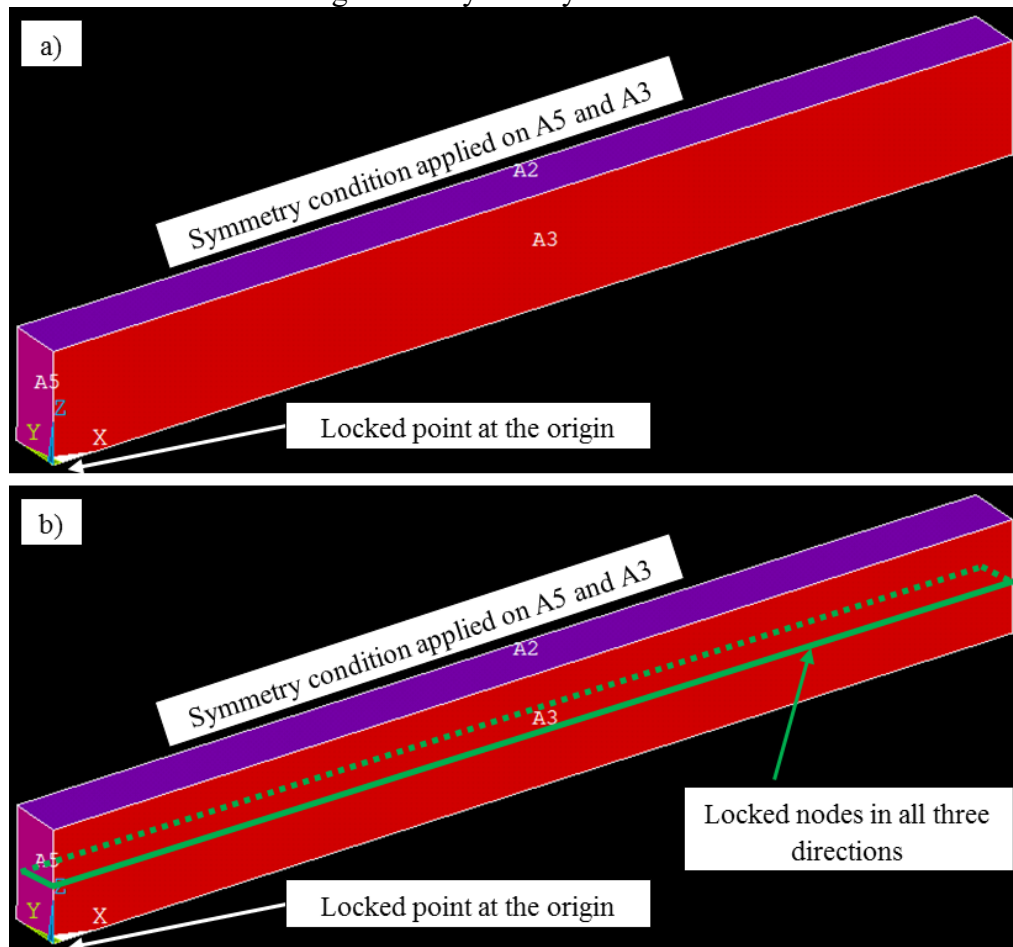
APPENDIX B - INTRODUCTION OF A RESIDUAL STRESS PROFILE WITH MECHANICAL APDL

There are quite a few different methods to introduce a stress profile on the workpiece. Tensile or compressive stresses can be employed to introduce a ‘constant’ stress profile through thickness. Either moment loads can be applied at the extreme ends of a beam or a loading device can be employed to introduce a bending moment on a beam or plate, whose resultant stresses vary ‘linearly’ through the thickness. There are also other possibilities available in Ansys, which are naturally more complex: (a) the introduction of an initial stress by using the INISTATE command; and (b) the introduction of a thermal gradient in order to produce a degree of thermal expansion or contraction generating, as a result, residual stresses.

Trials related to the introduction of a residual stress profile on a workpiece in the Mechanical APDL are presented in this Appendix. A few tests were performed to understand the behavior of the material under the presence of pre-stresses and the influence of the pre-stresses on the final equilibrium stress state acting in the material. A few possibilities were investigated, together with their advantages and drawbacks.

Two examples illustrate how the command INISTATE work. In both of them, a beam 0.55 (width)x1.1 (thickness)x20 (length) mm³ was analyzed. Figure 1a shows ¼ of the beam (case a). Symmetry condition was employed to reduce the computational cost, as shown in Figure 1. In Figure 1b an additional boundary condition was introduced along the longitudinal central plane for case b. Every node located within this plane was constrained in all three directions.

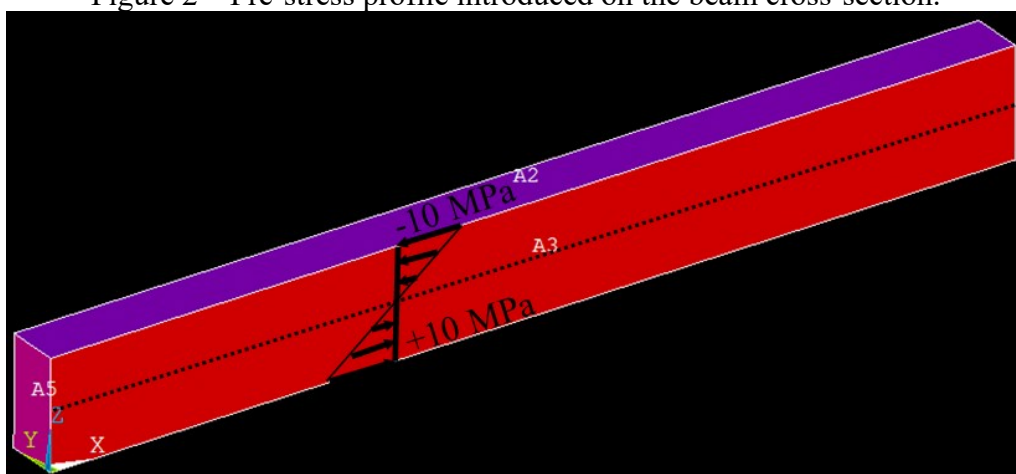
Figure 1 – Symmetry conditions.



Source: the author.

Besides the boundary conditions shown previously, an additional pre-stress was introduced along the entire beam length, as shown in Figure 2. This condition fairly represents a beam under pure bending conditions, where the upper side is subjected to tensile stresses and the lower side is subjected to compressive stresses.

Figure 2 – Pre-stress profile introduced on the beam cross-section.

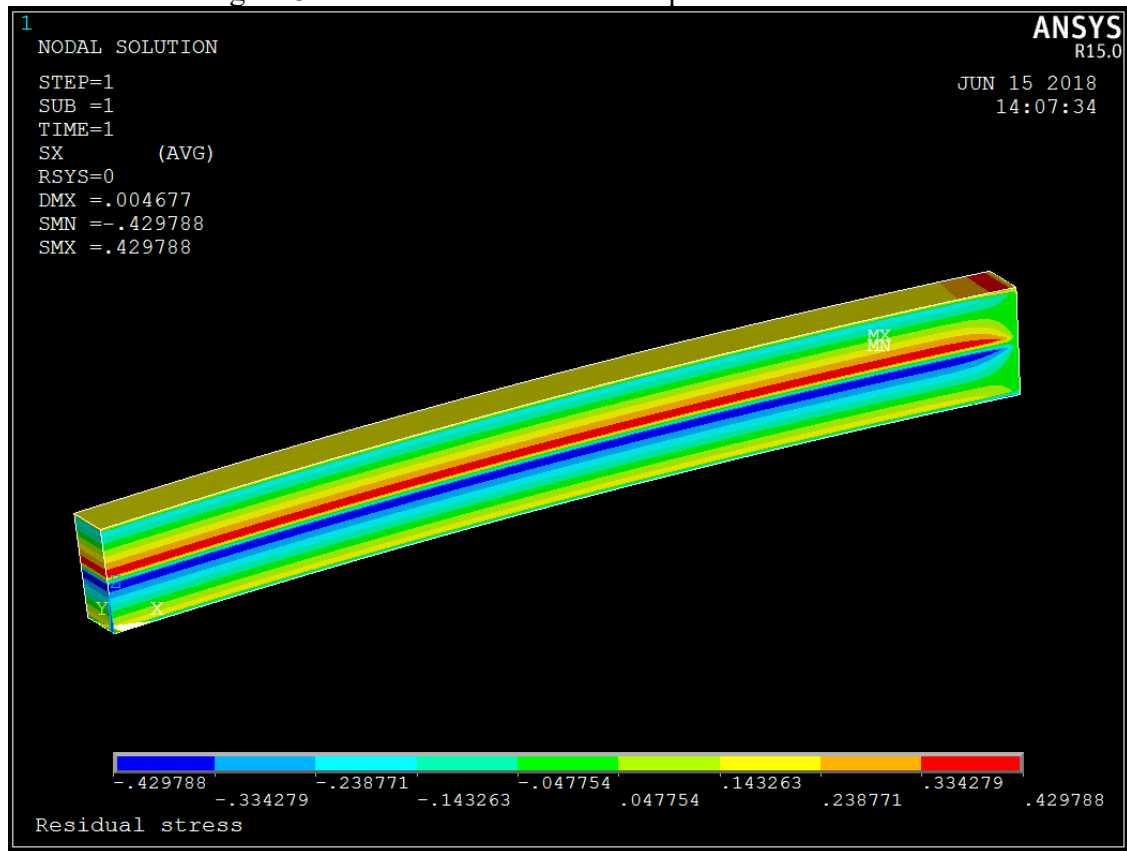


Source: the author.

For case a, the introduced stresses equilibrated within the beam, which resulted in a slightly bent beam, as shown in Figure 3. The bending itself relieved the stresses acting within the beam. It is

possible to see that the stress magnitude is 20 times smaller than the pre-stress values applied to the beam.

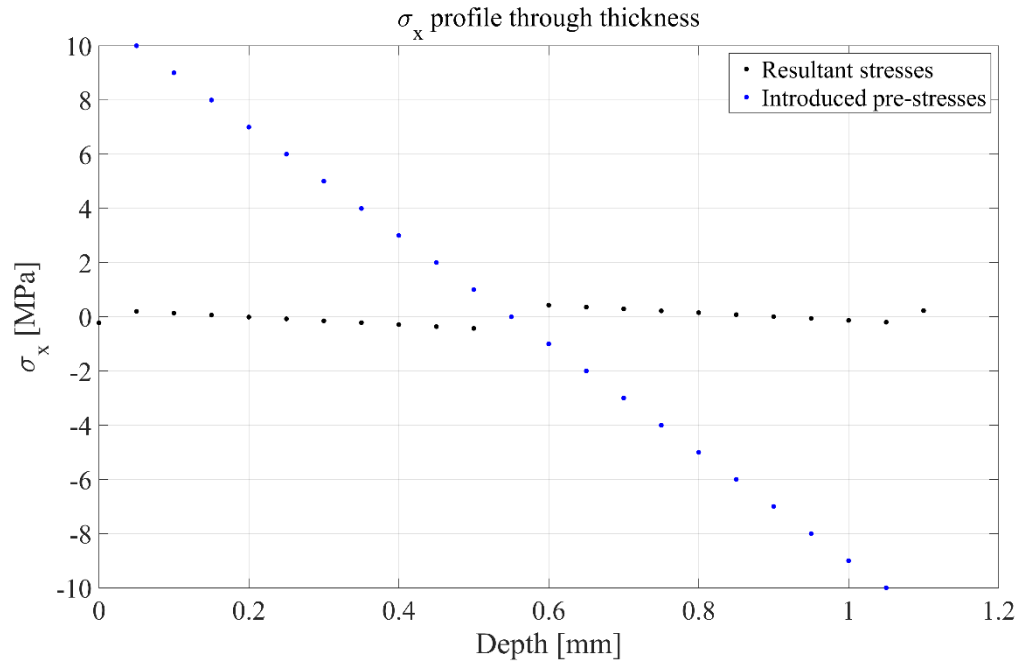
Figure 3 – Resultant stresses and displacement for case a.



Source: the author.

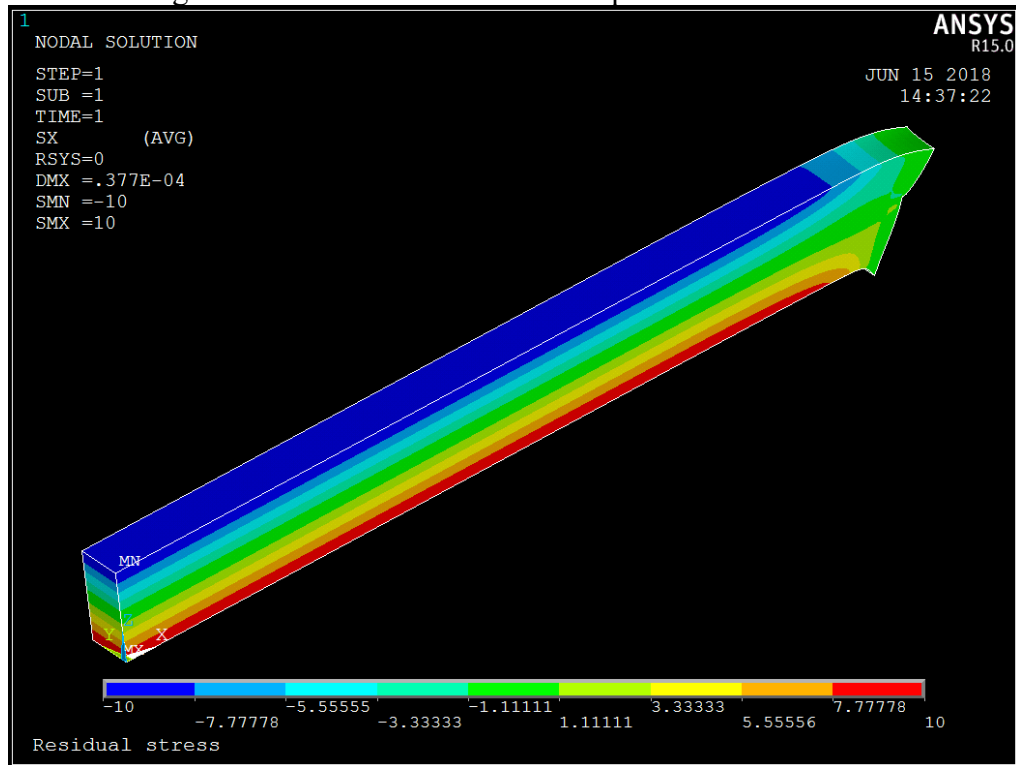
The stress relief generated by the bending incurs on resultant stresses which considerably deviate from the pre-stresses, as shown in Figure 4.

Figure 4 – Profile of the introduced and resultant stresses for case (a).



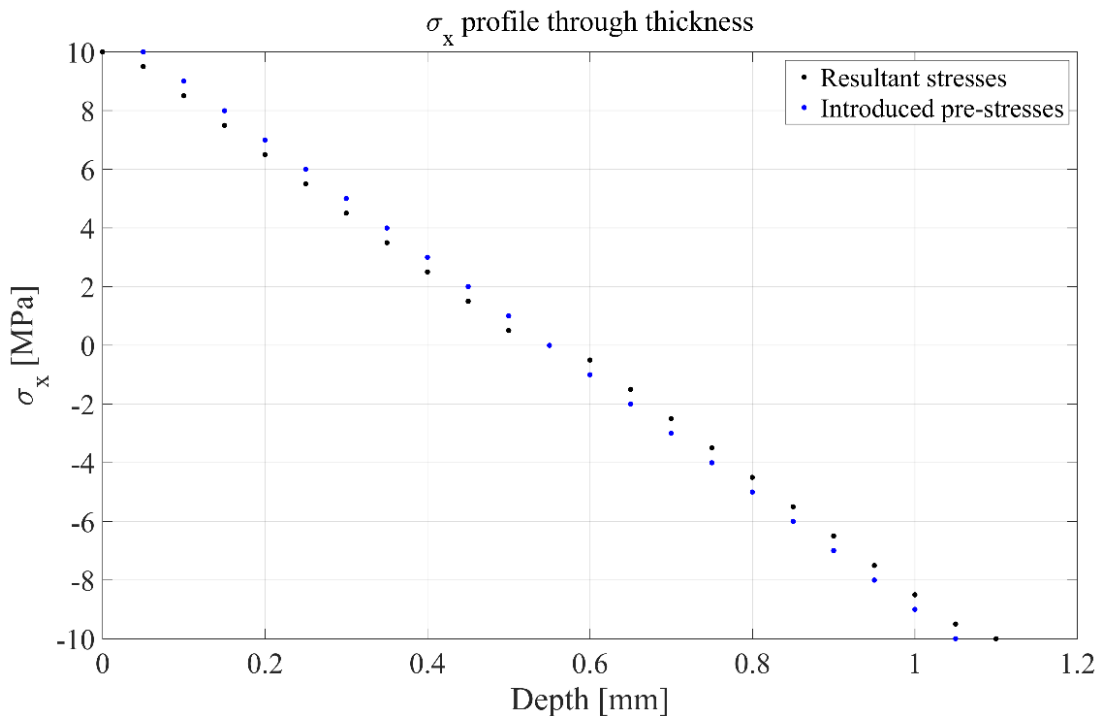
On the other hand, if the displacement of the beam is limited by encasing the central longitudinal plane, the pre-stresses are preserved within the workpiece, as shown in Figure 5. In this case, the stress relief does not occur, and the resultant stresses are much closer to the introduced pre-stresses, as shown in Figure 6.

Figure 5 – Resultant stresses and displacement for case b.



Source: the author.

Figure 6 – Profile of the introduced and resultant stresses for case (b).

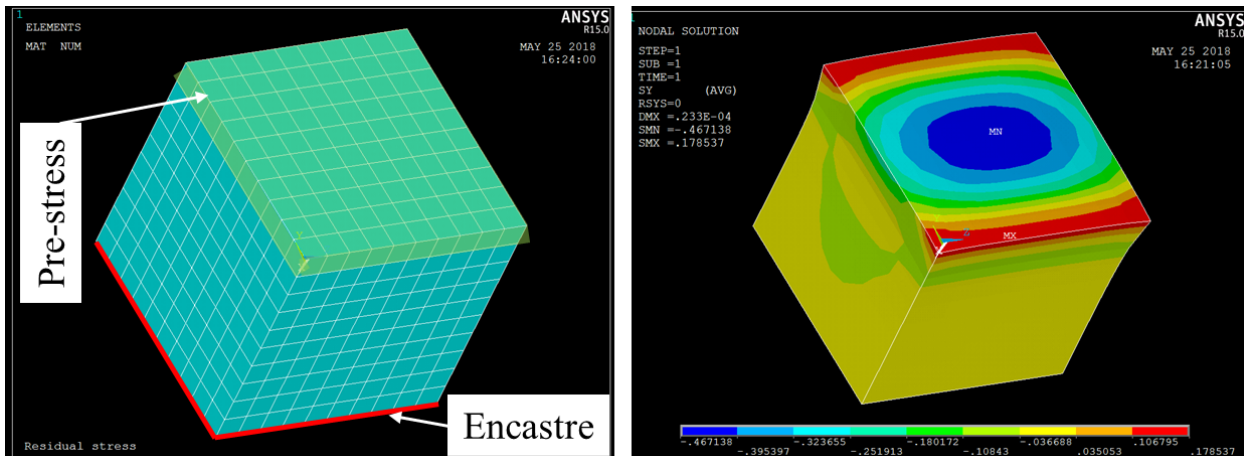


Source: the author.

In the same way, the resultant stress levels generated by a specific pre-stress introduced on a workpiece will vary with the percentage of the volume where the pre-stress was applied. For instance, if pre-stresses are applied to a small percentage of the total volume of a workpiece, the stresses are going to equilibrate within the body resulting in considerable levels of resultant stresses. But if the

pre-stresses are applied on the entire workpiece, they will simply produce a deformation and result in very small resultant stresses (unless the user applied constraints along the workpiece, as was done previously in case b). To clarify this idea, a compressive pre-stress of -1 MPa was applied within the first 2 mm of the $10 \times 10 \times 10 \text{ mm}^3$ block shown in Figure 7. The resultant stress magnitude is about half of the pre-stress.

Figure 7 – Resultant stresses generated by a -1MPa constant stress profile applied on the top surface.



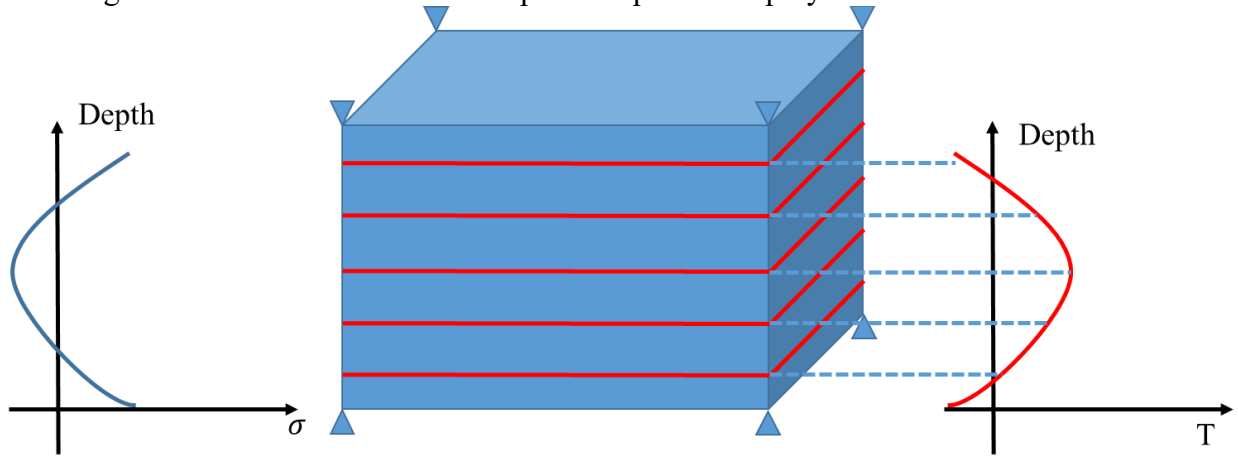
Source: the author.

The INISTATE command allows the user to define initial states to a component such as pre-stresses (residual stresses). It was shown that the resultant stresses do not correspond exactly to the introduced stresses – the resultant values are a result of stress equilibrium. Therefore, the workpiece geometry and the volume to which the pre-stresses were applied influence the resultant stresses. Further problems appear when working within the elastoplastic region. E-mails exchanged with a renowned researcher in the field led to the abandonment of this strategy. According to Prof. João P.S.G. Nobre, INISTATE did not work with elasto-plastic analysis. He found out that plastic yielding is obtained by ANSYS based on strains and not on stresses. In elastic analysis, it does not represent any problem. However, in plastic analysis, ANSYS used to impose pre-stresses without any associated strain, which does not work for plastic analyses.

APPENDIX C - TEMPERATURE GRADIENT

A residual stress profile can be introduced in a component by inducing a temperature gradient on a properly constrained model. The temperature values can be discretized and applied directly at the nodes. In this case, it is necessary to keep the same number of element divisions throughout the thickness of the model. This allows the nodes at each plane to receive a temperature value – the temperature is constant at each plane. A few planes are schematically presented in Figure 1. In this way, it is possible to induce a residual stress profile. This strategy is presented in the literature and was employed to induce an equibiaxial residual stress profile.

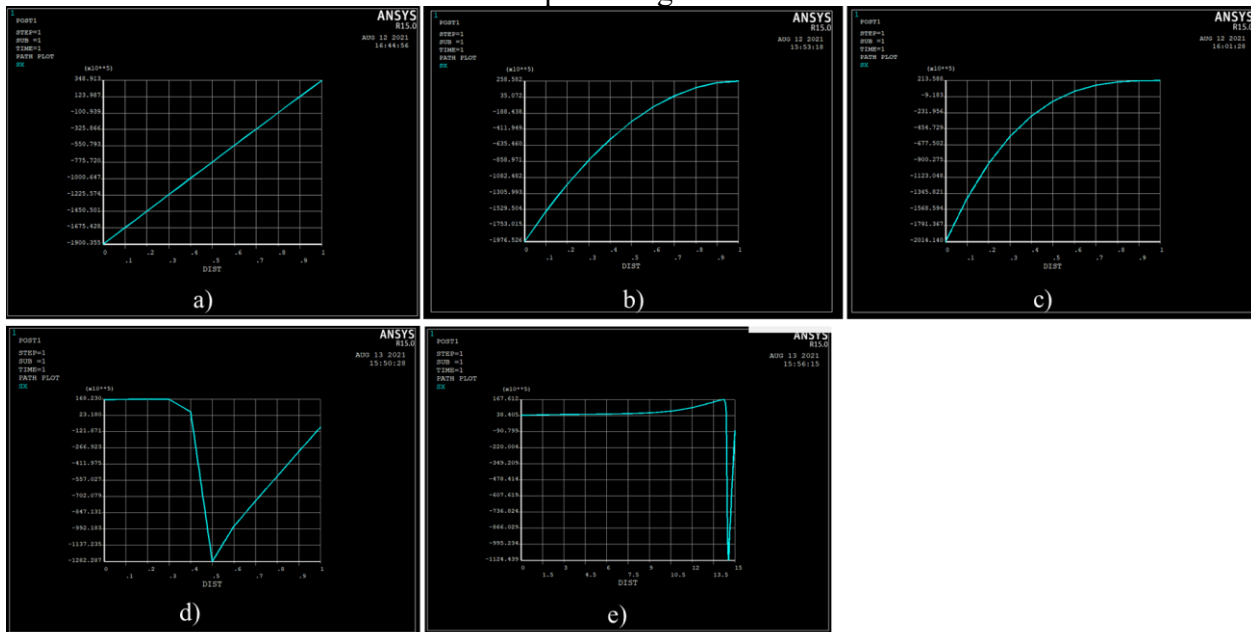
Figure 1 -Discretization of the temperature profile employed to induce residual stress.



This strategy can be employed to generate a uniaxial non-uniform residual stress profile. It is possible to define the temperature expansion coefficient (α) in both directions of the plane with constant temperature. Therefore, the artifact of setting the value of $\alpha_y=0$ would help generate the uniaxial stress state.

The model required two boundary conditions: restricting longitudinal displacement in the x-direction, and symmetry in both the x- and y-axes. These conditions allowed the generation of the residual stress profile. Three temperature profiles were applied as a function of depth: linear, quadratic, and cubic. These temperature profiles generated the stress profiles shown in Figure 2 (a), (b), and (c), respectively.

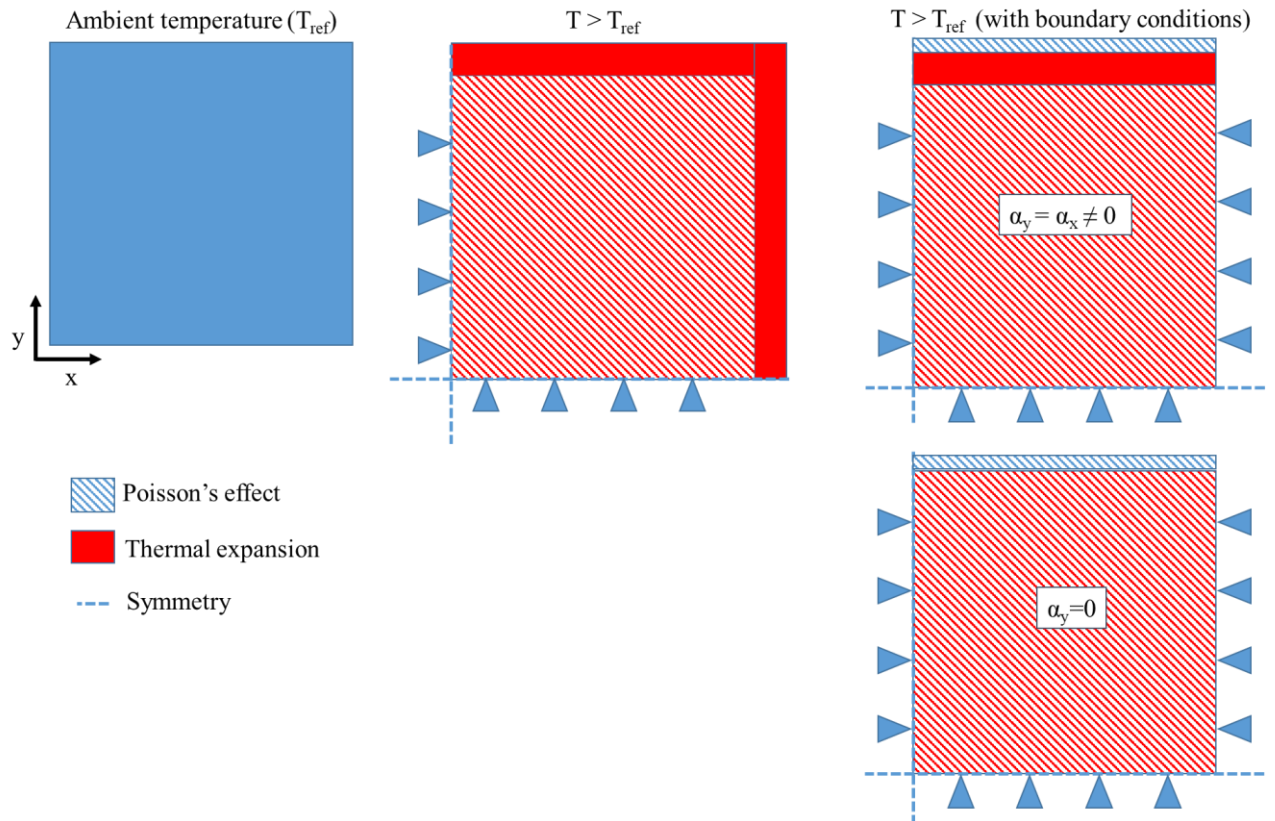
Figure 2 -Residual stress profiles induced applying different temperature gradient types: (a) linear, (b) quadratic, and (c) cubic. Details (c) and (d) illustrate the entire stress profile obtained for a linear temperature gradient.



A steep stress gradient is observed immediately below the region where the temperature gradient is applied, as shown in Figure 2 insets (d) and (e). This transition is quite aggressive because the base material is at reference temperature. One possibility of avoiding steep gradients within the measurement depth is to extend the applied temperature gradient further below the surface.

Only the stresses in the x direction were evaluated in Figure 2. The resultant stress profile was not purely uniaxial due to the Poisson effect, which is illustrated in Figure 3. Therefore, stresses in both directions were induced at different levels proportional to the Poisson's ratio. Indeed, the eliminated portion ($\alpha_y=0$) relative to the thermal expansion in the y direction is higher than the expansion induced by the Poisson's effect.

Figure 3 -Thermal expansion and expansion generated by the Poisson's effect.



These results indicate that the thermal gradient strategy is a promising strategy for introducing a non-uniform residual stress profile with depth.

APPENDIX D – DOG-BONE-SHAPED SPECIMEN LOADING

Figure 1 – Strain measurements for dog-bone-shaped specimen loading under bending (downwards).

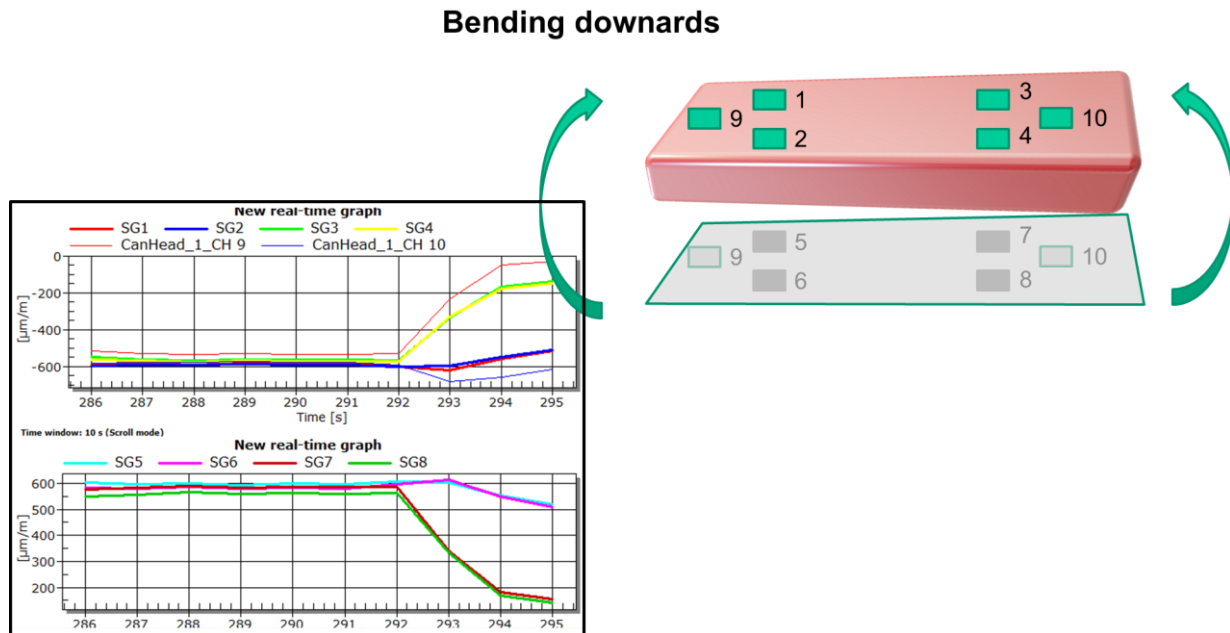


Figure 2 – Strain measurements for dog-bone-shaped specimen loading under bending (upwards).

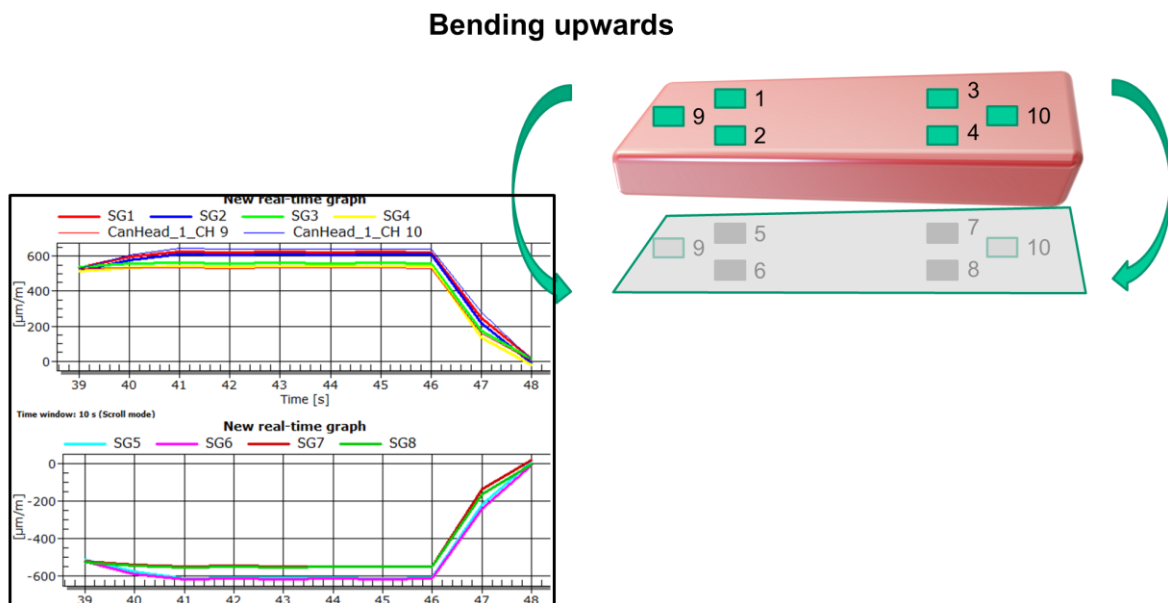


Figure 3 – Strain measurements for dog-bone-shaped specimen loading under lateral bending (front).

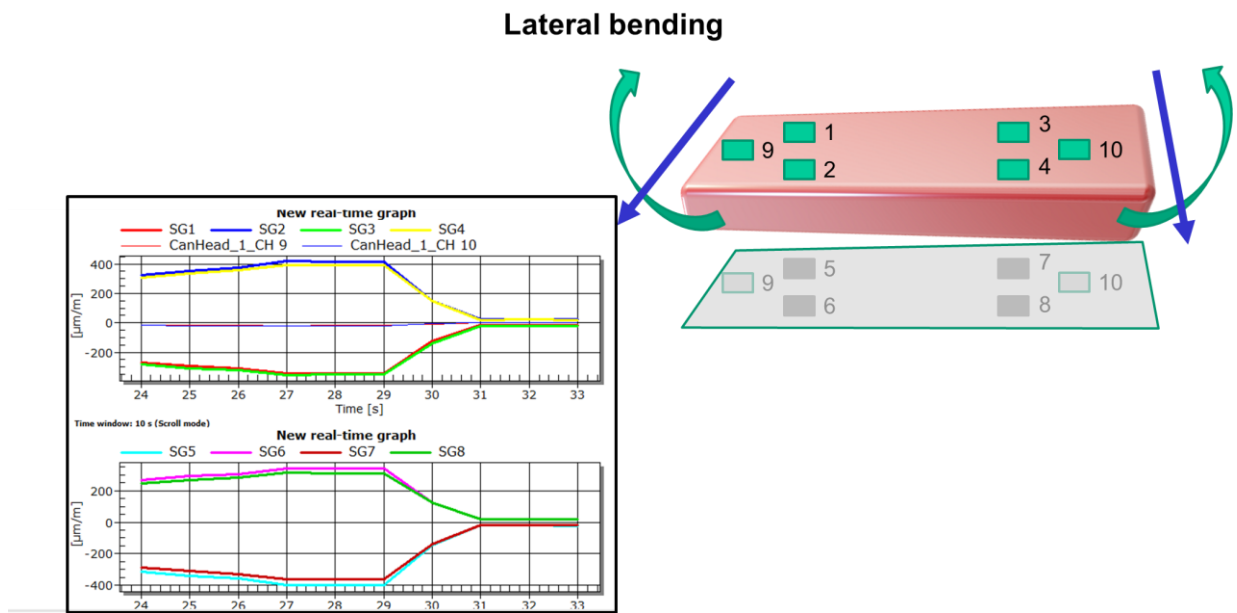


Figure 4 – Strain measurements for dog-bone-shaped specimen loading under lateral bending (rear).

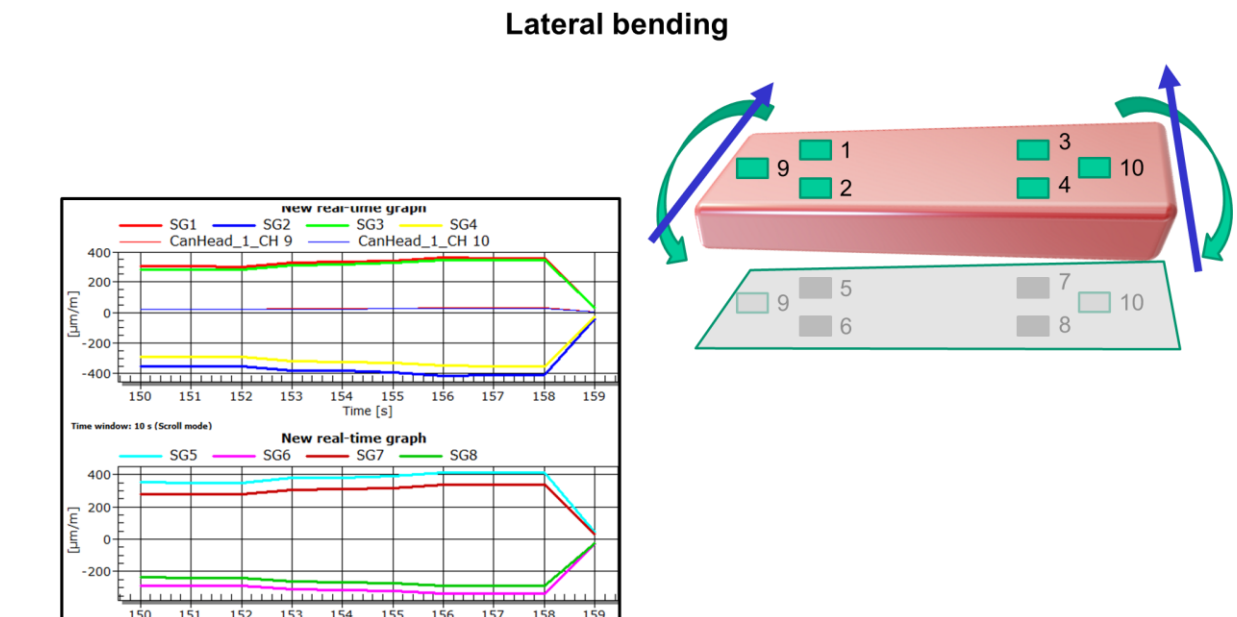


Figure 5 – Strain measurements for dog-bone-shaped specimen loading under uniaxial stress (tensile).

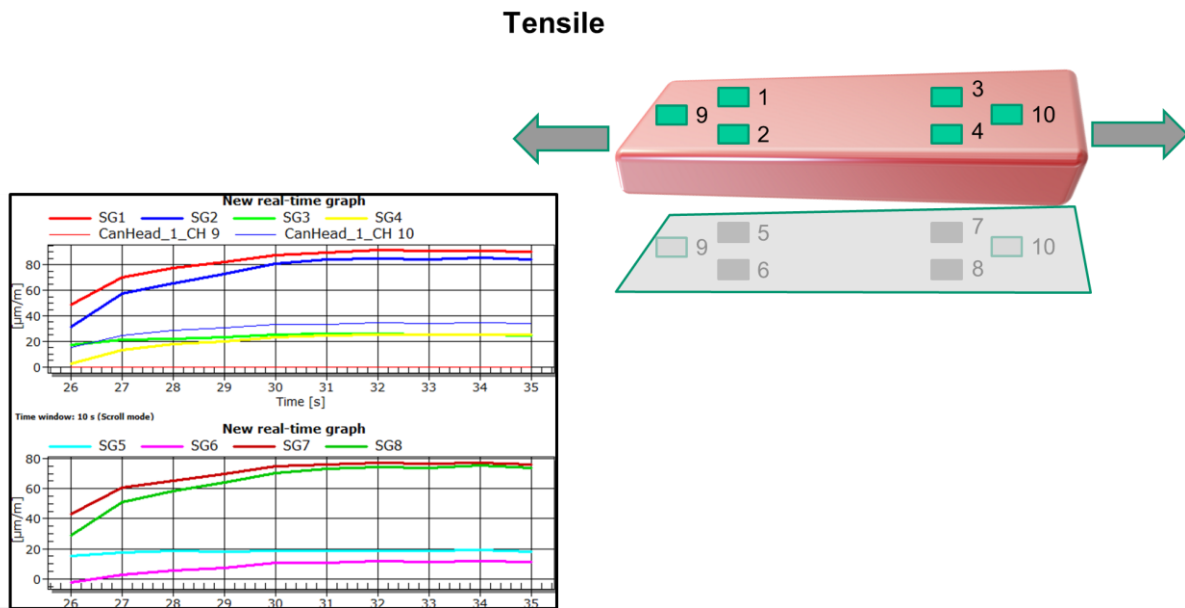


Figure 6 – Strain measurements for dog-bone-shaped specimen loading under uniaxial stress (compressive).

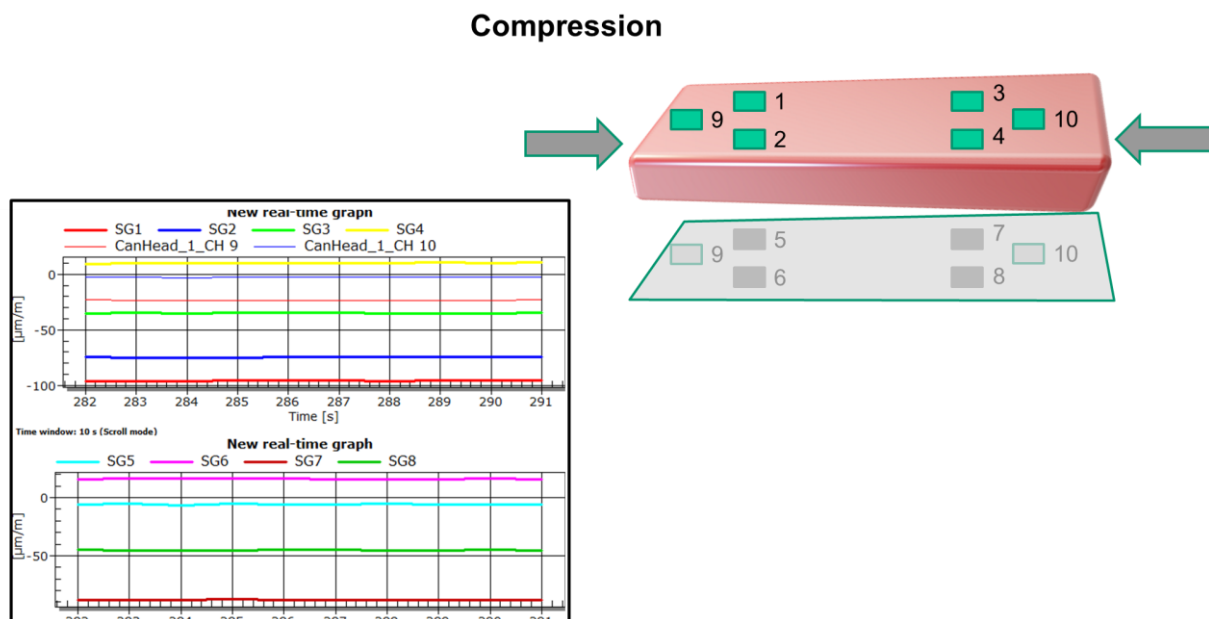
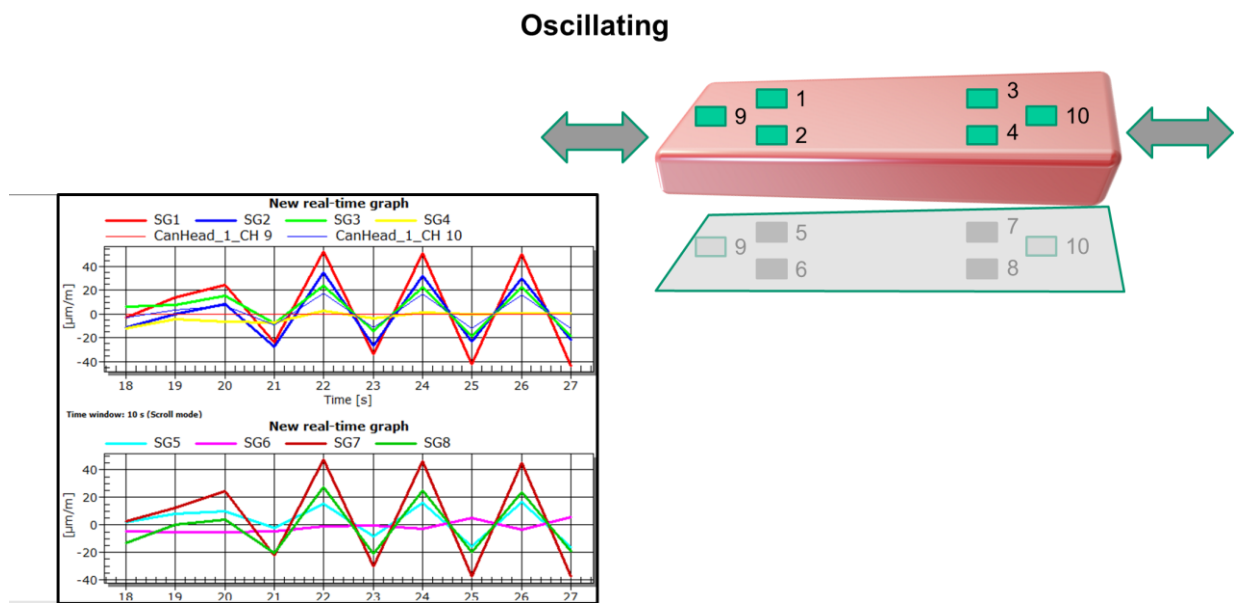


Figure 7 – Strain measurements for dog-bone-shaped specimen loading under uniaxial stress (tensile/compressive).



APPENDIX E – CONVERGENCE BEHAVIOR WITH MOVING AVERAGE

Figure 1 – Stresses in the x and y direction for depth 0.3 mm obtained considering the moving average of the last two points as the initial guess vector.

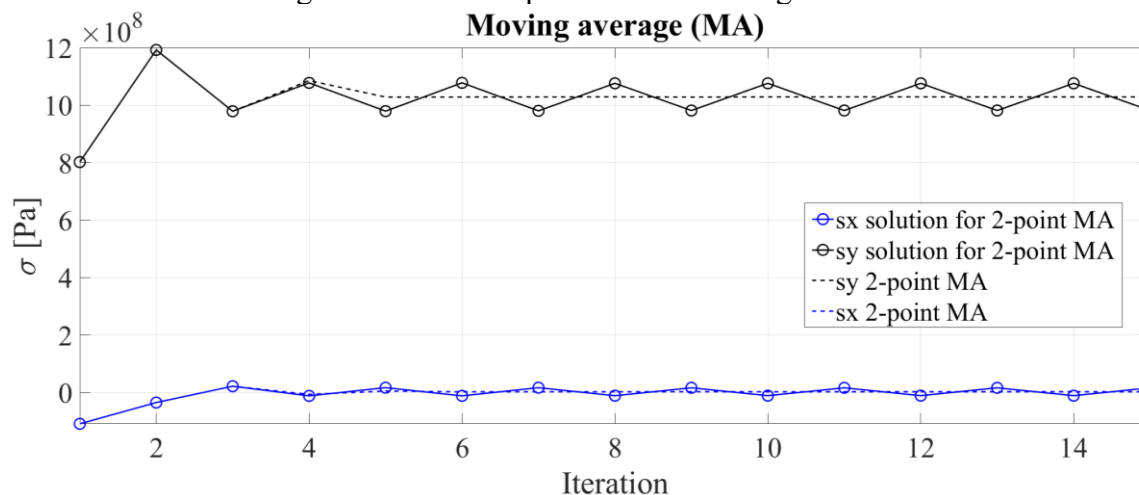


Figure 2 – Stresses in the x and y direction for depth 0.3 mm obtained considering the moving average of the last three points as the initial guess vector.

