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Quark matter models: consequences on astrophysical properties and the QCD phase diagram

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Quark matter models: consequences on astrophysical properties and the QCD phase diagram

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Certificamos que esta é a **versão original e final** do trabalho de conclusão que foi julgado adequado para obtenção do título de doutora em física nuclear e de hádrons.



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This work is dedicated to all who seek knowledge.

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"Fall in love with some activity, and do it! Nobody ever figures out what life is all about, and it doesn't matter. Explore the world. Nearly everything is really interesting if you go into it deeply enough. Work as hard and as much as you want to on the things you like to do the best. Don't think about what you want to be, but what you want to do. Keep up some kind of a minimum with other things so that society doesn't stop you from doing anything at all."
(FEYNMAN, Richard P.)

ABSTRACT

We propose an improvement of the already well-known MIT Bag model to describe quark matter. We implement a vector interaction in this model that simulates a meson exchange interaction and add an self-interacting term to control the repulsion interaction at high densities. We fix the quark-meson coupling constants via symmetry group theory and obtain the stability window according to the strange matter hypothesis. Astrophysical properties and QCD phase diagram are studied first with the use of this model alone and later also in combination with a QHD-based model. In the end we also investigate the consequences of dark matter added to hybrid stars.

Keywords: MIT Bag model. Strange quark stars. QCD phase diagram. Hybrid stars. Dark matter admixed hybrid stars.

RESUMO

Propomos uma melhoria do já conhecido modelo da sacola do MIT para descrever a matéria quark. Implementamos uma interação vetorial neste modelo que simula uma interação de troca de mésons e adicionamos um termo de autointeração para controlar a interação de repulsão em altas densidades. Fixamos as constantes de acoplamento quark-méson via teoria de grupos de simetria e obtemos a janela de estabilidade de acordo com a hipótese da matéria estranha. As propriedades astrofísicas e o diagrama de fases da QCD são estudados primeiro com o uso deste modelo sozinho e posteriormente também em combinação com um modelo do tipo QHD. No final também investigamos as consequências da adição de matéria escura às estrelas híbridas.

Palavras-Chave: Modelo da Sacola do MIT. Estrelas estranhas de quarks. Diagrama de fases da QCD. Estrelas híbridas. Estrelas híbridas misturadas com matéria escura.

RESUMO EXPANDIDO

Do ponto de vista teórico, ainda há muitas limitações para que a QCD seja resolvida, o que levou os físicos a abordar o problema sob duas perspectivas diferentes: através de cálculos de QCD na rede (LQCD) e através de modelos efetivos. Devido a restrições computacionais e dificuldades numéricas, a LQCD só funciona bem para potenciais químicos baixos (BORIÇI, 2004; ALEXANDRU et al., 2005; MENDES, 2007; BHAT-TACHARYA et al., 2014; GOY et al., 2017). Isso faz com que modelos efetivos sejam atualmente a única fonte para gerar resultados que cobrem o diagrama de fases da QCD em potenciais químicos mais altos.

Esses modelos efetivos devem satisfazer várias restrições. Nesta tese, propomos uma melhoria do já conhecido modelo da sacola do MIT para descrever matéria de quarks e exploramos vários de seus aspectos mais importantes além de também combiná-lo com outro modelo bem conhecido, mas para descrever matéria de hádrons, os chamados modelos baseados na hadrodinâmica quântica (QHD). Nós os confrontamos com diversas restrições nucleares e astrofísicas para testar sua validade.

No capítulo 2 revisitamos o modelo original da sacola do MIT, um dos modelos relativísticos mais simples para descrever matéria de quarks, e propomos algumas melhorias para que ele possa satisfazer mais restrições. Nos estudos apresentados neste capítulo, o objetivo principal é reconciliar a conjectura de Bodmer-Witten com a existência de estrelas massivas recentemente detectadas. Corrigimos o modelo da sacola do MIT Vetorial já proposto na literatura ao adicionar o termo de massa do campo vetorial. Além disso, além de considerar a constante universal de acoplamento méson-quark, usando argumentos de grupo de simetria, estudamos o caso em que o acoplamento do campo vetorial com o quark s é apenas 40% do acoplamento com os quarks u e d . Construimos a janela de estabilidade e conseguimos reproduzir estrelas com massas de $2,61 M_{\odot}$, mas com raio maior que o obtido em (CAPANO et al., 2020), porém, compatível com os valores inferidos das observações do NICER (MILLER et al., 2021). Também conseguimos obter uma massa máxima de $2,40 M_{\odot}$ que satisfaz todas as restrições astrofísicas, incluindo o raio das estrelas canônicas para (CAPANO et al., 2020) e (MILLER et al., 2021) ao mesmo tempo, mantendo também a consistência termodinâmica. Em todos os casos, conseguimos obter massas e raios que concordam com as medições do pequeno objeto HESS J1731-347.

Ainda no capítulo 2 propomos uma modificação no campo vetorial linear e introduzimos um termo quártico no canal vetorial. Este termo nos permite modificar a equação de estado mantendo a janela de estabilidade inalterada. Mostramos que um termo quártico positivo torna o EoS mais suave em altas densidades, enquanto um negativo os torna mais rígidos. Com a introdução deste termo, somos capazes de obter uma massa de $2,44 M_{\odot}$ para uma estrela estranha estável, satisfazendo todas as restrições astrofísicas.

No capítulo 3 continuamos a explorar alguns aspectos deste modelo de sacola do MIT modificado, adicionando temperatura ao modelo e estudando o diagrama de fases da QCD e a matéria de quarks quente. Começamos considerando matéria simétrica

consistindo apenas de quarks u e d com $\mu_u = \mu_d$ e concluímos que é difícil conciliar as restrições impostas pelos dados de *freeze-out* e transição do tipo líquido-gás com a existência de matéria estranha estável. Implementamos também uma Bag com dependência na temperatura de tal forma que seu valor aumenta com a temperatura. Usando esse modelo com a parametrização onde o valor Bag em $T = 0$ é $B^{1/4}(T = 0) = B_0^{1/4} = 165$ MeV, fomos capazes de obter uma boa descrição para o diagrama de fase QCD para temperaturas baixas e altas, satisfazendo todas as restrições impostas.

Também construímos os diagramas de fase QCD usando matéria estelar constituída por quarks u , d e s + léptons em equilíbrio químico e neutralidade de carga elétrica. Observamos uma diminuição considerável nas temperaturas de transição em $\mu \rightarrow 0$ (T_{max}) e nos potenciais químicos em $T \rightarrow 0$ (μ_{max}). Quando analisamos as relações massa-raio, constatamos que nenhuma parametrização usada nesse capítulo atende a todas as restrições impostas pelos dados astrofísicos.

No capítulo 4 continuamos a estudar o diagrama de fases da QCD, mas então com dois modelos efetivos: ainda o modelo modificado da sacola do MIT para descrever a matéria de quarks, mas também um modelo da hadrodinâmica quântica (QHD) para descrever a matéria hadrônica. As condições de Gibbs foram utilizadas para estabelecer os pontos de cruzamento da pressão em função dos potenciais químicos obtidos em ambas as fases.

Mais uma vez obtivemos primeiro os resultados para matéria simétrica para confrontá-los com resultados da LQCD e do *freeze-out*. E novamente, apenas com a Bag com dependência na temperatura, em que $B_0^{1/4} = 165$ MeV, conseguimos resultados satisfatórios para baixos potenciais químicos. Também observamos de novo que, quanto maior o valor da pressão da Bag, maiores os potenciais químicos obtidos. O mesmo acontece quando incluímos o campo vetorial, mas neste capítulo, os resultados são mais sensíveis aos diferentes valores de G_V do que no capítulo anterior.

Além disso, calculamos os diagramas de fases para a matéria estelar usando duas prescrições diferentes, uma onde impomos a conservação do sabor no ponto de transição de fase, uma prescrição nunca aplicada a temperaturas finitas antes, e a outra onde ambas as fases são β -estáveis e eletricamente neutras, sendo esta última a prescrição de Maxwell. Entre as duas prescrições, a conservação do sabor resulta em potenciais químicos mais altos em baixas temperaturas. Comparando esses resultados com a matéria simétrica, concluímos que a matéria estelar favorece a fase de quarks, sendo que as T_{max} s e os μ_{max} s são maiores para a matéria simétrica.

Na última seção desse capítulo, discutimos os resultados para a energia latente $L|_\epsilon$ e mostramos que eles são sempre positivos. Usando outra definição e chamando-a de calor latente $L|_S$, buscamos os pontos onde seu valor se torna negativo, o que pode corresponder ao ponto crítico (CEP) do diagrama de fases da QCD. Se isso for verdade, todos os diagramas de fase obtidos neste capítulo são válidos apenas até este ponto.

No capítulo 5 continuamos usando os dois modelos efetivos usados no capítulo an-

terior, mas retornamos à temperatura zero e exploramos a possibilidade de estrelas muito massivas detectadas recentemente serem, na verdade, estrelas híbridas, ou seja, estrelas compactas com um núcleo composto de quarks desconfinados cercados por matéria hadrônica comum. Como critério para construir as equações de estado híbridas a partir dos dois modelos, usamos a construção de Maxwell.

Mostramos que o modelo original da sacola do MIT, sem interação entre quarks, não é suficiente para explicar estrelas híbridas que satisfazem as restrições impostas por PSR J0740+6620 (RILEY, T. E. et al., 2021) ou PSR J0952-0607 (ROMANI et al., 2022). No entanto, quando introduzimos o canal vetorial fomos capazes de reproduzir estrelas híbridas dentro dos limites impostos pelas observações astrofísicas consideradas. Além disso, ainda com o modelo do MIT vetorial, obtivemos núcleos de quarks hipermassivos, correspondendo de 25% até 84% da massa total da estrela.

Quando também introduzimos o termo autointerativo proporcional à constante b_4 , verificamos que, se mantivermos o mesmo valor de pressão da sacola, precisamos aumentar significativamente o valor de G_V para obter potenciais químicos na interface hádron-quark maiores que 1050 MeV. Não fomos capazes de reproduzir estrelas híbridas dentro das restrições astrofísicas.

E finalmente no capítulo 6, investigamos como a matéria escura (DM) afeta a composição de estrelas de nêutrons (NSs) e anãs brancas (WDs) e focamos principalmente em como a DM pode desencadear o aparecimento de matéria de quarks desconfinada no núcleo de NSs.

Para construir as estrelas híbridas, usamos os mesmos modelos do capítulo anterior, ou seja, modelo baseado em QHD e modelo baseado em MIT, mas com uma parametrização diferente para o último. Para DM, usamos a equação de estado de (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006) para uma partícula de DM massiva não aniquiladora e exploramos diferentes forças de interação e duas massas diferentes para a partícula DM, $m_D = 5$ GeV e $m_D = 100$ GeV. Consideramos que a DM e matéria ordinária interagem apenas gravitacionalmente.

Analisamos a relação massa-raio para os ramos de NS e WD e mostramos que quando uma das pressões é muito maior que a outra, o fluido com a maior pressão central domina e a configuração se comporta como um único fluido. A quantidade de DM necessária para a dominância dela depende da força de interação e da massa de sua partícula.

Então, nos concentramos apenas no ramo NS e mostramos que a DM aumenta a pressão central de uma estrela já compacta e que isso pode permitir o aparecimento ou mesmo o aumento na quantidade de matéria de quarks desconfinada em seu núcleo. Mostramos que, exceto para a DM de interação forte e uma massa de $m_D = 5$ GeV, podemos obter estrelas de mesma massa com quase o mesmo raio, mas com diferentes pressões centrais, dependendo da quantidade de DM adicionada, o que, por sua vez, pode permitir o aparecimento de QM desconfinada. Concluimos que a DM está mascarando estrelas híbridas. Também mostramos que a quantidade de DM

necessária para aumentar a aparência de matéria de quarks depende tanto da força de interação quanto da massa da partícula DM, sendo menor para $m_D = 5$ GeV do que para $m_D = 100$ GeV e maior para DM de interação fraca.

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1 INTRODUCTION

It is currently understood that, at an elementary level, all matter is composed of leptons and quarks. Quarks are subject to the four fundamental forces as they have mass and electrical charge, which subject them to gravitational and electromagnetic forces, respectively; they also have a color charge, which subjects them to the strong nuclear force, and they can change flavour in decays, which is due to the weak nuclear force. Quantum chromodynamics (QCD) is the relativistic quantum field theory that explains the interaction between quarks through the strong force.

Ordinary matter, however, is made up of hadrons. That is, under normal conditions, quarks are always confined within hadrons. However, the QCD has a property called asymptotic freedom, discovered and described in 1973 by Frank Wilczek and David Gross (GROSS; WILCZEK, 1973) and, independently, by David Politzer in the same year (POLITZER, 1973), which says that the bond to which quarks are subjected decreases in intensity at small distances or high energies. This means that in situations of high-energy, such as those found in particle accelerators or the early universe, or at very high densities, such as those found inside compact stars, quarks can become deconfined.

In fact, since the discovery of asymptotic freedom, the idea of the existence of a quark-gluon plasma (QGP) has been increasingly reinforced. The first experimental corroboration of the existence of the QGP occurred at RHIC in 2005 (ADAMS, J. et al., 2005).

From the theoretical point of view, there are still many limitations for the QCD to be solved, which led physicists to attack the problem from two different perspectives: through QCD calculations on the lattice, the so called lattice QCD (LQCD), and through effective models. Due to computational constraints and numerical difficulties, such as the sign problem, the LQCD method only achieves results for low chemical potentials (BORIÇI, 2004; ALEXANDRU et al., 2005; MENDES, 2007; BHATTACHARYA et al., 2014; GOY et al., 2017). This makes effective models currently the only source to generate results that cover the QCD phase diagram at higher chemical potentials.

These effective models must satisfy several constraints. In this thesis we explore several aspects of some effective models and confront them with various constraints.

In chapter 2 we revisit the original MIT bag model, one of the simplest relativistic models to describe quark matter (QM), and propose some improvements to it to enable it to satisfy more constraints. In this chapter the main objective is to reconcile the Bodmer-Witten conjecture, i.e, the strange matter (SM) as being the true ground state of matter, with the existence of massive stars recently detected.

In chapter 3 we continue to explore some aspects of this modified MIT bag model in that we add temperature to the model and study the QCD phase diagram and hot

QM.

In chapter 4 we continue to study the QCD phase diagram, but then with two effective models: still the modified MIT bag model to describe the QM and now also a quantum hadrodynamics (QHD) model to describe the hadron matter.

In chapter 5 we continue using the two effective models, but return to zero temperature and explore the possibility of the very massive stars detected recently be actually hybrid stars, i.e., compact stars with a core composed of deconfined quarks surrounded by ordinary hadron matter.

In chapter 6 we investigate how the dark matter (DM) affects the composition of compact objects (COs), such as neutron stars (NSs) and white dwarfs (WDs), and focus mainly on how the DM can trigger the appearance of deconfined QM in the core of NSs.

Following we present some of the aspects that will be explored more profoundly throughout this work.

1.1 QCD PHASE DIAGRAM

There are still a lot of uncertainties about the QCD phase diagram and only conjectured forms can be found in the literature, as the one showed below in Fig. 1. For low chemical potentials and temperatures, there is a hadronic phase of color-neutral bound states. At high temperatures the effective degrees of freedom are the quarks and gluons, this phase is called the quark–gluon plasma (QGP) (GUENTHER, 2021).

Calculations of the Lattice QCD (LQCD) suggest that at low baryon chemical potentials μ_B the transition from QGP to the hadron phase is a smooth crossover¹ (FUKUSHIMA; HATSUDA, 2010; BELLWIED et al., 2015; AOKI et al., 2006; STEPHANOV, 2005), while at large μ_B and low temperatures various models seem to suggest the transition to be of first order. The first order phase transition ends at the so called QCD critical point or critical endpoint (CEP), which is a second order phase transition, before it becomes a crossover. For very large chemical potentials a color superconducting phase (RISCHKE, 2004) is expected.

Lots of experiments probe various regions of the QCD phase diagram, as indicated in Fig. 1. At low μ_B the probes are done at the Large Hadron Collider (LHC) and Relativistic Heavy Ion Collider (RHIC). The beam energy scan (BES) program at RHIC is specially designed to probe the location of the CEP. This program will be complemented by the Nuclotron-based Ion Collider Facility (NICA), located in Dubna, Russia,

¹ A crossover type of phase transition is a very smooth transition, that happens gradually and, therefore, is very difficult to precise where exactly the transition happens, but you can point to where it is the strongest. That is why in this region of the QCD phase diagram one often talks about pseudo phase transition line. It differs from a first order phase transition, which is a more abrupt transition and one can precise exactly where it happens.

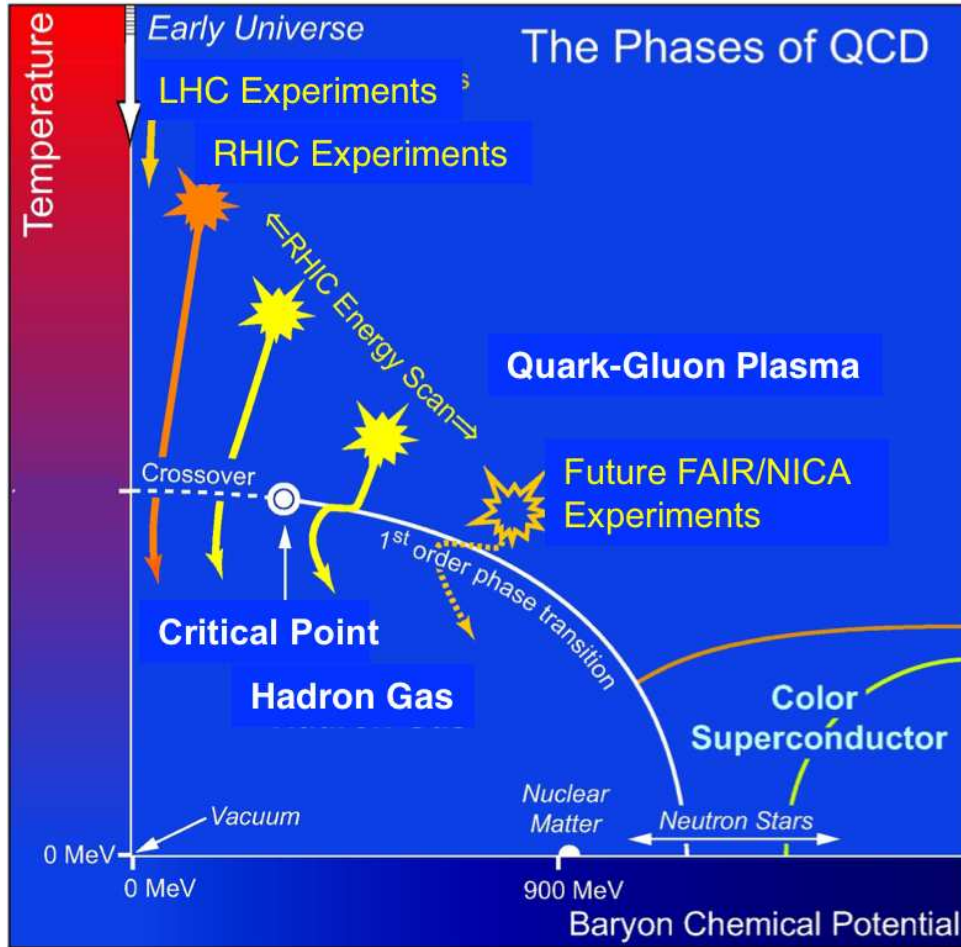


Figure 1 – A schematic QCD phase diagram in the temperature (T) and baryonic chemical potential (μ_B) plane. The regions probed by different accelerator facilities are indicated (NAYAK, 2020).

and Facility for Antiproton and Ion Research (FAIR) at GSI, located in Darmstadt, Germany, which will explore regions of larger μ_B .

Studying Au+Au and Pb+Pb collisions, Cleymans (CLEYMANS et al., 2006) was able to trace the chemical freeze-out line, obtained when inelastic collisions between particles cease such that the abundance ratios do not change anymore. In such collisions a hot fireball is produced, that then cools and expands until it thermally freezes out and free-streaming hadrons reach the detector. The spectroscopy of these hadrons and the much rarer photons, electrons, and muons allow conclusions about the state of the matter inside the fireball, such as its temperature and density (BRAUN-MUNZINGER; WAMBACH, 2009). Although the chemical freeze-out is not directly related to the hadron-quark phase transition, the hadron multiplicities in central high energy nucleus-nucleus collisions are established very close to the phase boundary between hadronic and quark matter (BRAUN-MUNZINGER; STACHEL; WETTERICH, 2004), especially at very low baryon chemical potential, when the chemical freeze-out temperature and the critical temperature are expected to lie at the same error

bar (BRAUN-MUNZINGER; STACHEL; WETTERICH, 2004; KARSCH; LAERMANN; PEIKERT, 2001). Moreover, the chemical freeze-out is expected to be a pure hadronic process, therefore, its trace needs to be in the hadron phase. This feature, therefore, acts as a constraint for hadron-quark phase transition modeling and will be used as such in chapters 3 and 4 of this thesis.

Another important region in the QCD phase diagram is the liquid-gas instability region related to nuclear fragmentation (FINN et al., 1982; D'AGOSTINO et al., 2005). For low temperatures and chemical potential, the nucleons are confined into the nuclei, which can be regarded as a liquid phase. As the temperature increases, the nuclei start to dissolve into a diluted interacting gas of nucleons. A critical temperature, above which only the gas phase survives, is expected (ELLIOTT et al., 2013). Like the chemical freeze-out, the liquid-gas phase transition is a pure hadronic process, where the contribution from the quark degrees of freedom can be neglected. Nevertheless, the region expected to undergo this phase transition can also be used as a constraint.

Furthermore, at $T = 0$ we expect the phase transition to occur at chemical potential values at least greater than 1050 MeV, as shown in (FUKUSHIMA; HATSUDA, 2010) using the Polyakov loop formalism. In the works (BUBALLA, 2005; RUESTER et al., 2005), using Nambu Jona-Lasinio (NJL) type models, this transition occurs around $\mu = 1080$ MeV to $\mu = 1100$ MeV. There is no experimental evidence about the maximum chemical potential that preserves the hadronic phase, but in the study presented in (ANNALA et al., 2020) it is argued that QM inside massive neutron stars is not only possible, but also probable and (AYRIYAN et al., 2018), using a relativistic density functional approach, indicates that this transition should occur around $\mu = 1200$ MeV for β -stable matter.

We will study in details the QCD phase diagram in chapters 3 and 4 of this thesis. In chapter 3 we will use only the MIT Bag models to obtain those diagrams, making $P = 0$ as the condition for the deconfinement of the quarks whereas in chapter 4 we will also use QHD type models and the conditions to phase transition will be the Gibbs' conditions of thermodynamic equilibrium. The above discussed restraints imposed by the chemical freeze-out line traced by Cleymans and by liquid-gas phase transition will be taken into consideration. And also, for $T = 0$, based on the discussions above, for symmetric matter of two flavours, we seek that the maximum chemical potential is between $1050 < \mu < 1400$ MeV.

In this thesis we will only discuss the $T \times \mu_B$ plane of the QCD phase diagram, but more dimensions could be added as the hadron-quark phase transition also depends on quantities like the magnetic field, the strangeness and the isospin. Some examples that explore more dimensions can be found in (ARYAL et al., 2020; DEXHEIMER et al., 2020), for instance.

1.2 STRANGE MATTER HYPOTHESIS

Strange matter is QM containing strange quarks. This matter could be created when nuclear matter, composed of protons and neutrons, is compressed past a critical density. Bodmer and Witten (BODMER, 1971; WITTEN, 1984), however, argue that the strange matter is actually the true ground state of all matter, and therefore more stable than ordinary nuclear matter. This idea is called Strange Matter Hypothesis or Strange Matter Conjecture or even Bodmer-Witten Conjecture.

The Pauli exclusion principle forbids equal fermions, like quarks, to occupy the same energy level. So, when the particle density is high enough and all energy levels below the available thermal energy are already occupied, new fermions are forced to occupy higher energy levels. But, at some point, when the required energy level is too high, it becomes energetically more favorable that some of the quarks convert into strange quarks. The rest mass of the s quark is much higher than the of the quarks up and down,² which costs some energy, but the additional energy levels make that the average energy per particle can be lower so that the strange matter is more stable than non-strange quark matter. The other heavier quarks would only occur at much higher densities.

In practical terms, this means that the strange quark matter (SQM) needs to have a binding energy per baryon lower than the energy per nucleon of the ^{56}Fe , the element that has the highest binding energy per nucleon and, at the same time, that the two-flavoured quark matter has a higher binding energy per baryon than the energy per nucleon of the ^{56}Fe :

$$\frac{E}{A} = \frac{\mathcal{E}_{(u,d,s)}}{n_{B(u,d,s)}} \leq 930 \text{ MeV} \quad \text{while} \quad \frac{\mathcal{E}_{(u,d)}}{n_{B(u,d)}} > 930 \text{ MeV}, \quad (1)$$

where E is the binding energy, A is the baryonic number and $\mathcal{E}$ and n_B are the energy and baryonic densities, respectively.

If this second condition were not obeyed the ordinary atomic nuclei could be made only from deconfined matter, which contradicts experience.

If the strange matter hypothesis is true, the nuclear matter is actually only metastable and the lifetime for spontaneous decay is very long, so that we do not see this decay happening.

If we are taking into account the Bodmer-Witten conjecture we should also discuss the stability window, which is the window of values like the Bag constant B , the meson-quark coupling constant G_V , the constant of the self-interacting term b_4 and eventually also the mass of the quark s (m_s), which allow the description of stable

² Throughout this work we will use $m_u = m_d = 4 \text{ MeV}$ and $m_s = 95 \text{ MeV}$.

strange matter. We will construct this stability window in Chapter 2 and apply its consequences throughout this entire work.

The possibility of quark matter with only u and d quarks (udQM) is usually discarded because of the observed stability of ordinary nuclei. In (HOLDOM; REN; ZHANG, 2018), however, the authors find that udQM generally has lower bulk energy per baryon than normal nuclei and SQM and, taking into account the finite size effects, they argue that udQM can be the ground state of baryonic matter only for baryon number $A > A_{min}$ with $A_{min} \gtrsim 300$. This would ensure the stability of ordinary nuclei and point to a new form of stable matter just beyond the periodic table. In this thesis, however, we will not aboard this possibility.

1.3 COMPACT OBJECTS AND THE TOLMAN-OPPENHEIMER-VOLKOFF EQUATIONS

Compact objects (COs), or compact stars, are white dwarfs (WD), neutron stars (NS) or even black holes, this last one not being treated in this thesis, are one of the possible end results of the stellar evolution. All stars maintain itself stable through the balance between gravitational force and the pressure generated by the fusion of its atoms, but when the fuel runs out, the star will collapse, and, depending on its initial mass, the outcomes are different. If $M_{initial} \lesssim 8M_{\odot}$ the outcome is a WD, if $M_{initial} \lesssim 25M_{\odot}$ the end result is a NS and if $M_{initial} \gtrsim 25M_{\odot}$ it results in a black hole (ANNALA, 2020).

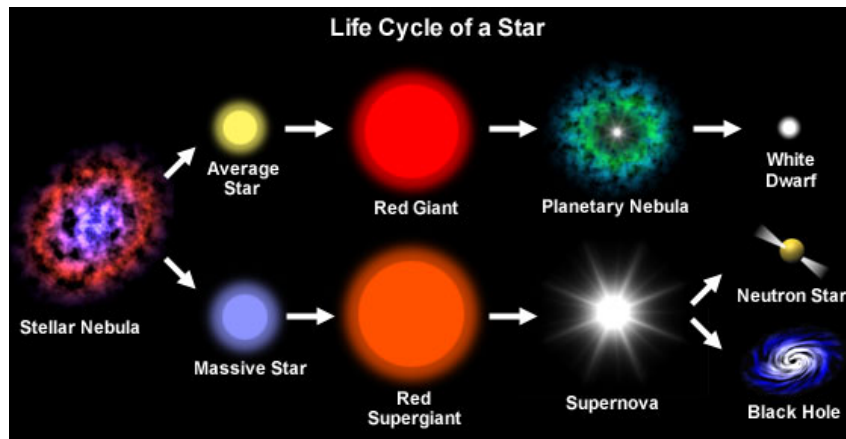


Figure 2 – Life Cycle of a Star. Taken from (LIFE CYCLE OF A STAR. . . , n.d.)

White dwarfs are composed mostly of electron-degenerate matter. Its masses lie between $0.17M_{\odot}$ and $1.33M_{\odot}$, but most of them have between 0.5 and $0.7M_{\odot}$ (KEPLER et al., 2007). The radii are typically 0.8–2% of the radius of the Sun (SHIPMAN, 1979), which is comparable to the Earth's radius (6371 km). Therefore, a WD packs a mass comparable to the Sun's mass into a volume that is around a million times smaller than

the Sun's volume. In this thesis we will only aboard these objects very briefly in chapter 6 and concentrate most of the time on the NSs.

Neutron stars were proposed by Baade and Zwicky in 1933 (BAADE; ZWICKY, 1934), one year after the neutron was discovered by Chadwick (CHADWICK, 1932). They were searching for an explanation for the origin of a supernova, and proposed that in supernova explosions ordinary stars are turned into stars that consist of extremely closely packed neutrons that they called neutron stars. The first NS was not observed until 1967 when the first radio pulsar was discovered by Jocelyn Bell Burnell.

Most of the basic models for NS imply that they are composed almost entirely of neutrons, because the extreme pressure causes the electrons and protons to combine producing neutrons through the Urca process. What prevents those stars to further collapse is the neutron degeneracy pressure, just as WD are supported against collapse by electron degeneracy pressure. But this is not enough to explain NSs with masses beyond $0.7M_{\odot}$, for that repulsive nuclear forces come into play. The most massive NS detected so far, PSR J0952–0607, with a mass of $2.35 \pm 0.17M_{\odot}$ (ROMANI et al., 2022), but most of those stars have $1.4M_{\odot}$ (which is the so called canonical mass) with radius of around 10 km.

At the extraordinarily high densities of NSs, ordinary matter is squeezed to nuclear densities. Specifically in the inner core such high densities may allow the appearance of exotic states of matter, as for example hyperons³ or even deconfined quarks. So NSs may actually be hadron stars, because they include in its composition besides the nucleons also hyperons, hybrid stars, i.e., hadron stars with a core of deconfined QM, or even strange quark stars, formed totally by deconfined QM. We will aboard all of them in this thesis.

The equation of state (EoS) for NSs encapsulates information about their internal structure and so tells us on how matter behaves under extreme densities within these stellar remnants. By placing constraints on this equation, we gain insights into the workings of the strong force as described by the standard model, with far-reaching implications for nuclear and atomic physics. This makes neutron stars natural laboratories for probing fundamental physics.

Different EoS give different values of observables. An EoS is only directly relating the density and pressure, but it also leads to calculating observables like the speed of sound, mass, radius, and tidal deformability. Because the EoS is unknown, there are many proposed ones and it is an active area of research. In this thesis we will propose new ones to describe QM.

The connection between nuclear physics and compact stars is made through the Tolman-Oppenheimer-Volkoff (TOV) equations, i.e., once we have the EoS describing

³ Hyperons are baryons made up of at least one quark *s*, but no quark *c*, *t* or *b*.

the hadron and/or quark matter⁴ we are able to obtain the macroscopic values of the compact star, as its mass-radius relation, by the use of the TOV equations.

1.3.1 Composition of Neutron Stars

There is no consensus on the constitution of NSs. It could be that those objects do not have enough density to deconfine quark matter. In this case these stars would be constituted entirely by hadronic matter and would be called only neutron stars or hadron stars. It could also be that these stars are hybrid, that is, that they have a core of deconfined QM, but that the outer layer is made up of hadrons. We will explore this possibility in chapter 5. And one other possibility is that the compact stars are made up completely of deconfined quarks, being then called quark stars or strange stars. This last possibility we will explore in chapters 2 and 3.

The hybrid and hadron compact stars have a crust which represents only around 10% in terms of the radius and less than 1% of the mass, but that needs to be taken into account when calculating the macroscopic properties of those stars. Although the method to connect the crust and the core can influence the theoretical prediction of the NS radius by 3-5% (FORTIN et al., 2016), here we will simply take the numerical data from the literature and connect it to the core EoS studied here. Strange stars do not have crusts.

Beyond that, as neutron stars present internal densities that can be up to 10 times higher than the nuclear saturation density ($n_0 \approx 0.16 \text{ fm}^{-3}$), the onset of hyperons is expected because their appearance is energetically favorable as compared with the inclusion of more nucleons in the system (MENEZES, Débora Peres, 2021). Taking this into account, in this work we will consider the six lightest hyperons, Λ^0 , Σ^+ , Σ^0 , Σ^- , Ξ^0 e Ξ^- as part of the hadron matter, together with the nucleons, forming, so, baryon octet.

In table 1 the general characteristics of the baryonic octet are shown.

Barion	quarks	m (MeV)	Spin	I_3	q_B
p	uud	938,28	1/2	1/2	+1
n	udd	939,57	1/2	-1/2	0
Λ	uds	1115,6	1/2	0	0
Σ^+	uus	1189,4	1/2	1	+1
Σ^0	uds	1192,5	1/2	0	0
Σ^-	dds	1197,3	1/2	-1	-1
Ξ^0	uss	1314,9	1/2	1/2	0
Ξ^-	dss	1321,3	1/2	-1/2	-1

Table 1 – Baryons of the baryon octet, quark composition, its masses (m), spin, isospin (I_3) and electrical charge (q_B).

⁴ One may also include dark matter, as we will see further on.

1.3.2 TOV equations

The TOV equations are obtained from the Einstein field equations, assuming that the star is spherical, static, and made of an ideal and isotropic fluid. They are the basis for calculating the properties of compact stars, such as the mass-radius relations and their tidal deformability. They are two differential equations that depend upon three functions ($P(r)$, $\varepsilon(r)$, $M(r)$), which means that, to solve this system we need another equation that relates these functions. This relation comes from the microscopic composition of the star, i.e., from the EoS that gives us $\varepsilon = \varepsilon(P)$. With this it is possible to solve the system of equations and obtain the characteristics of a star from a fluid described by the EoS used. The dimensionless TOV equations are as the following:

$$\begin{aligned} \frac{dP'}{dr} &= -(P' + \varepsilon') \frac{M + 4\pi r^3 P'}{r(r - 2M)} \quad \text{and} \\ \frac{dM}{dr} &= 4\pi r^2 \varepsilon', \end{aligned} \quad (2)$$

where P' , ε' , M and r are the dimensionless pressure, energy density, mass and radius, respectively. The mass in the units of solar masses ($M_\odot$) and the radius in km will be given by (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006)

$$M = 1.632 \quad M_\odot \cdot M \left(\frac{1 \text{ GeV}}{m_p} \right)^2 \quad (3)$$

$$R = 2.410 \quad \text{km} \cdot M \left(\frac{1 \text{ GeV}}{m_p} \right)^2, \quad (4)$$

where m_p is the mass of the particle used to turn P and ε dimensionless.

So, with the TOV equations, for each value of pressure and the associated energy density given by the EoS we obtain a value for the radius and the mass of the star which has as central pressure and central energy density equal to the value used as input to the TOV equations⁵.

These are the equations for one-fluid stars which will be used in chapters 2, 3 and 5 of this thesis. In chapter 6 we will introduce the TOV equations for a two-fluid system.

1.3.3 Tidal deformability

When an object is subject to a non homogeneous external gravitational field, i.e., in presence of a non zero gradient of the gravitational field, that object undergoes

⁵ For smaller values of the star radius ($r < R$) the associated pressure and energy density also follows the values given by the EoS

a tidal deformation. And the tidal deformability strongly depends on the structure and the matter of the object.

The tidal deformability of a compact object is a single parameter λ (sometimes called Love parameter) that quantifies how easily the object is deformed when subjected to the external tidal field. A larger tidal deformability indicates that the object is easily deformable. On the opposite side, a compact object with a smaller tidal deformability parameter is smaller, more compact, and it is more difficult to deform it.

The tidal deformability is defined as the ratio between the induced quadrupole moment $Q_{i,j}$ and the perturbing tidal field $\mathcal{E}_{i,j}$ that causes the perturbation:

$$\lambda = -\frac{Q_{i,j}}{\mathcal{E}_{i,j}}. \quad (5)$$

However, in the literature is more commonly found the dimensionless tidal deformability parameter (Λ) defined as:

$$\Lambda \doteq \frac{\lambda}{M^5} \equiv \frac{2k_2}{3C^5}, \quad (6)$$

where M is the compact object's mass and $C = M/R$ is its compactness. The parameter k_2 is called Love number, which provides us with a measure of the difficulty in deforming a the object in the face of an external tidal field, and is given by

$$\begin{aligned} k_2 = \frac{8C^5}{5} (1-2C)^2 [2 + 2C(y_R - 1) - y_R] \times \left\{ 2C[6 - 3y_R + 3C(5y_R - 8)] \right. \\ \left. + 4C^3 [13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R)] \right. \\ \left. + 3(1-2C)^2 [2 - y_R + 2C(y_R - 1)] \ln(1-2C) \right\}^{-1}. \end{aligned} \quad (7)$$

The quantity $y_R = y(R)$ is obtained by solving, together with the TOV equations, the differential equation for y^6 :

$$r \frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2 Q(r) = 0, \quad (8)$$

where

$$F(r) = \frac{r - 4\pi r^3(\varepsilon - P)}{r - 2m} \quad (9)$$

and

$$Q(r) = \frac{4\pi r}{r - 2m} \left[5\varepsilon + 9P + \frac{\varepsilon + P}{v_s^2} - \frac{6}{4\pi r^2} \right] - 4 \left[\frac{m + 4\pi r^3 P}{r^2 \left(1 - \frac{2m}{r} \right)} \right]^2, \quad (10)$$

⁶ Eq. (8) is only valid when the EoS presents no discontinuity. In the case of Strange Stars, when $P \rightarrow 0$ at the surface does not correspond to $\varepsilon \rightarrow 0$, the gap in the energy density must be taken into account. See more details in (LOPES, L. L.; MENEZES, Debora P, 2022).

with $v_s^2 = dp/d\varepsilon$ being the squared speed of sound. The initial conditions for y in the center of the star is $y(r = 0) = 2$.

The Gravitational-Wave Interferometer LIGO/VIRGO observatories and the GW170817 event put some constraints on the tidal deformability. In (ABBOTT, B. P. et al., 2018), the authors constraint the tidal parameter for the canonical mass in the range between $70 < \Lambda_{1.4} < 580$. Although we have to emphasize that the authors still use some nuclear model to fix the parameters and, therefore, the tidal deformability restraint they find is dependent of this model must be faced with care.

1.3.4 Restraints for stars

We know of the existence of two solar masses neutron stars since the observation of PSR J1614-2230 (DEMOREST et al., 2010) with a mass of $1.97 \pm 0.04 M_\odot$ made more than a decade ago. But since then even more massive objects have been observed, as, for example, PSR J0348+0432, with a mass of $2.01 \pm 0.04 M_\odot$ (ANTONIADIS et al., 2013), and PSR J0740+6620 with a mass of $2.14^{+0.10}_{-0.09} M_\odot$ (CROMARTIE et al., 2020). In 2021, though, the measurement of the mass and radius of PSR J0740+6620 were refined by the Neutron Star Interior Composition Explorer (NICER) telescope to the range of $2.072^{+0.067}_{-0.066} M_\odot$ for the mass and between 11.41 km and 13.70 km for the radius (RILEY, T. E. et al., 2021). And even more recently was detected PSR J0952-0607, with a mass of $2.35 \pm 0.17 M_\odot$ (ROMANI et al., 2022), yet to be confirmed. These two measurements will be used as restraints throughout this work.

In chapters 2 and 3 we will also check if we are able to fulfill the restraints imposed by HESS J1731-347, a tiny neutron star with mass of $M=0.77^{+0.20}_{-0.17} M_\odot$ and radius $R=10.4^{+0.20}_{-0.78}$ km (DOROSHENKO et al., 2022), that may be a strange star, although the results are still a matter of debate.

Besides the constraint regarding the mass and the radius of the PSR J0740+6620, NICER telescope has also constrained the radius of the canonical $1.4 M_\odot$ star. For instance, in (MILLER et al., 2021) it is suggested that $11.80 < R_{1.4} < 13.85$ km. We also take into account the restraint of $R_{1.4} < 11.9$ km suggested by (CAPANO et al., 2020).

And finally, we compute the dimensionless tidal deformability and check if we meet the restraint $70 < \Lambda_{1.4} < 580$, as imposed by the LIGO/VIRGO gravitational wave observatories and the GW170817 event (ABBOTT, B. P. et al., 2018).

1.4 DARK MATTER

Astrophysical and cosmological observations indicate that most of the mass of the Universe appears in the form of non-baryonic mass/energy (BERTONE; HOOPER; SILK, 2005; DEL POPOLO, 2014). In (ADE et al., 2016) is said that the Universe is composed of only 4.9% of baryonic matter, 26.4% of invisible form of matter whose

existence is inferred from its gravitational effects, called dark matter (DM). Another component, we call dark energy (DE), that is hypothesized to permeate all space, and whose existence is related to the accelerated expansion of the Universe, is said to form 68.7% of the total mass.

The nature of DM is, however, still puzzling. A lot of candidates have been proposed, with masses ranging from 10^{-33} GeV (Fuzzy DM) to 10^{15} GeV (Wimpzillas). Among those, WIMPs (Weakly interacting massive particles), Axions, and sterile neutrinos, have been favored (DELIYERGIYEV et al., 2019).

Whereas there are direct methods for detecting DM using particle accelerators or analyzing DM scattering off nuclear targets in terrestrial detectors, no evidence of its existence has been produced so far. An alternative for testing the possible effects of DM are the compact objects (COs), such as WDs and neutron stars (NS). Because of its extreme densities the probability of the interaction of DM with ordinary matter and the DM capture is increased (GOLDMAN; NUSSINOV, 1989; KOUVARIS, 2008).

Based on the idea that the current DM abundance has a similar origin of visible matter, an appealing alternative to WIMPs is the asymmetric dark matter (ADM) model. WIMPs are supersymmetric particles, based on the assumption of a symmetry between bosons and fermions. If, at the other hand, nature is parity symmetric, we have a different form of DM, mirror matter (DELIYERGIYEV et al., 2019).

Because ADM is non-annihilating it can accumulate, producing changes in mass and radius of the stars, possibly forming extraordinary compact NSs.

Comparing the mass-radius relation predicted by star models with ordinary matter and with ordinary matter admixed with DM in NSs, it is possible to extract information on DM and the equation of state (EoS) of the NSs (CIARCELLUTI; SANDIN, 2011; DELIYERGIYEV et al., 2019; BARBAT; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2024; TOLOS; SCHAFFNER-BIELICH, Jürgen, 2015; DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2021).

In the chapter 6 of this thesis we will study some aspects of COs admixed with ADM. We will discuss some main effects the DM has on the mass-radius relation of NSs and WDs, but will be concentrating mainly on effects this DM has on hybrid star and on the hadron-quark phase transition.

2 THE MIT BAG MODELS

In this chapter we seek to reconcile the Bodmer-Witten conjecture (BODMER, 1971; WITTEN, 1984), also called the strange matter hypothesis, which says that the matter composed of the quarks u , d and s is actually the true ground state of matter, with the existence of compact stars with more than $2 M_{\odot}$, as imposed by the NICER results for PSR J0740 + 6620, with $M=2.072^{+0.067}_{-0.066} M_{\odot}$ and radius between 11.41 km and 13.70 km (RILEY, T. E. et al., 2021) and the recently detected PSR J0952-0607, with a mass of $2.35 \pm 0.17 M_{\odot}$ (ROMANI et al., 2022), yet to be confirmed. We also confront our results with the measurements of the HESS J1731-347, a tiny neutron star with mass of $M=0.77^{+0.20}_{-0.17} M_{\odot}$ and radius $R=10.4^{+0.20}_{-0.78}$ km (DOROSHENKO et al., 2022), that may be a strange star, although the results are still a matter of debate. Besides that, we also look at the radius of the canonical stars, the ones with $1.4 M_{\odot}$, and check if our results agree with the ones from (MILLER et al., 2021), where it is suggested that $11.80 < R_{1.4} < 13.85$ km. We also take into account the restraint of $R_{1.4} < 11.9$ km suggested by (CAPANO et al., 2020).

And finally, we compute the dimensionless tidal deformability and check if we meet the restraint $70 < \Lambda_{1.4} < 580$, as imposed by the LIGO/VIRGO gravitational wave observatories and the GW170817 event (ABBOTT, B. P. et al., 2018).

To do all of this, we start by revisiting the original MIT Bag Model in the first section. In search of a better model, in the next section we explore and propose an improvement on the MIT Vector Bag Model. And finally, inspired by the Boguta-Bodmer nonlinear terms in quantum hadrodynamics (BOGUTA; BODMER, 1977; BODMER, 1991), in the subsequent section we study the introduction of a self-interacting vector field to the MIT Vector Bag Model.

In all sections we also compute the strangeness fraction in order to better analyse each model and the modifications thereof proposed.

The discussions presented in this chapter are strongly based on the article entitled "Modified MIT Bag models - part I: Thermodynamic consistency, stability windows and symmetry group", published in *Physica Scripta* (LOPES, L. L.; BIESDORF, Carline; MENEZES, Débora P, 2021).

2.1 THE ORIGINAL MODEL

The MIT bag model is one of the simplest models used to describe quark matter. But even in its simplicity, it's able to incorporate two of the main characteristics of the QCD: asymptotic freedom and confinement. It was originally developed at the Massachusetts Institute of Technology in the 1970s (CHODOS et al., 1974) as an alternative to explaining the mass of hadrons in terms of quarks. It has since been used extensively to describe quark matter. In this model, the hadrons are considered as bags

inside which the quarks are confined. On the bag exterior, there is only the vacuum of QCD which exerts pressure on the bag. This pressure is also called Bag pressure value or vacuum pressure (and denoted by B , or, more commonly, by $B^{1/4}$ due to its unit in the energy density and pressure equations, as is shown in the sequel). The quarks are represented by a free Fermi gas and so, through its kinetic movement, generate a pressure that counterbalances the vacuum pressure.

The Lagrangian density of the original MIT Bag model is given by:

$$\mathcal{L} = \sum_q \left\{ \bar{\psi}_q [i\gamma^\mu \partial_\mu - m_q] \psi_q - B \right\} \Theta(\bar{\psi}_q \psi_q) - \frac{1}{2} \bar{\psi}_q \psi_q \delta_S(\bar{\psi}_q \psi_q), \quad (11)$$

where m_q is the mass of the quark q of flavour u , d or s , ψ_q is the Dirac field of the quark, B is the Bag constant (Bag pressure value), $\Theta(\bar{\psi}_q \psi_q)$ is the Heaviside step function that guarantees that the quarks only exist confined inside the bag, and δ_S is the Dirac delta function that guarantees the continuity of the quark fields on the surface of the bag.

Applying the Euler-Lagrange equations for quarks, we get:

$$[i\gamma^\mu \partial_\mu - m_q] \psi_q = 0, \quad (12)$$

which gives us the energy eigenvalue for the quark q :

$$E_q = \sqrt{m_q^2 + k_q^2}, \quad (13)$$

where k_q is the momentum.

As the quarks are fermions with spin 1/2, the energy density and the number density of quarks can be obtained via Fermi-Dirac statistics (GREINER; NEISE; STÖCKER, 1995). At zero temperature there are no antiparticles and the Fermi-Dirac distribution function of the particles becomes a Heaviside step function:

$$\Theta(\mu_q - E_q) = \begin{cases} 0, & \mu_q - E_q < 0, \\ 1, & \mu_q - E_q > 0. \end{cases} \quad (14)$$

And so, at the upper limit, we can identify the Fermi energy as $E_F = \mu = \sqrt{m_q^2 + k_F^2}$, where k_F is the Fermi momentum. This makes the calculations of the EoSs easier because then the integrals can be calculated between the limits 0 and k_F . In this case, the total number density of quarks is given by:

$$n_Q = \sum n_q = 2 \times 3 \sum \int_0^{k_{Fq}} \frac{d^3 k_q}{(2\pi)^3} = 2 \times 3 \sum \int_0^{k_{Fq}} \frac{4\pi k_q^2 dk_q}{(2\pi)^3} = \sum \frac{k_{Fq}^3}{\pi^2} \quad (15)$$

remembering that the baryon density is $n_B = n_Q/3$, and the total energy density:

$$\varepsilon = \sum \varepsilon_q = \frac{3}{\pi^2} \sum \int_0^{k_{Fq}} E_q k_q^2 dk_q, \quad (16)$$

where the 2×3 corresponds to the spin degeneracy and the number of colors of the quarks, respectively, and the sum is over all the quarks considered.

However we did not yet take into account the Bag contribution. This is easily done by the use of the relation $\mathcal{H} = -\langle \mathcal{L} \rangle$, where $\langle \cdot \rangle$ stands for expectation value. Hence, the total energy density is the sum over all the quarks considered in the model plus the Bag constant contribution:

$$\begin{aligned}\varepsilon &= \sum_q \varepsilon_q + B \\ &= \sum_q \left(\frac{3}{\pi^2} \int_0^{k_{Fq}} dk_q \cdot k_q^2 \sqrt{m_q^2 + k_q^2} \right) + B.\end{aligned}\quad (17)$$

The pressure of a Fermi gas of quarks is given by:

$$P = \sum_q n_q \mu_q - \varepsilon = -\Omega_q, \quad (18)$$

where n_q is the number density, μ_q is the chemical potential and Ω is the thermodynamic potential. We can also write the pressure (as well the thermodynamic potential) as the derivative of the energy with respect to the volume:

$$P_q = -\Omega_q = -\left(\frac{\partial E_q}{\partial V} \right)_T = \frac{1}{\pi^2} \int_0^{k_{Fq}} \frac{k_q^4 dk_q}{E_q}. \quad (19)$$

The total pressure in the MIT Bag model is, therefore, the sum over every flavour of quarks plus the Bag constant ($p = \langle \mathcal{L} \rangle$):

$$\begin{aligned}P &= \sum_q P_q - B \\ &= \sum_q \left(\frac{1}{\pi^2} \int_0^{k_{Fq}} dk_q \frac{k_q^4}{\sqrt{m_q^2 + k_q^2}} \right) - B.\end{aligned}\quad (20)$$

So, to sum it up, for the original MIT Bag model the expressions for the number density, energy density and pressure for $T = 0$ are given by Eqs. (15), (17) and (20), respectively.

2.1.1 Stability Window

The Bag constant value cannot assume any value. Its value needs to be so that a two-flavoured quark matter is unstable and that it is not the ground state of the hadrons, i.e., its energy per baryon must be higher than 930 MeV at zero pressure, otherwise the protons and neutrons would decay into u and d quarks. On the other hand, if the Bodmer-Witten conjecture (BODMER, 1971; WITTEN, 1984), that states that the true

ground state of the hadronic matter does not consist of baryons, but strange matter with $\mu_U = \mu_D = \mu_S$, is true, the three-flavored quark matter needs to be stable (energy per baryon lower than 930 MeV), while the two-flavoured quark matter is unstable.

Therefore, to respect the Bodmer-Witten conjecture, the Bag can only assume a range of values, which we call the stability window. These values also depend on the mass of the quarks, as can be seen in (TORRES; MENEZES, 2013). In the present work, we only use the values $m_U = m_D = 4$ MeV and $m_S = 95$ MeV, following the results of (TANABASHI et al., 2018). In the table 2 the values forming the stability window of this model are presented.

MIT	$B_{min}^{1/4}$	$B_{max}^{1/4}$
-	148 MeV	159 MeV

Table 2 – Stability Window for the MIT Bag model considering the quark masses $m_U = m_D = 4$ MeV and $m_S = 95$ MeV.

2.2 VECTOR MIT BAG MODEL

The previous section was devoted to an idealized case of noninteracting quarks. In a more realistic consideration, the interparticle interactions must be included, too. In the following, we address the impact of interactions on the stability window, strange fraction and mass-radius relations. In this section we first introduce a vector field that produces repulsion between the quarks.

The inclusion of vector channels in the MIT Bag model, however, is not new, as can be seen in (GOMES, R. et al., 2019; GOMES, R.; CHAR; SCHRAMM, 2019; FRANZON; GOMES, R. d. O.; SCHRAMM, Stefan, 2016; KLÄHN; FISCHER, 2015; CIERNIAK et al., 2019; WEI et al., 2019). Unfortunately in previous articles the mass term of the vector channel was missing. As we show below, in the mean-field approximation, the vector channel becomes zero if the mass is zero. Because of this, here we correct the model by introducing the mass term of the field V_μ (ω_μ). The Lagrangian density of this new model is given by

$$\mathcal{L} = \sum_q \left\{ \bar{\psi}_q \left[\gamma^\mu (i\partial_\mu - g_{qqV} V_\mu) - m_q \right] \psi_q + \frac{1}{2} m_V^2 V_\mu V^\mu - B \right\} \Theta(\bar{\psi}_q \psi_q) - \frac{1}{2} \bar{\psi}_q \psi_q \delta_S(\bar{\psi}_q \psi_q), \quad (21)$$

in which the interaction between the quarks is mediated by the vector channel V_μ analogous to the meson ω in QHD (SEROT, Brian D, 1992; SEROT, B.; WALECKA, 1986). In fact, in this work, we consider that the vector field is the ω meson itself.

Applying the mean field approximation (MFA)¹ ($V^\mu \rightarrow \langle V \rangle \rightarrow \delta_{0,\mu} V^0$) we get the eigenvalues for the energy of the quarks and the equation of motion for the V field, respectively:

$$\begin{aligned} E_q = \mu_q &= \sqrt{m_q^2 + k_q^2} + g_{qq} V^0, \\ m_V^2 V_0 &= \sum_{u,d,s} g_{qq} V n_q. \end{aligned} \quad (22)$$

From the above expression, it is clear that if the mass of the vector field is zero, the vector field itself must be zero. The energy density of the quarks is given by

$$\varepsilon_q = \frac{3}{\pi^2} \int_0^{k_{Fq}} E_q k_q^2 dk_q, \quad (23)$$

which is equal to Eq. (16), but remembering that E_q here is different. In the mean field approximation, we have $\varepsilon = \langle \mathcal{L} \rangle$. Therefore, the total energy density is given by:

$$\begin{aligned} \varepsilon &= \sum \varepsilon_q - \frac{1}{2} m_V^2 V_0^2 + B \\ &= \sum \left(\frac{3}{\pi^2} \int_0^{k_{Fq}} dk_q \cdot k_q^2 \sqrt{m_q^2 + k_q^2} + g_{qq} V_0 \right) - \frac{1}{2} m_V^2 V_0^2 + B \\ &= \sum \left(\frac{3}{\pi^2} \int_0^{k_{Fq}} dk_q \cdot k_q^2 \sqrt{m_q^2 + k_q^2} \right) + \frac{1}{2} m_V^2 V_0^2 + B, \end{aligned} \quad (24)$$

where we used the equation of motion for V to rewrite the expression on the last line. The penultimate term of Eq. (24) does not appear in the works of (GOMES, R. et al., 2019; GOMES, R.; CHAR; SCHRAMM, 2019; FRANZON; GOMES, R. d. O.; SCHRAMM, Stefan, 2016). The pressure can be obtained by the thermodynamic relationship $p = n\mu - \varepsilon$, to ensure thermodynamic consistency:

$$P = \sum \left(\frac{1}{\pi^2} \int_0^{k_{Fq}} dk_q \frac{k_q^4}{\sqrt{m_q^2 + k_q^2}} \right) + \frac{1}{2} m_V^2 V_0^2 - B. \quad (25)$$

So, to sum it up, for the vector MIT Bag model the expressions for energy density and pressure, at $T = 0$ are given by Eqs. (24) and (25), respectively, and the number density continues to be given by Eq. (15), as in the original MIT Bag model, but here we have to take into account also Eqs. (22).

¹ In the relativistic mean-field approximation, also called the relativistic Hartree approximation, it is considered that at sufficiently large values of baryonic density, there will be so many quanta (mediator mesons) present in a given volume that it becomes reasonable to approximate the expected values of the quantum fields as classic fields. In this situation, nuclear matter is considered to be infinite.

2.2.1 Stability Window

To construct the new stability window we need to establish values for the coupling constant g_{qqV} and for the mass of the vector field m_V . As for the absolute value of g_{qqV} , there are still few studies trying to limit its value and its uncertainty is still very high (RESTREPO et al., 2015; KITAZAWA et al., 2002). Most models consider it a free parameter (MENEZES, Debora P et al., 2014; HANAUSKE et al., 2001; SHAO et al., 2012). As for the relative value of g_{qqV} for the different flavours of quarks, in the literature, a universal value is always assumed, that is, the same for all the quark flavours (GOMES, R. et al., 2019; GOMES, R.; CHAR; SCHRAMM, 2019; FRANZON; GOMES, R. d. O.; SCHRAMM, Stefan, 2016; RESTREPO et al., 2015; KITAZAWA et al., 2002; MENEZES, Debora P et al., 2014; HANAUSKE et al., 2001; SHAO et al., 2012; SUGANO et al., 2014; LOPES, L. L.; MENEZES, Debora P, 2020; FLORES et al., 2020). Here we innovate again and use symmetry group to establish this relative value. And we get the following relation:

$$g_{ssV} = \frac{2}{5}g_{uuV} = \frac{2}{5}g_{ddV}. \quad (26)$$

All calculations are detailed in the appendix of (LOPES, L. L.; BIESDORF, Carline; MENEZES, Débora P, 2021).

Although the use of symmetry groups to establish coupling constants is quite common for baryons (LOPES, L. L.; MENEZES, Debora P, 2020; FLORES et al., 2020; LOPES, L. L.; MENEZES, Debora P, 2014; DOVER; GAL, 1984; DE SWART, 1963; PAIS, 1966; WEISSENBERG; CHATTERJEE; SCHAFFNER-BIELICH, Juergen, 2012, 2012; LOPES, Luiz; MENEZES, Debora, 2015), as far as we know, it is not for quarks.

Now we redefine the coupling constant as:

$$G_V \equiv \left(\frac{g_{uuV}}{m_V} \right)^2, \quad (27)$$

we define X_V as the ratio between the meson-quark coupling constant g_{ssV} and the meson-quark coupling g_{uuV} :

$$X_V = \frac{g_{ssV}}{g_{uuV}} \quad (28)$$

and we build the stability window varying G_V from 0 to 0.3 fm^2 for $X_V = 1.0$, corresponding to what is generally found in the literature, and $X_V = 0.4$, which is the value predicted by the symmetry group. The results are shown in Fig. 3 and also in the table 3.

As G_V increases, the stability window is shifted to smaller values. This happens because the increase in G_V increases the repulsion between quarks, which, in turn, makes the energy per baryon at zero pressure increase as well and so smaller Bag

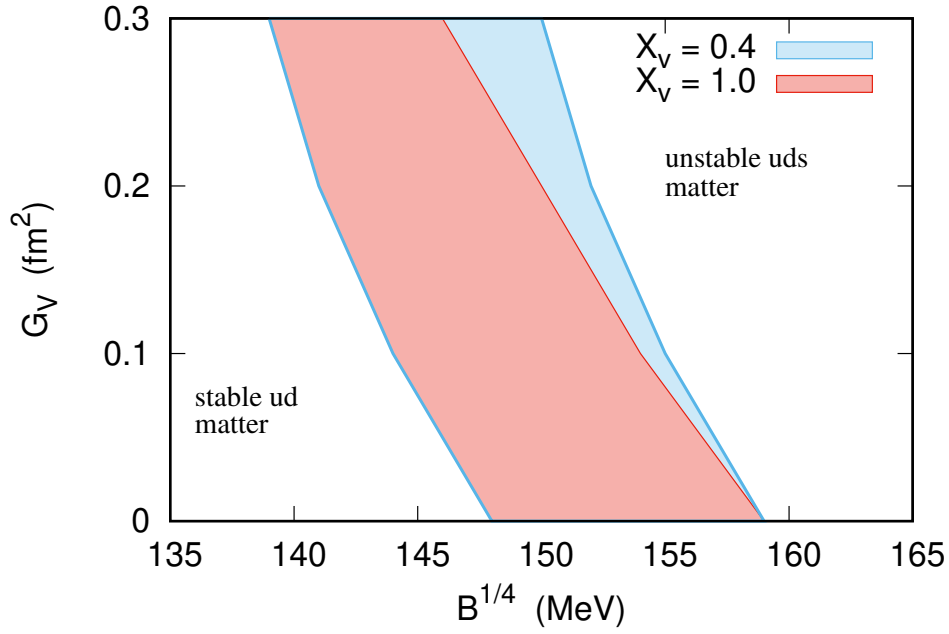


Figure 3 – Stability window for the MIT Bag model with vector interaction and different values of X_V .

$G_V \text{ (fm}^2\text{)}$	X_V	$B_{min}^{1/4}$	$B_{max}^{1/4}$
0.0	-	148 MeV	159 MeV
0.1	1.0	144 MeV	154 MeV
0.1	0.4	144 MeV	155 MeV
0.2	1.0	141 MeV	150 MeV
0.2	0.4	141 MeV	152 MeV
0.3	1.0	139 MeV	146 MeV
0.3	0.4	139 MeV	150 MeV

Table 3 – Stability window for vector MIT Bag model.

pressure values are allowed. For $X_V = 1.0$ the window becomes also narrower with the increase of G_V , but for $X_V = 0.4$ the same does not happen and the difference between the minimum and maximum values of the Bag remains 11 MeV for all analyzed values of G_V .

The narrowing of the window occurs for $X_V = 1.0$ because for this value there is a greater repulsion between the quarks. The lower limit of the stability window is the same for $X_V = 1.0$ and $X_V = 0.4$ because this limit represents where the energy per baryon at zero pressure gets higher than 930 MeV for two-flavoured matter, i.e., the strange quark and, therefore, X_V plays no role in its determination.

It is worth mentioning that the regions outside the stability window on the left (stable $u-d$ matter) and on the right (unstable $u-d-s$ matter) of Fig. 3 have very different meanings and consequences. If we use values of the constants describing stable $u-d$ matter, we are describing an universe that is not composed of protons and

neutrons, as is ours, as these nucleons would decay into the $u-d$ matter. But if we use values that describe unstable $u-d-s$ matter, this would only imply that strange matter is not the ground state of matter, which may or may not be true, as the Bodmer-Witten conjecture is still only a hypothesis. In short, the use of values that describe stable $u-d$ matter is never allowed, but the of unstable $u-d-s$ matter may be allowed, in fact this last condition enables the existence of deconfined quark matter in the cores of massive hybrid stars, which we will discuss in Chapter 5.

2.2.2 Strangeness Fraction

Here we analyze how the strangeness fraction varies when we vary the parameter G_V . The strangeness fraction is defined as:

$$f_s = \frac{n_s}{\sum n_q} = \frac{1}{3} \frac{n_s}{n}, \quad (29)$$

where n is the total baronic density, n_s is the density of quarks s and the sum is over the three flavours of quarks.

As can be seen from Eq. (15), the density of particles is not affected by the Bag pressure value, and so the strangeness fraction also remains unaffected when we change the value of B .

In Fig. 4 we plot the strangeness fraction f_s for different values of G_V , for $X_V = 0.4$ and $X_V = 1.0$ at the left and right, respectively.

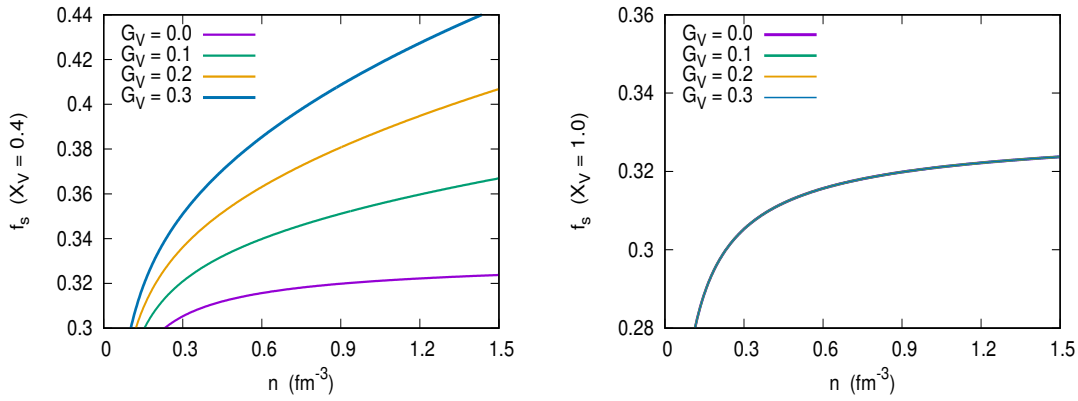


Figure 4 – Strangeness fraction (f_s) as a function of G_V for $X_V = 0.4$ (left) and $X_V = 1.0$ (right).

It can be noticed that f_s has no dependence on G_V when we use the universal coupling constant, i.e., $X_V = 1.0$. This fact is easily explained by observing Eq. (22): as all g_{qqV} have the same value, the chemical potential of all quarks is changed by the same amount.

On the other hand, if the coupling constants are not all the same, i.e., $X_V = 0.4$, the chemical potential of quark s undergoes a smaller variation because its coupling

constant is smaller. And, in this case, as we increase the value of G_V , the difference between the chemical potentials of quark s and the others increases and, consequently, the strangeness fraction increases as well.

2.2.3 Mass and Radius of Strange Stars

We now want to study the mass and the radius relation of strange stable stars. To do so we first need electrically neutral and β -stable quark matter and for that we add leptons as a free Fermi gas to the quark matter:

$$\mathcal{L}_{lep} = \sum_l \bar{\psi}_l [i\gamma^\mu \partial_\mu - m_l] \psi_l, \quad (30)$$

where the sum is over the two lightest leptons (e and μ), and impose chemical equilibrium and charge neutrality, respectively:

$$\begin{aligned} \mu_s &= \mu_d = \mu_u + \mu_e \quad \text{and} \quad \mu_e = \mu_\mu, \\ n_s + n_\mu &= \frac{1}{3}(2n_u - n_d - n_s). \end{aligned} \quad (31)$$

We take the lepton masses as being $m_e = 0.511$ MeV and $m_\mu = 105.66$ MeV. As we want to obtain stars as massive as possible², we always use the minimum value of the Bag constant allowed. This because, as the Bag constant subtracts from the pressure exerted by the quarks (and meson), the lower the Bag value, the stiffer is the EoS and stiffer EoSs result in more massive stars.

Once we have the EoSs of electrically neutral and β -stable quark matter we use them as input for the Tolman-Oppenheimer-Volkoff (TOV) equations given by Eq. (66).

In Fig. 5 we present the EoSs and the corresponding mass-radius relation obtained with vector MIT Bag model³. Notice that the restraints shown in the mass-radius diagrams in this chapter have been updated from the paper this chapter is based on.

As we can see, the increase of G_V stiffens the EoS. We must also remember that a higher G_V value means a smaller Bag pressure value and that a smaller Bag pressure value also makes EoS stiffer. As a result of these combined effects, we are able to produce very massive stable quark stars, fully compatible with the PSR J0740+6620 and PSR J0952–0607. At the same time, we are also able to produce stars that agree with measurements of HESS J1731-347.

² In reality we have to respect Ruffini's limit, which, based on Einstein's theory of relativity, the principle of causality, and Le Chatelier's principle, establishes that the maximum mass of the equilibrium configuration of a neutron star cannot be larger than $3.2M_\odot$ (RHOADES JR; RUFFINI, 1974). But, as we will see in the following, we never surpass this limit.

³ Note that we also display unstable results in the mass-radius diagram, making no differentiation between stable and unstable ones. Throughout this whole chapter, all results after the maximum mass are dynamically unstable. See details about stability in the appendix B.

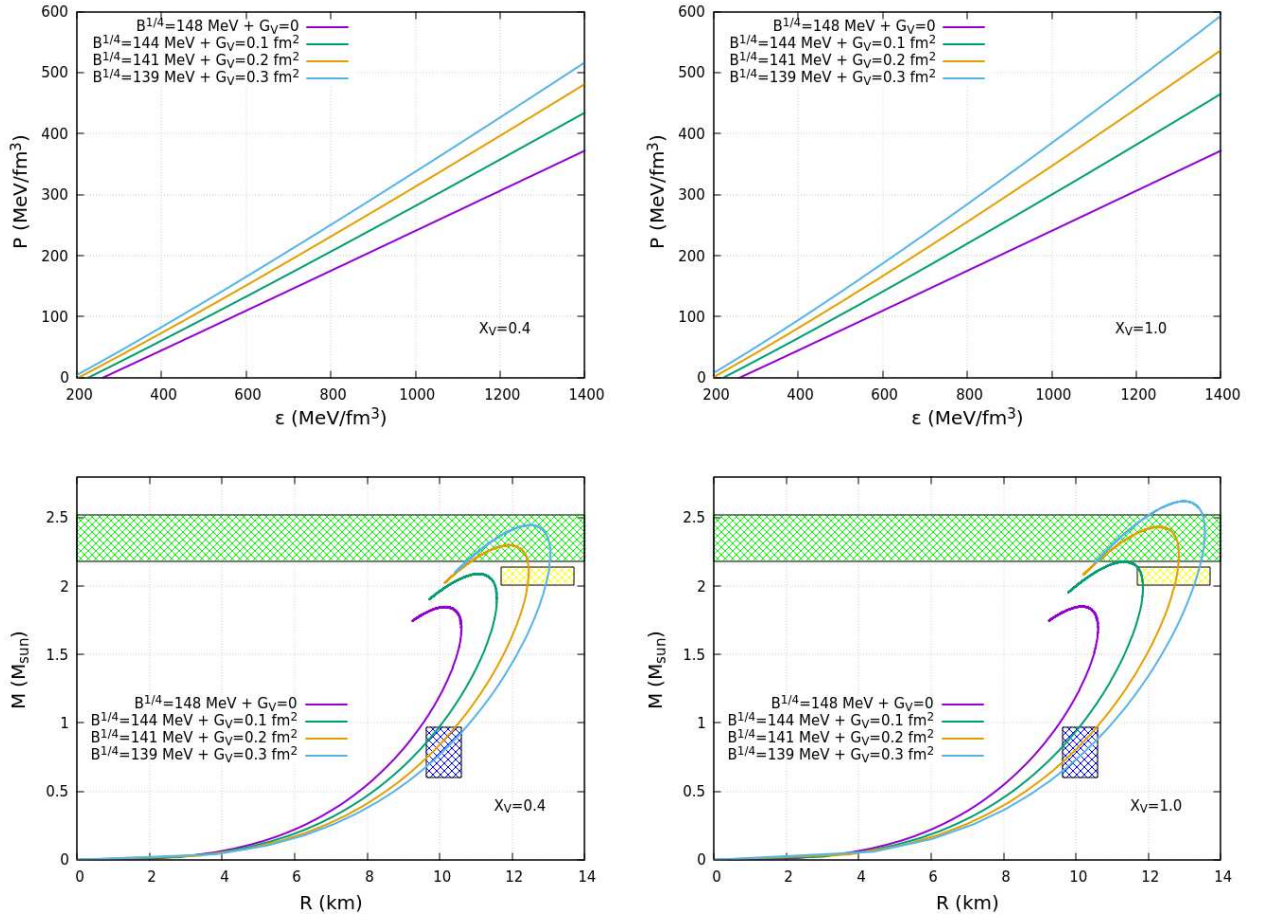


Figure 5 – EoS (top) and mass-radius relation (bottom) for $X_V = 0.4$ (left) and $X_V = 1.0$ (right) with the minimum Bag constant value that produces stable strange stars for different values of G_V . The yellow area represents the uncertainties on both, the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021), the green area refers to the mass of the PSR J0952–0607 (ROMANI et al., 2022) with its error bars and the blue area the uncertainties on both, the radius and mass of the HESS J1731-347 (DOROSHENKO et al., 2022).

For $X_V = 1.0$ and $G_V = 0.3$, which corresponds to a Bag pressure value of $B^{1/4} = 139$ MeV, we obtain a maximum mass of $2.61 M_{\odot}$, which is the largest mass we obtain in this scenario. For the same values of $B^{1/4}$ and G_V , but for $X_V = 0.4$, the maximum mass is smaller, $2.41 M_{\odot}$, something expected because $X_V = 0.4$ produces a smaller repulsion between the quarks, which results in a less stiff EoS.

In table 4 we add more details of the macroscopic properties of the quarks stars. There, in addition to the results for the minimum Bag pressure value, we also present the results for the maximum Bag pressure value allowed by the stability window. As we can see, the maximum values of the masses of the stars are lower when using the maximum Bag pressure value, as expected. It is also interesting to note that the difference between the maximum masses for $X_V = 1.0$ and $X_V = 0.4$, as G_V increases,

$G_V (fm^2)$	X_V	$M/M_\odot - B_{min}^{1/4}$	R (km)	$\varepsilon_c (MeV/fm^3)$	f_s	$R_{1.4}$
0.0	-	1.85	10.17	1286	0.322	10.38
0.1	0.4	2.09	11.11	1071	0.354	11.02
0.2	0.4	2.28	11.83	962	0.379	11.50
0.3	0.4	2.41	12.33	893	0.402	11.81
0.1	1.0	2.16	11.29	1051	0.320	11.11
0.2	1.0	2.43	12.24	921	0.318	11.66
0.3	1.0	2.61	12.97	795	0.317	12.08
$G_V (fm^2)$	X_V	$M/M_\odot - B_{max}^{1/4}$	R (km)	$\varepsilon_c (MeV/fm^3)$	f_s	$R_{1.4}$
0.0	-	1.61	8.93	1726	0.323	9.20
0.1	0.4	1.82	9.63	1450	0.379	9.85
0.2	0.4	2.01	10.36	1248	0.388	10.38
0.3	0.4	2.14	10.86	1154	0.413	10.72
0.1	1.0	1.92	9.98	1331	0.321	10.08
0.2	1.0	2.19	11.00	1112	0.320	10.79
0.3	1.0	2.40	11.85	978	0.319	11.34

Table 4 – Quark star main properties for different values of G_V and X_V .

is even bigger when we use the maximum Bag pressure value than when we use the minimum.

As for the radius of the stars, all our results, except when we use $B^{1/4} = 139$ MeV, $X_V = 1.0$, and $G_V = 0.3 \text{ fm}^2$, are in agreement with (CAPANO et al., 2020), which, as we mentioned at the beginning of this chapter, states that the radius of a canonical star cannot exceed 11.9 km, a value that should be looked at with a grain of salt since the NICER result for the canonical star points to a larger radius. However, if this value is taken as correct, it indicates that the Bag constant $B^{1/4} = 139$ MeV is too small and/or the use of $X_V = 0.4$ is best suited to describe stellar quark matter. On the other side, if we consider the NICER results, i.e., $11.80 < R_{1.4} < 13.85$ km (MILLER et al., 2021), only the results when we use $G_V = 0.3 \text{ fm}^2$, for both $X_V = 1.0$ and $X_V = 0.4$, produce a radius big enough, which indicates that more interaction between the quarks is preferable. By all means, it is clear from table 4, that, with this model, we are able to produce stable strange stars with masses above $2.4 M_\odot$ that satisfy all astrophysical restrictions mentioned in this chapter.

2.3 SELF-INTERACTING VECTOR FIELD

The vector channel in the mean field approximation takes into account only the valence quarks. This scenario is called ‘no sea approximation’, once the Dirac sea of quarks is completely ignored (FURNSTAHL; SEROT, Brian D; TANG, 1997). As the vector field is borrowed directly from quantum hadrodynamics (QHD), the vector MIT Bag model also becomes renormalizable (SEROT, Brian D, 1992). However, instead of transforming the mean field approximation (MFA) into a more complex relativistic Hartree

or Hartree–Fock approximation, we can take the Dirac sea into account throughout modifications on the effective Lagrangian density as done in (FURNSTAHL; SEROT, Brian D; TANG, 1997).

In this section we want to introduce a quartic contribution of the vector field, $(V_\mu V^\mu)^2$, as a correction to the EoS at high densities that mimics the contribution from the Dirac Sea, which is absent in the mean field approximation.

Adding the vector channel to the Lagrangian of the MIT Bag model creates a repulsive term in the quark-quark interaction, which increases the pressure (as well as the chemical potential and energy density). This EoS hardening grows linearly with density. The quartic vector field makes the EoS more malleable.

The introduction of self-interacting fields is not something new in relativistic models. Boguta and Bodmer (BOGUTA; BODMER, 1977) also introduced a self-interacting term in the scalar sector to correct the compressibility of symmetric nuclear matter. Bodmer also introduced a quartic interaction in the vector sector (BODMER, 1991), and other authors (SUGAHARA; TOKI, 1994; TOKI et al., 1995) used quartic terms to correct the behavior of nuclear matter at densities above $2n_0$, where n_0 is the saturation density of nuclear matter.

Here we introduce a self-interacting vector field into the vector MIT Bag model:

$$U(V^\mu) = b_4 \frac{(g_{uuV}^2 V_\mu V^\mu)^2}{4}, \quad (32)$$

where b_4 is a dimensionless parameter (FURNSTAHL; SEROT, Brian D; TANG, 1997). Depending on the value of this parameter, we can make the EoS stiffer or softer, but, as in the hadronic case (SUGAHARA; TOKI, 1994), no significant change in the EoS is expected for densities below $2n_0$.

The complete Lagrangian density so becomes:

$$\begin{aligned} \mathcal{L} = \sum_q \left\{ \bar{\psi}_q \left[\gamma^\mu (i\partial_\mu - g_{qqV} V_\mu) - m_q \right] \psi_q + \frac{1}{2} m_V^2 V_\mu V^\mu + \right. \\ \left. + b_4 \frac{(g_{uuV}^2 V_\mu V^\mu)^2}{4} - B \right\} \Theta(\bar{\psi}_q \psi_q) - \frac{1}{2} \bar{\psi}_q \psi_q \delta S(\bar{\psi}_q \psi_q). \end{aligned} \quad (33)$$

Using the mean-field approximation and solving the Euler-Lagrange equations of motion, we obtain the energy eigenvalues for the quarks and for the V field as follows:

$$\begin{aligned} E_q = \mu = \sqrt{m_q^2 + k^2} + g_{qqV} V_0, \\ g_{uuV} V_0 + \left(\frac{g_{uuV}}{m_V} \right)^2 \left(b_4 (g_{uuV} V_0)^3 \right) = \left(\frac{g_{uuV}}{m_V} \right) \sum_{u,d,s} \left(\frac{g_{qqV}}{m_V} \right) n_q. \end{aligned} \quad (34)$$

Due to the quartic nature of this non-linear term, the higher the density, the greater the deviation from the linear case. Furthermore, as the nonlinear term given by Eq. (32) also has a quartic dependence on the coupling constant g_{uuV} , the greater G_V , the greater the influence of the quartic term. The advantage of using this non-linear term is that we can modify the EoS at high densities, but keep the stability window unchanged.

The energy density is now given by:

$$\begin{aligned}\varepsilon_q &= \frac{3}{\pi^2} \int_0^{k_f} E_q k^2 dk, \\ \varepsilon &= \sum \varepsilon_q + B - \frac{1}{2} m_V^2 V_0^2 - U(V_0) \\ &= \sum \left(\frac{3}{\pi^2} \int_0^{k_f} dk \cdot k^2 \sqrt{m_q^2 + k^2} \right) + \frac{1}{2} m_V^2 V_0^2 + \frac{3}{4} b_4 (g_{uuV} V_0)^4 + B. \quad (35)\end{aligned}$$

And the pressure is obtained through the thermodynamic relationship, $p = n\mu - \varepsilon$, to ensure thermodynamic consistency:

$$P = \sum \left(\frac{1}{\pi^2} \int_0^{k_f} dk \frac{k^4}{\sqrt{m_q^2 + k^2}} \right) + \frac{1}{2} m_V^2 V_0^2 + \frac{1}{4} b_4 (g_{uuV} V_0)^4 - B. \quad (36)$$

So, to sum it up again, for the vector MIT Bag model with the self-interacting vector field the expressions for energy density and pressure, at $T = 0$ are given by Eqs. (35) and (36), respectively, and the number density continues to be given by Eq. (15), as in the original MIT Bag model and the vector MIT Bag model without the self-interacting term, but here we have to take into account also Eqs. (34).

2.3.1 Stability Window

In Fig. 6 we present the stability windows for different values of the Bag constant, of G_V and b_4 . For each value of G_V we vary b_4 from -0.4 to 1.0 . Each colored wall is a stability window for a fixed G_V with the vertical axes corresponding the values of $B^{1/4}$. As we can see, the value of b_4 does not affect the stability window; the results are the same as the ones presented on the previous section in the table 3.

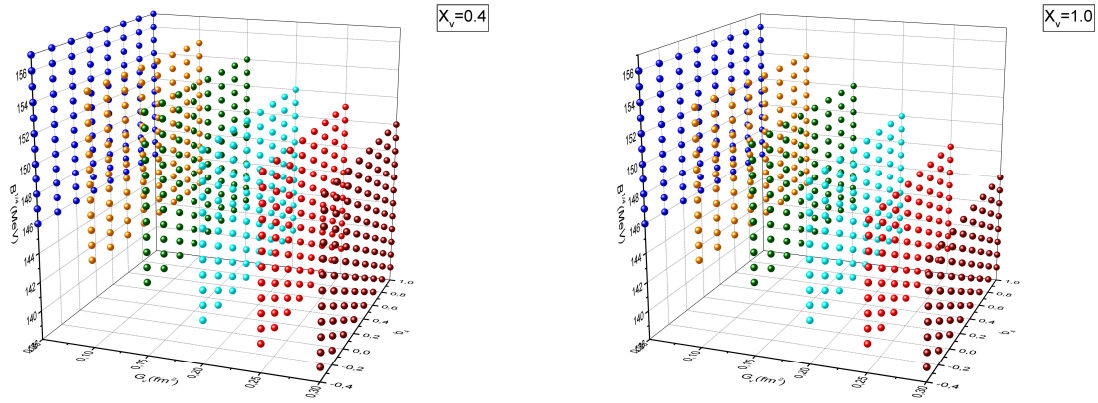


Figure 6 – Stability windows in 3D for $X_V = 0.4$ (left) and $X_V = 1.0$ (right). Each wall of different color represents a value of G_V and the vertical axes correspond to the Bag pressure value $B^{1/4}$. As expected, the results are independent of b_4 .

2.3.2 Strangeness Fraction

As previously discussed, the bigger the value of G_V , the greater is the influence of the self-interaction term. Therefore, here we present only the results for $G_V = 0.3 \text{ fm}^2$, being that for smaller values the influence is significantly smaller. We could also use larger values, but the results would be qualitatively the same.

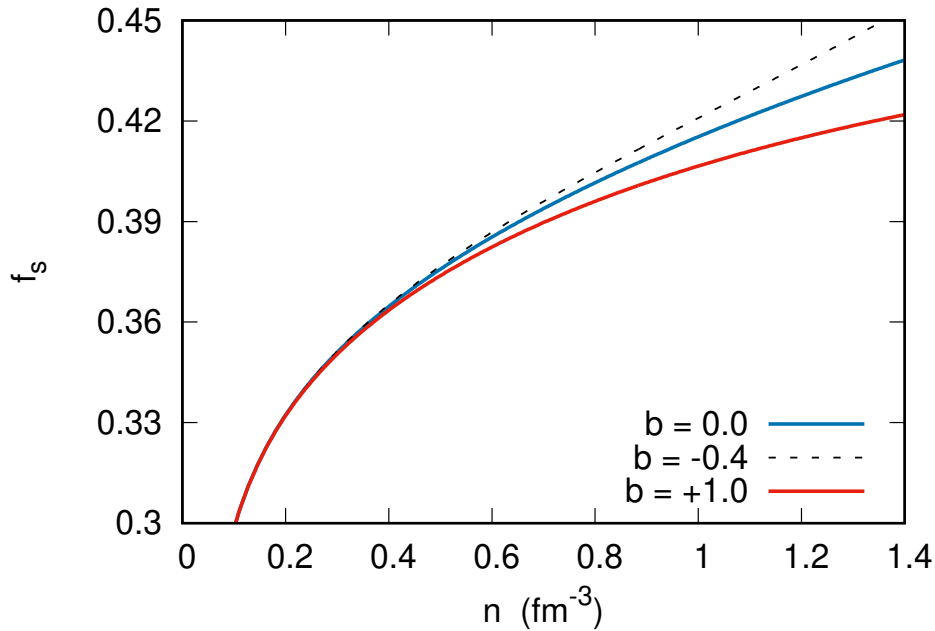


Figure 7 – Strangeness fraction for $G_V = 0.3 \text{ fm}^2$ and $X_V = 0.4$ for different values of b_4 .

When b_4 is negative the vector field V_0 increases with density compared to the linear case ($b_4 = 0$) and, when b_4 is positive, the vector field V_0 decreases with density.

This also affects the strangeness fraction, as can be seen in Fig. 7 where we present the strangeness fraction for $G_V = 0.3 \text{ fm}^2$, $X_V = 0.4$ and different values of b_4 . For $X_V = 1.0$, as discussed earlier, the strangeness fraction is not affected by the vector field.

We can notice that the strangeness fraction is the same in all cases up to densities around 0.4 fm^{-3} , where the results start to diverge from the linear case. As for $b_4 < 0$ there is an increase in the vector field, which in turn makes the differences in the coupling constants more evident, in which case there is an increase in the fraction of s quarks. On the other hand, as $b_4 > 0$ decreases the value of the vector field, in this case a decrease in the fraction of s quarks is observed.

2.3.3 Mass and Radius of Strange Stars

Once again we add leptons (Eq. (30)) to the quark matter and impose chemical equilibrium and charge neutrality (Eq. (31)) to study the mass-radius relations. In Fig. 8 we present the EoSs and TOV equations results for $G_V = 0.3 \text{ fm}^2$, $B^{1/4} = 139 \text{ MeV}$, which is the corresponding minimum Bag pressure value inside stability window, and three values for b_4 .

The EoS for the different values of b_4 are practically equal until the energy density reaches a value around $400 - 500 \text{ MeV/fm}^3$, where, as in the case of the strangeness fraction, they start to deviate from the linear case ($b_4 = 0$). Furthermore, the influence of the quartic term is greater for $X_V = 1.0$, since in this case, V_0 is greater.

The results for the EoS are reflected in the mass-radius relation. As a stiffer EoS produces a bigger maximum mass and vice versa, we have that the maximum mass for $b_4 = -0.4$ is bigger and smaller for $b_4 = 1.0$.

b_4	X_V	$M/M_\odot - B_{min}^{1/4}$	R (km)	$\varepsilon_c \text{ (MeV/fm}^3\text{)}$	f_s	$R_{1.4}$
-0.4	0.4	2.43	12.45	903	0.407	11.84
0.0	0.4	2.41	12.33	893	0.402	11.81
+1.0	0.4	2.39	12.35	869	0.396	11.80
-0.4	1.0	2.65	13.02	797	0.317	12.13
0.0	1.0	2.61	12.97	795	0.317	12.08
+1.0	1.0	2.57	12.96	791	0.317	12.08
b_4	X_V	$M/M_\odot - B_{max}^{1/4}$	R (km)	$\varepsilon_c \text{ (MeV/fm}^3\text{)}$	f_s	$R_{1.4}$
-0.4	0.4	2.16	10.88	1188	0.417	10.88
0.0	0.4	2.14	10.86	1154	0.413	10.86
+1.0	0.4	2.10	10.77	1185	0.406	10.69
-0.4	1.0	2.44	11.85	986	0.319	11.36
0.0	1.0	2.40	11.85	978	0.319	11.34
+1.0	1.0	2.34	11.73	994	0.319	11.27

Table 5 – Main properties of quark stars for different values of b_4 and X_V .

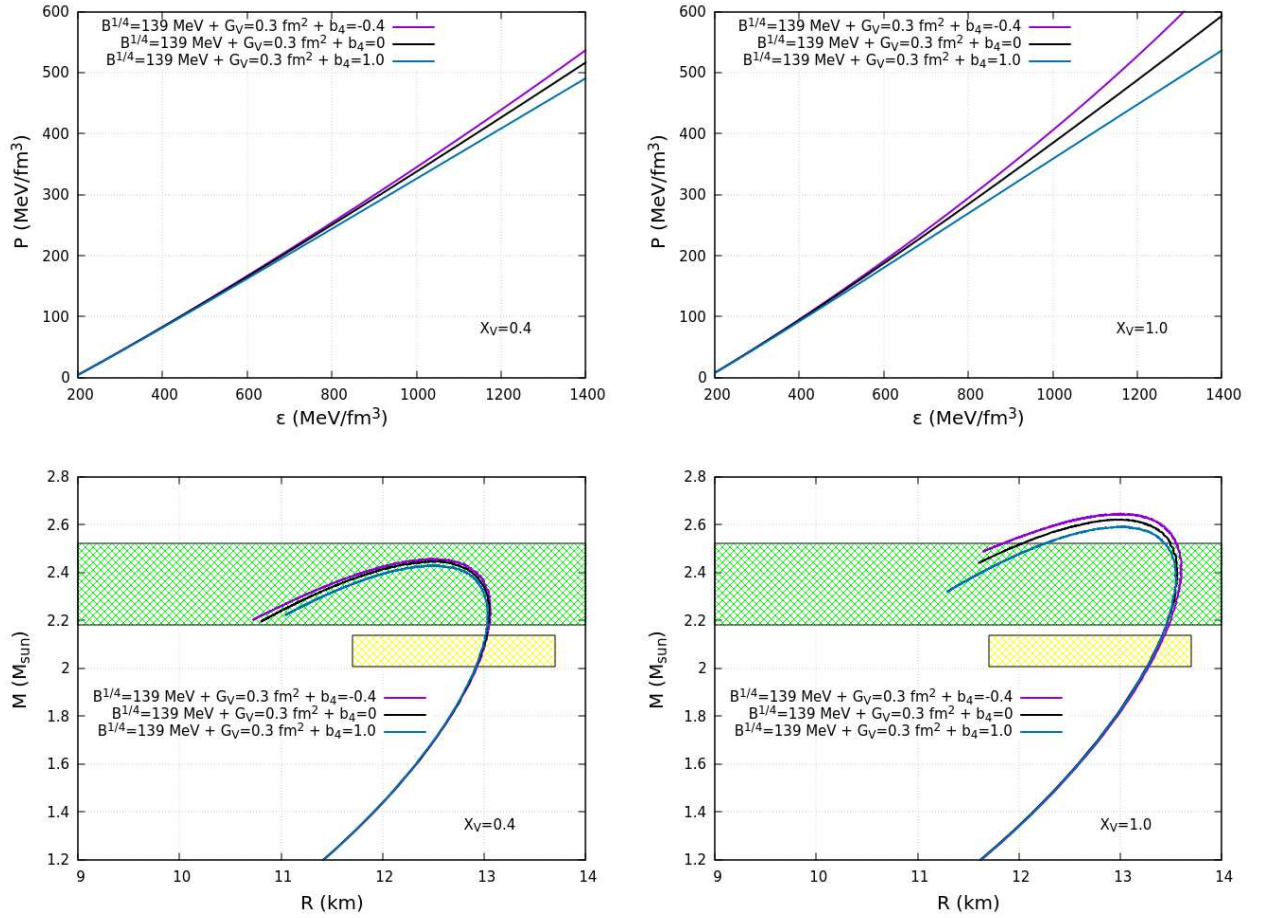


Figure 8 – EoS (top) and mass-radius relation (bottom) for $X_V = 0.4$ (left) and $X_V = 1.0$ (right) with $G_V = 0.3 \text{ fm}^2$ and $B^{1/4} = 139 \text{ MeV}$, the minimum Bag pressure value that produces stable strange stars, for different values of the parameter b_4 . The yellow area represents the uncertainties on both, the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021) and the green area refers to the mass of the PSR J0952–0607 (ROMANI et al., 2022) with its error bars.

Also from Fig. 8 and table 5 we can see that the influence of the quartic term is bigger if the maximum Bag constant is used than when the minimum Bag constant is used. The influence is also bigger when using $X_V = 1.0$ than when using $X_V = 0.4$. The variation on the radius is not significant. As far as strangeness is concerned, for $X_V = 0.4$ the fraction of s quarks in the center of the maximum mass star increases if b_4 is negative and decreases if b_4 is positive.

As for the radius of canonical stars, except the case in which $B^{1/4} = 139 \text{ MeV}$, $G_V = 0.3 \text{ fm}^2$ and $X_V = 1.0$ were used, we were able to reproduce stable strange stars according to the studies carried out by (CAPANO et al., 2020). Moreover, all the results obtained with the minimum Bag pressure value agree with the results of NICER (MILLER et al., 2021), whereas none with the maximum Bag pressure value agree. And, finally, as there is no change in the region of the HESS J1731-347, we continue to satisfy its constraints here as well.

2.4 TIDAL DEFORMABILITY

We next check if our model can satisfy yet another restraint, the one imposed by the tidal deformability parameter, as given by Eq. (6), more specifically the tidal parameter for the canonical stars. So we check if we can satisfy the restrictions $70 < \Lambda_{1.4} < 580$, as imposed by the LIGO/VIRGO gravitational wave observatories related to the GW170817 event (ABBOTT, B. P. et al., 2018).

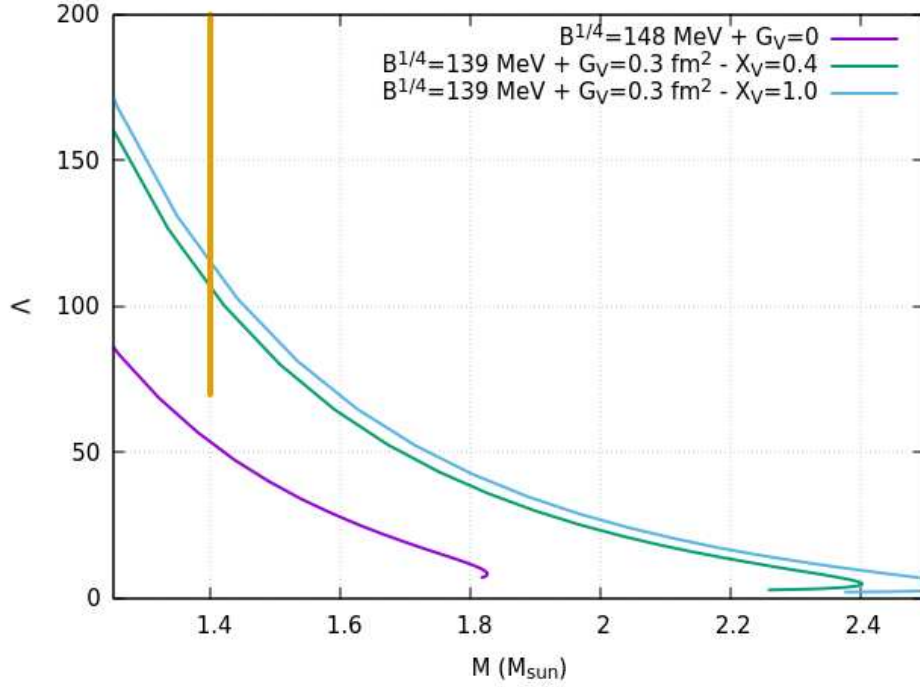


Figure 9 – Dimensionless tidal parameter as a function of the gravitational mass for different modified MIT models. The yellow line is the LIGO/VIRGO constraint $70 < \Lambda_{1.4} < 580$ for the $1.4M_{\odot}$ star.

In Fig. 9 we present the dimensionless tidal deformability as a function of the star masses for some representative parameters of the models used up to now. As can be seen, only when we use the modified MIT Bag model we are able to satisfy the constraint $70 < \Lambda_{1.4} < 580$. Here we don't show the results including also the self-interacting vector field to not oversaturate the graphic, but as shown in the previous section, its results don't deviate much from the ones using only the vector field, so, its results for $\Lambda_{1.4}$ also agree with the above mentioned constraints.

The values for $\Lambda_{1.4}$ are all lower than 290, the most probable value discussed in ref. (ABBOTT, B. P. et al., 2018). This is the opposite of what normally happens with commonly used models of neutron stars, which generally present a higher value for the $\Lambda_{1.4}$ as shown in ref. (LOPES, L. L.; MENEZES, Débora P, 2021). In (HINDERER, 2008), it is shown that the Love number k_2 has a higher dependence on the microscopic EoS than on the macroscopic mass and radius, and our results seem to agree with

this.⁴

2.5 CONCLUSIONS

In this chapter, we reviewed the MIT Bag model and proposed some modifications to it. Starting with the original model, we presented its Lagrangian density and obtained the stability window.

For the vectorial MIT Bag model, we corrected the one already proposed in the literature in that we added the vector field mass term. Furthermore, in addition to considering the universal meson-quark coupling constant, i.e., one that is equal for the three quarks, using symmetry group arguments, we studied the case in which the coupling of the vector field with the s quark is only 40% of the coupling with the quarks u and d . We built the stability window and were able to reproduce stars with masses of $2.61 M_{\odot}$, but with a radius greater than that obtained in (CAPANO et al., 2020), however, compatible with the values inferred from the NICER observations (MILLER et al., 2021). We also managed to get a maximum mass of $2.40 M_{\odot}$ which satisfies all astrophysical constraints, including the radius of the canonical stars for both (CAPANO et al., 2020) and (MILLER et al., 2021) at the same time while also maintaining thermodynamic consistency. In all cases, we were able to obtain masses and radii that agree with the measurements of the tiny object HESS J1731-347.

Inspired by the QHD models, we also proposed a modification to the linear vector field and introduced a quartic term in the vector channel. This term allowed us to modify the EoS while keeping the stability window unchanged. We conclude that a positive quartic term makes the EoS softer at high densities while a negative one makes them stiffer. With the introduction of this term we were able to obtain a mass of $2.44 M_{\odot}$ for a stable strange star, satisfying all astrophysical constraints.

In the end we also computed the dimensionless tidal deformability parameter and concluded that the modified MIT Bag models agree with the restraints imposed by this parameter as well.

So, to finalize, we conclude that the original MIT Bag model is not good enough to reconcile the Bodmer-Witten conjecture with the existence of compact stars with more

⁴ After the defense of this thesis it was brought to my attention that the way to calculate the tidal deformability here is not completely right because one should also take into account the discontinuity present at the surface of the strange star. When discontinuity is present (as in the case of self-bounded strange stars), the value of y_R calculated in Eq. (8) should be corrected:

$$y_R \rightarrow y_R - \frac{4\pi R^3 \Delta_{\epsilon s}}{M}, \quad (37)$$

where R and M are the star radius and mass, respectively, and $\Delta_{\epsilon s}$ is the difference between the energy density at the surface ($p = 0$) and the exterior of the star (which implies $\epsilon = 0$).

In (LOPES, L. L.; MENEZES, Debora P, 2022) was showed that the results for Λ are shifted to higher values and that for $X_V = 0.4$, only when $G_V = 0.32 \text{ fm}^2$, the constraint for $\Lambda_{1.4}$ is fulfilled and for $X_V = 1.0$ only for the maximum allowed Bag pressure values.

than $2M_{\odot}$, as imposed by recent measurements and all other astrophysical restraints discussed in this chapter, and that the Modified MIT Bag models proposed here are an excellent alternative, once they do agree with the restraints imposed. These results are also corroborated by the recent work (SILVA et al., 2023), where a comparative analysis between the density-dependent quark model and the vector MIT bag model (the one proposed here), using Bayesian analysis was performed. In this work, it is shown that the vector MIT bag model performs slightly better.

In the next chapter we go one step further and include finite temperature to the Modified MIT Bag models discussed in the present chapter.

3 PHASE DIAGRAMS AND HOT QUARK STARS VIA MIT BAG MODELS

In this chapter we use the modified versions of the MIT Bag model presented and discussed in the previous chapter, as well as in the article (LOPES, L. L.; BIESDORF, Carline; MENEZES, Débora P, 2021), to study the QCD phase diagram and proto-quark stars.

We start by considering symmetric quark matter consisting of two flavours, with $\mu_u = \mu_d$, and confront the results with the freeze-out line obtained by Cleymans (CLEY-MANS et al., 2006), which is known to be a fully hadronic process, and, therefore, needs to be completely in the hadronic phase. The liquid-gas phase transition (FINN et al., 1982), which is also a pure hadronic process that takes place at low temperatures, must be confined to the hadronic part of the QCD phase diagram as well, therefore being another restraint to our models. Furthermore, based on the results of (FUKUSHIMA; HATSUDA, 2010), we seek that the chemical potential at $T = 0$ is at least larger than 1050 MeV and as an upper limit, we impose $\mu = 1400$ MeV, even though there is no real indication of its value.

Besides these restraints, we also check if it is possible to satisfy these conditions together with the existence of stable strange quark matter (SQM), as proposed by the Bodmer-Witten conjecture.

Subsequently, we construct the phase diagrams with what we will call stellar matter, which is strange quark matter, i.e., composed of the quarks u , d and s , added with leptons and with the imposition of chemical equilibrium and charge neutrality. Although compact stars are generally considered cold, that is, with $T = 0$, here we extrapolate this study to finite temperatures and check if we can, at the same time, obtain phase diagrams according to the imposed restrictions and reproduce massive stars within the limits proposed by astrophysical observations.

We also perform a brief study of *hot* quark stars and compare the mass-radius diagrams obtained for $T=40$ MeV with the ones obtained for $T=0$. The same is done for the study of the speed of sound.

The discussions presented in this chapter are taken from the article entitled "Modified MIT Bag Models - part II: QCD phase diagram and hot quark stars" published in *Physica Scripta* (LOPES, L. L. et al., 2021).

3.1 THE MIT BAG MODELS AT FINITE TEMPERATURE

We will work here with the original MIT Bag Model, as discussed in section 2.1, with the Vector MIT Bag Model, as discussed in section 2.2, and also with the Vector MIT Bag Model with the addition of the Self-Interacting Vector Field, as discussed in section 2.3. We can, therefore, take equations (15), (35) and (36), considering also Eq. (34), as our EoSs and evaluate the influence of the self-interacting term on the

phase diagrams. We then fix $b_4 = 0$ to evaluate the influence of the vector field, and fix $G_V = b_4 = 0$ to evaluate the results obtained using the original MIT model. But first, to describe the QCD phase diagram, we need to add temperature to these equations.

As discussed in the previous chapter, at zero temperature the Fermi-Dirac distribution function turns into a Heaviside function, which makes the energy eigenvalue equal to the chemical potential and the integrals run from 0 to the Fermi momentum k_f . However, for finite temperature, the Fermi-Dirac distribution functions appear and we have the following equations for quark density, energy density, and pressure, respectively:

$$n_Q = \sum \frac{3}{\pi^2} \int dk_q \cdot k_q^2 \cdot (f_{q+} - f_{q-}), \quad (38)$$

$$\varepsilon = \sum \left(\frac{3}{\pi^2} \int dk_q \cdot k_q^2 \sqrt{m_q^2 + k_q^2} \cdot (f_{q+} + f_{q-}) \right) + \frac{1}{2} m_V^2 V_0^2 + \frac{3}{4} b_4 (g_{uuV} V_0)^4 + B, \quad (39)$$

$$P = \sum \left(\frac{1}{\pi^2} \int dk_q \frac{k_q^4}{\sqrt{m_q^2 + k_q^2}} \cdot (f_{q+} + f_{q-}) \right) + \frac{1}{2} m_V^2 V_0^2 + \frac{1}{4} b_4 (g_{uuV} V_0)^4 - B, \quad (40)$$

where f_{q+} and f_{q-} are the Fermi-Dirac distribution functions of quarks and anti-quarks, respectively, and are given by:

$$f_{q\pm} = \frac{1}{1 + \exp[(E_q^* \mp \mu_q^*)/T]}, \quad (41)$$

where we have that $\mu_q^* = \mu_q - g_{qqV} V_0$ is the effective chemical potential and $E_q^* = \sqrt{k_q^2 + m_q^2}$.

The equation of motion for the V field reads:

$$g_{uuV} V_0 + \left(\frac{g_{uuV}}{m_V} \right)^2 (b_4 (g_{uuV} V_0)^3) = \left(\frac{g_{uuV}}{m_V} \right) \sum \left(\frac{g_{qqV}}{m_V} \right) n_q. \quad (42)$$

In the equations above, the sums run over the quark flavours $q = u, d$, and s , when this quark flavour is taken into account.

Notice that in the equation for the quark density (38) the contribution of the anti-quarks is subtracted from that of the quarks. In this sense it is not hard to see that $n_Q = 0$ does not necessarily mean the absence of particles, but rather that the density of quarks is equal to the density of anti-quarks.

3.1.1 Conditions for the Phase Transition

In models like the MIT Bag model, we have that, for low values of temperature and chemical potential, the quarks are confined, which is indicated by the negative

value of the pressure given by the Bag constant. This is the hadron phase. When matter is heated, light hadrons, preferably pions, are thermally created, which fills the space between the nucleons. These pions and hadrons begin to overlap with each other and with the bags of original nucleons, so that a network of zones with quarks, anti-quarks and gluons is formed. At a certain critical temperature these zones fill the entire volume and we have a new state of matter which is the quark-gluon plasma (QGP). When matter is strongly compressed, the nucleons overlap at a certain critical density and form a degenerate QGP consisting mostly of quarks.

Therefore, in this work, we consider that the transition from hadronic matter to quark matter occurs when $P = 0$, that is, when the pressure exerted by the quark gas (plus, eventually, the meson ω and the leptons) is equal to the pressure given by the Bag constant.

It should also be mentioned that here we only deal with first-order phase transitions, although LQCD calculations indicate that for low chemical potentials a transition of the crossover type occurs (AOKI et al., 2006; BELLWIED et al., 2015).

3.1.2 Symmetric Matter

We start with the two-flavoured symmetric matter with $\mu_u = \mu_d$ to make a direct comparison with the chemical freeze-out results, as presented in (CLEYMANS et al., 2006), and also with the liquid-gas phase transition.

We use three different values for the Bag constant, $B^{1/4} = 148$ MeV, 165 MeV and 205 MeV. The smallest value is also the smallest value allowed by the stability window for the original model, that is, smaller values would allow for a stable $u - d$ matter if we were to use it for the original model, remembering that the Vector MIT Bag model allows for lower values of the Bag constant (see Fig. 3).

To describe massive quark stars, we also check EOS with the following parameter: for $B^{1/4} = 148$ MeV we use $G_V = 0.3 \text{ fm}^2$ and $b_4 = 60$ and for $B^{1/4} = 165$ MeV and $B^{1/4} = 205$ MeV we use $G_V = 0.8 \text{ fm}^2$ and $b_4 = 20$. In the previous chapter we used only low values of b_4 in order not to affect the stability window, but here we relax this condition so as to study the strong coupling effects.

The results are presented in Fig. 10. Notice that, for the same type of line, purple corresponds to the original MIT model, green to the Vector MIT bag model and, finally, yellow corresponds to the Vector MIT bag model with Self-interacting Field.

We can see that when there is no interaction term, increasing the Bag pressure value favors the hadron phase. The bigger the $B^{1/4}$ value, the higher the maximum transition temperature T_{max} and the maximum transition chemical potential μ_{max} . The vector field increases the repulsion between the quarks which increases the maximum transition chemical potential. However, as shown in Eq. (42), the vector channel couples with the density and, therefore, causes little effect at vanishing chemical potentials. The

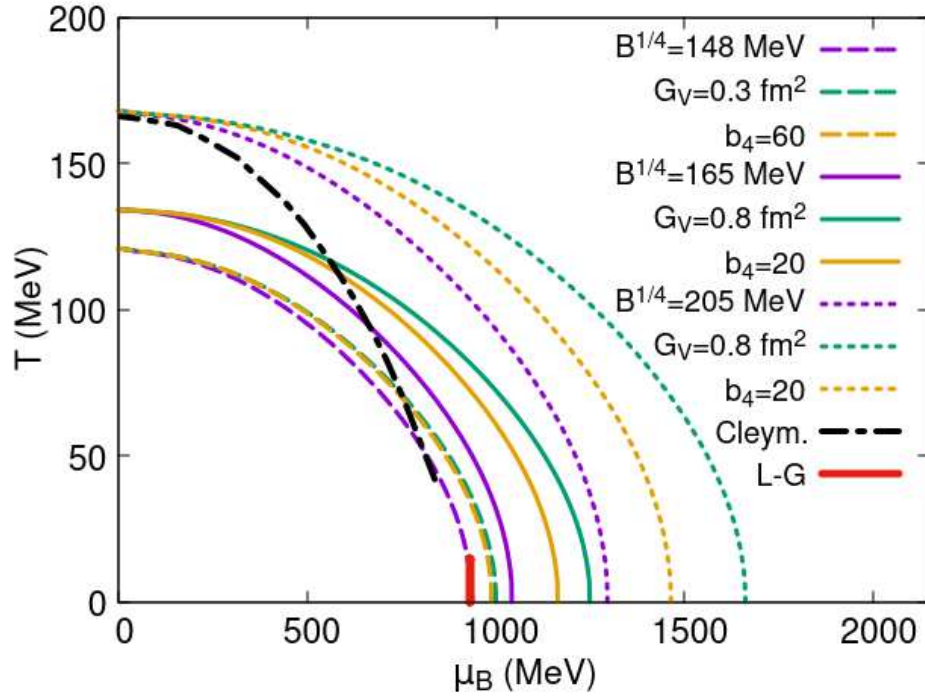


Figure 10 – QCD phase diagram for different values of $B^{1/4}$, G_V and b_4 considering two-flavoured symmetric matter. Cleym line is the experimental data line of the freeze-out proposed in (CLEYMANS et al., 2006) and L-G is the region where a liquid-gas phase transition is expected (FINN et al., 1982).

vector field does not change the transition temperature for $\mu \rightarrow 0$, and the value of this temperature depends exclusively on the Bag pressure value. The self-interaction term reduces the repulsion between quarks at high densities and, as a consequence, there is a decrease in the maximum chemical potential with respect to the previous case.

We see that for $B^{1/4} = 148$ MeV, which is the only value that lies within the stability window, the maximum transition temperature obtained is only $T = 121$ MeV. This value is far below the experimental results of the chemical freeze-out and LQCD calculations. As for $T \rightarrow 0$ we see that without the vector field not even the region related to the liquid-gas phase transition is fully contained inside the hadron phase. With the addition of the vector field, chemical potentials bigger than 1000 MeV are obtained, but still smaller than the minimum limit of 1050 MeV that we are looking for.

For $B^{1/4} = 165$ MeV the highest transition temperature obtained is $T = 134$ MeV, which is still below the results of LQCD and freeze-out estimations. Without the vector field, the maximum chemical potential is 1040 MeV, but with the vector term and also with the self-interacting term, this chemical potential is always in the range $1050 < \mu < 1400$ MeV.

Finally, for $B^{1/4} = 205$ MeV, our results for low chemical potentials agree with the freeze-out line and the LQCD results, such that Cleyman's line lies completely inside the hadronic phase. The liquid-gas phase transition is also completely contained inside

the hadronic phase, but when we add the vector field and the self-interacting field to the model, the chemical transition potentials reach values above 1400 MeV. Hence, for $B^{1/4} = 205$ MeV only the original MIT Bag model result falls within $1050 < \mu < 1400$ MeV.

The results of this section are also summarized in the top part of Table 6.

In short, we need to use a Bag pressure value of at least $B^{1/4} = 205$ MeV to satisfy the restraints at $\mu \rightarrow 0$, but with this value for $B^{1/4}$ we can not add interactions between quarks as the μ_{max} increases too much when we add the vector field. The best results for $T \rightarrow 0$ are obtained when we use $B^{1/4} = 165$ MeV, but this value, as already stated, does not work for higher values of the temperature. This shows that we need to improve our model. Let us first investigate how this model behaves when used to describe stellar matter.

3.1.3 Stellar Matter

Although $T = 0$ is generally used for calculations involving stable compact stars, finite temperatures can be important in the early stages of protostars. For example, the deconfinement of neutron matter occurs on a strong interaction time scale of 10^{-23} s (LUGONES; BENVENUTO, 1998), while chemical equilibrium occurs on a weak interaction time scale of 10^{-8} s. On the other hand, the cooling of a newborn neutron star by diffusion of neutrinos takes a few seconds (LATTIMER, James M, 2018). Therefore, the phase diagram for values up to $T = 50$ MeV for β -stable matter is important (MENEZES, Debora P; PROVIDÊNCIA, 2004). For higher temperatures, the discussion is more pedagogical, since β -stable matter is not expected in this region (GUPTA, V. et al., 2003; CAVAGNOLI; MENEZES, Debora P, 2008).

Based on that, we construct here phase diagrams for strange stellar matter, i.e., we allow the appearance also of the quark s , add the leptons e and μ in the form of a free Fermi gas and impose chemical equilibrium and charge neutrality, as given by Eqs. (31) and done in the previous chapter, but here we do it for finite temperature. We use the same parameters used on the previous section for symmetric matter. The results are shown in Fig. 11.

As we can see, the stellar matter disfavors the hadron phase compared to the symmetric matter presented in Fig. 10, as it always produces a smaller transition temperature value for $\mu \rightarrow 0$ as well as a smaller transition chemical potential when $T = 0$. Furthermore, comparing the results for $X_V = 0.4$ and $X_V = 1.0$, we see that larger values of X_V result in larger values of chemical potentials in the region where $T \rightarrow 0$, which is no surprise being that bigger values for X_V mean more repulsion between the quarks.

Although we have already studied the TOV solutions for the models used here, in this chapter we use different parameters that are better suited for the QCD phase

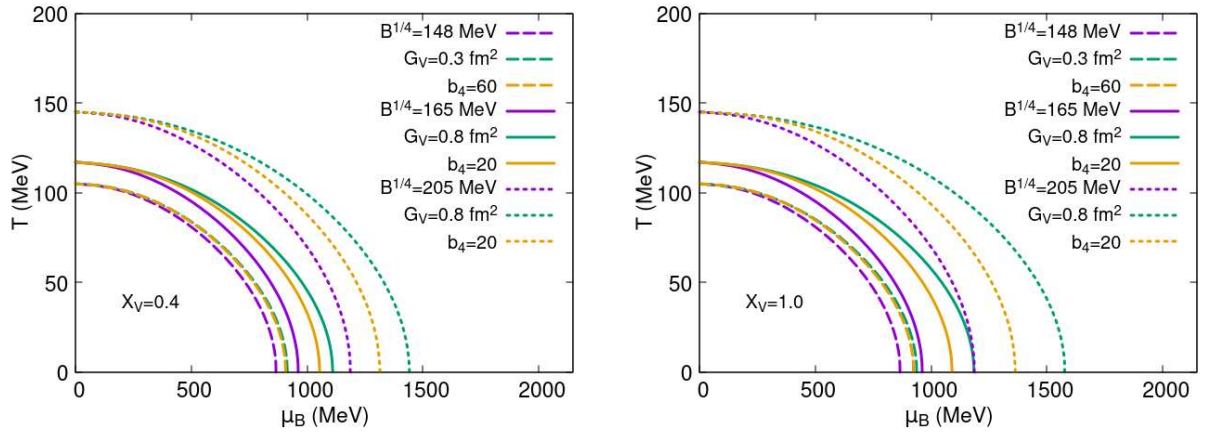


Figure 11 – Phase diagrams for $X_V = 0.4$ (left) and $X_V = 1.0$ (right) for stellar matter.

diagrams, so we check the TOV solutions again now for the parameters we use in this chapter to see if we can, at the same time, obtain QCD phase diagrams and mass-radius relations that satisfy all the restraints imposed. So, for the same parameters used above we now take $T = 0$ and calculate the mass-radius diagrams using the TOV equations (66). The results are shown in Fig. 12.

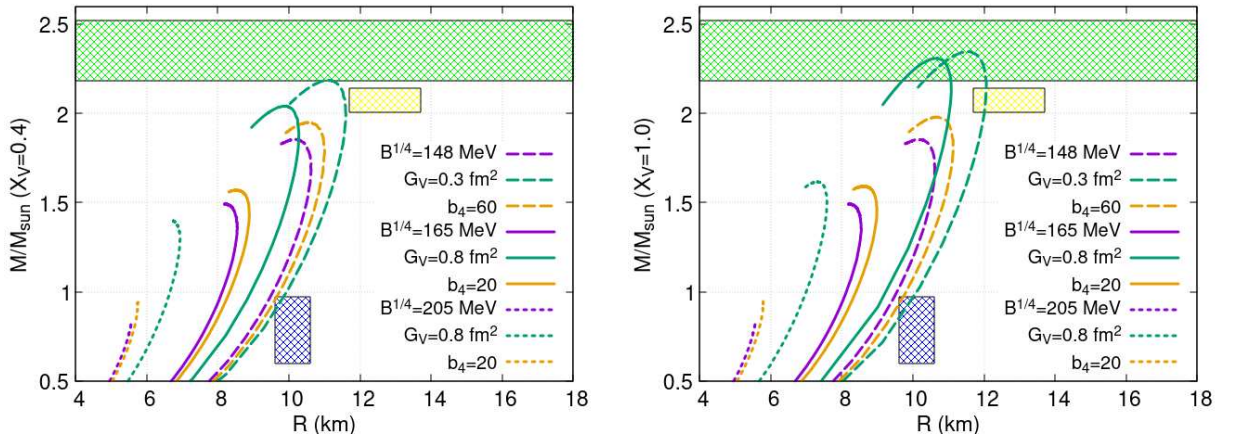


Figure 12 – Mass-radius relation for different values of $B^{1/4}$, G_V and b_4 with $X_V = 0.4$ (left) and $X_V = 1.0$ (right). The yellow area represents the uncertainties on both, the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021), the green area refers to the mass of the PSR J0952–0607 (ROMANI et al., 2022) with its error bars and the blue area the uncertainties on both, the radius and mass of the HESS J1731-347 (DOROSHENKO et al., 2022).

Even though the Bag value $B^{1/4} = 205$ MeV allows a reasonably well description of the QCD phase diagram at the vanishing chemical potential region, it can be seen from Fig. 12 that the same value does not perform well at the mass-radius diagram and that it actually performs badly: only very low masses and radius are obtained, reaching, in most cases, barely, $1.0M_\odot$ and not meeting any of the restraints coming from the astrophysical observations. This was already expected as the bigger values of $B^{1/4}$ turn

the EoS softer.

With $B^{1/4} = 148$ MeV, $G_V = 0.3 \text{ fm}^2$ and $X_V = 0.4$, still respecting the stability window, we managed to obtain a maximum mass of $2.19 M_\odot$ and, outside the window, that is, with $G_V = 0.3 \text{ fm}^2$ and $X_V = 1.0$, we get a maximum mass of $2.36 M_\odot$, which is also the highest mass we are able to obtain with the parameters used in this chapter. Both values meet the restraints of the PSR J0952–0607 and HESS J1731–347, but only with the universal coupling $X_V = 1.0$ we also meet the restraint imposed by PSR J0740+6620. Unfortunately, as we saw in Figs. 10 and 11, these parameters give us bad results for the QCD phase diagram.

For $B^{1/4} = 165$ MeV we get a maximum mass of $2.31 M_\odot$, which is very close to the $2.36 M_\odot$ obtained with $B^{1/4} = 148$ MeV, however, the radii of canonical stars for these two parameterizations are very different. For $B^{1/4} = 148$ MeV we obtain a radius of 11.15 km whereas for $B^{1/4} = 165$ MeV the radius is only 10.20 km, almost 1 km smaller. In fact, the radius is very sensitive to the Bag pressure value. Unfortunately, when we use $B^{1/4} = 165$ MeV we are never able to meet the restraints imposed by PSR J0740+6620 and HESS J1731–347.

We also see that the maximum mass is very sensitive to the value of X_V when the self-interacting term is not present. When this term is present, the effect of X_V is secondary. In fact, the self-interacting term makes the results more similar to those obtained with the original Bag model than with those obtained with the model plus the addition of the vector field only. This is something new we can observe here and that was not so clear in the previous chapter as here we use much higher values for b_4 .

Another important aspect is the radius of the canonical star. We see that the only models where the radii are $10.4 < R_{1.4} < 11.9$ km, and so agree with (CAPANO et al., 2020), are those where we use $B^{1/4} = 148$ MeV, which is within or very close to the stability window of stable SQM. Bag pressure values that are too far from the stability window produce very small radii. On the other side, no results agree with the NICER results, i.e., $11.80 < R_{1.4} < 13.85$ km (MILLER et al., 2021).

The results of this section are summarized in the bottom of table 6.

Symmetric matter with $\mu_u = \mu_d$								
$B^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	T_{max}	μ_{max}	Cleym.?	L-G?	μ_{max} ?
148	-	0.0	0.0	121	938	no	no	no
165	-	0.0	0.0	134	1040	no	yes	no
205	-	0.0	0.0	168	1296	yes	yes	yes
148	-	0.3	0.0	121	1000	no	yes	no
148	-	0.3	60	121	988	no	yes	no
165	-	0.8	0.0	134	1249	no	yes	yes
165	-	0.8	20	134	1163	no	yes	yes
205	-	0.8	0.0	168	1662	yes	yes	no
205	-	0.8	20	168	1464	yes	yes	no

Strange stellar matter							
$B^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	M (M _⊙)	R _{1.4} (km)	T_{max}	μ_{max}
148	-	0.0	0.0	1.85	10.31	105	866
165	-	0.0	0.0	1.50	8.58	117	962
205	-	0.0	0.0	0.98	-	145	1187
148	0.4	0.3	0.0	2.19	10.93	105	915
148	0.4	0.3	60	1.96	10.67	105	908
165	0.4	0.8	0.0	2.04	9.78	117	1111
165	0.4	0.8	20	1.56	8.86	117	1055
205	0.4	0.8	0.0	1.41	6.72	145	1442
205	0.4	0.8	20	1.00	-	145	1315
148	1.0	0.3	0.0	2.36	11.15	105	938
148	1.0	0.3	60	1.99	10.73	105	924
165	1.0	0.8	0.0	2.31	10.20	117	1186
165	1.0	0.8	20	1.58	8.94	117	1091
205	1.0	0.8	0.0	1.62	7.60	145	1575
205	1.0	0.8	20	1.02	-	145	1364

Table 6 – Maximum temperatures T_{max} , maximum chemical potentials μ_{max} , main properties of quark stars and some restrictions for symmetric matter (top) and stellar strange matter (bottom) for different values of $B^{1/4}$, X_V , G_V and b_4 . Temperatures and chemical potentials are all given in MeV.

3.2 TEMPERATURE-DEPENDENT BAG MODEL

It is clear from the results of the last sections that we are not able to reproduce the maximum freeze-out temperature and at the same time reproduce massive stars that satisfy the astrophysical constraints. With a Bag pressure value of 205 MeV we managed to get the freeze-out line to be completely on the hadronic side of the diagram, but the maximum masses and radii of the stars resulting from this parameterization are very low, far below what has been observed. With the $B^{1/4}$ of 148 MeV and 165 MeV it is possible to obtain very massive stars and also reasonable results for the phase diagram at $T \rightarrow 0$, but the maximum temperatures are far from the freeze-out temperature. Therefore, it is impossible to use a single Bag pressure value to describe the entire energy spectrum of the QCD phase diagram. Therefore, we propose an

alternative, a temperature dependent Bag model.

Such a proposal is not new, as can be seen in (DEY, J.; DEY, M., 1986; MILLER et al., 2019; LI, S.; BHALERAO; BHADURI, 1991). Here we propose a Bag pressure value similar to the one proposed in (DEY, J.; DEY, M., 1986), but with its value increasing with temperature instead of decreasing:

$$B(T) = B_0 \left[1 + \left(\frac{T}{T_0} \right)^4 \right]. \quad (43)$$

T_0 is set to reproduce the transition temperature at $\mu = 0$ given by LQCD and freeze-out. So, here we have:

$$\begin{aligned} B_0^{1/4} = 148 \text{ MeV} &\longrightarrow T_0 = 131 \text{ MeV} \\ B_0^{1/4} = 165 \text{ MeV} &\longrightarrow T_0 = 155 \text{ MeV} \end{aligned}$$

We completely discard the $B_0^{1/4} = 205 \text{ MeV}$ value as it performs very badly at low temperatures. But it is easy to check that at temperatures around $T = 168 \text{ MeV}$, which is what the LQCD and freeze-out foresees for T_{max} , $B^{1/4}(T)$ assumes values very close to 205 MeV.

The QCD phase diagrams for two-flavoured symmetric matter with $\mu_u = \mu_d$ are shown in Fig. 13.

As we can see, we manage to obtain a maximum transition temperature around 168 MeV for both values of B_0 . However, for $B_0^{1/4} = 148 \text{ MeV}$, the freeze-out line is not entirely inside the confined (hadron) phase. We checked and even if we use the upper limit of the stability window, we would encounter the same problem. This indicates that, for this model, we are not able to reproduce, at the same time, stable SQM and the QCD phase diagram obeying the restrictions that we impose. Furthermore, since when $T \rightarrow 0$ we have $B(T) \rightarrow B_0$, causing us to fall back on what was discussed in the section 3.1.2, we have the same problem as the one found in that section: for $B_0^{1/4} = 148 \text{ MeV}$ the maximum chemical potentials are too low.

As for $B_0^{1/4} = 165 \text{ MeV}$ we are able to fit the freeze-out line completely inside the hadron phase when we use the modified versions of the MIT Bag model; with $G_V = b_4 = 0.0$ we are still unable to satisfy this restriction. Moreover, for this value of $B(T)$ we have $1050 < \mu < 1400 \text{ MeV}$ for temperatures close to zero. So in this case we have a good description of the QCD phase diagram. Furthermore, as already shown in Fig. 12, we managed to reproduce massive stars satisfying at least the restrictions imposed by PSR J0952+0607 when $G_V = 0.8 \text{ fm}^2$. One should bear in mind that this Bag pressure value is far from the stability window predicted for zero temperature¹.

¹ As shown in (DEXHEIMER; TORRES; MENEZES, 2013), for finite temperature one should use the free energy per baryon instead of the binding energy per baryon, as done in the previous chapter, to obtain the upper limits of the stability window. If so done, a wider range of constants becomes possible to ensure stable matter.

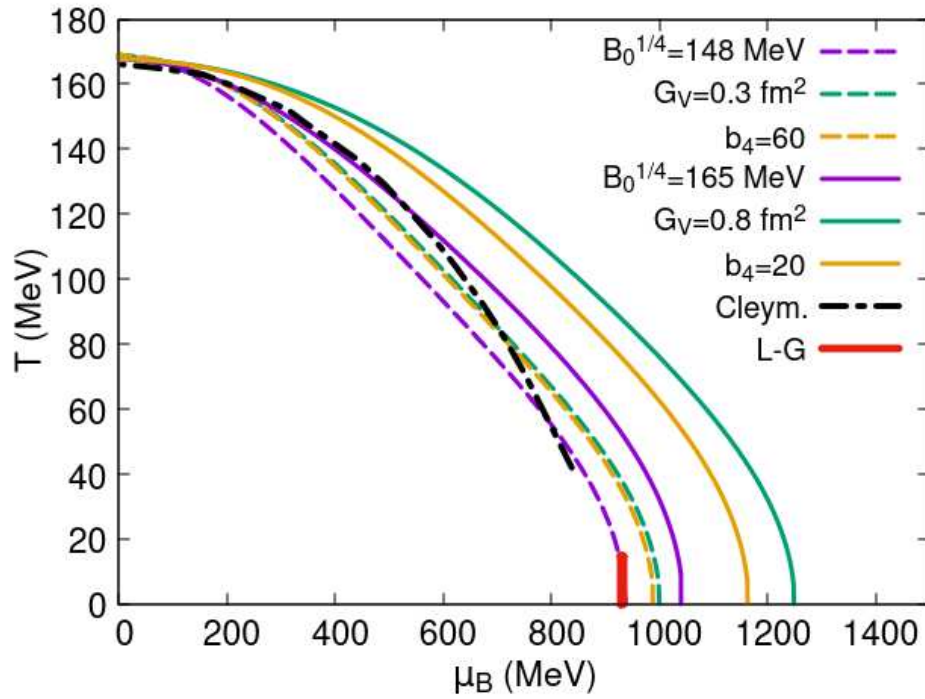


Figure 13 – QCD phase diagram for temperature-dependent Bag model considering two-flavoured symmetric matter. Cleym’s line is the experimental data line of freeze-out (CLEYMANS et al., 2006) and L-G is the region where a liquid-gas phase transition is expected (FINN et al., 1982).

For completeness, in Fig. 14 we also present the diagrams for strange stellar matter.

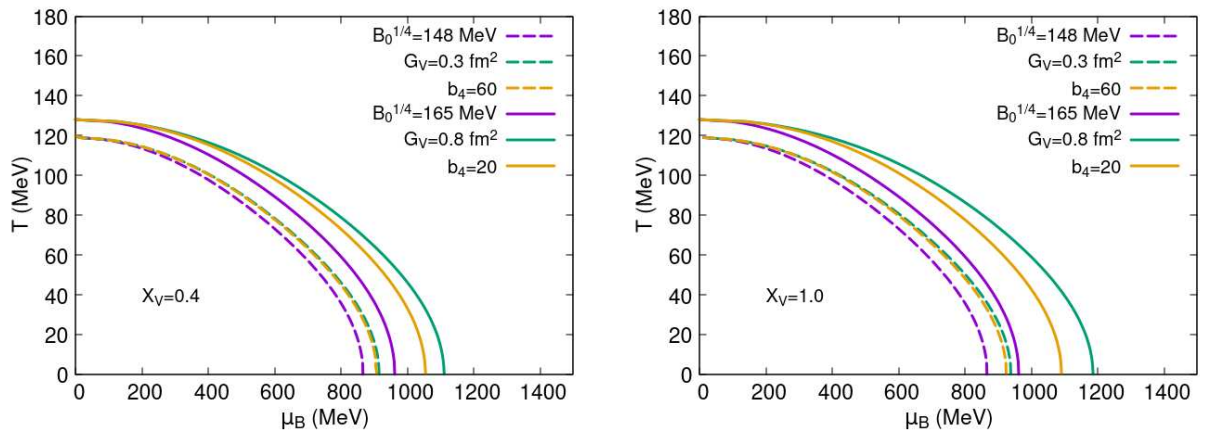


Figure 14 – Phase diagrams for $X_V = 0.4$ (left) and $X_V = 1.0$ (right) for stellar matter using temperature-dependent Bag model.

For low temperatures, the figures 14 and 11 are very similar, but for low chemical potentials there is an increase in the transition temperature, which is expected due to the temperature dependence of the Bag pressure value. When compared with symmetric matter, we see that the maximum transition temperature is lower in the latter situation.

We then conclude that, within this Bag formalism with temperature dependence, using $B_0^{1/4} = 165$ MeV, we obtain a good description of the QCD phase diagram and, at the same time, can reproduce massive stars compatible with the PSR J0952–0607 measurements (ROMANI et al., 2022) when we use $G_V = 0.8$ fm². However, we do not meet the restraints of PSR J0740+6620 and HESS J1731-347.

We also remember that, for this Bag pressure value, the radii of the canonical stars are very small, in disagreement with (CAPANO et al., 2020) and (MILLER et al., 2021).

In Table 7 the numerical data obtained in this section are presented.

Symmetric matter with $\mu_u = \mu_d$								
$B_0^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	T_{max}	μ_{max}	Cleym.?	L-G?	μ_{max} ?
148	-	0.0	0.0	169	938	no	no	no
165	-	0.0	0.0	168	1040	no	yes	no
148	-	0.3	0.0	169	1000	no	yes	no
148	-	0.3	60	169	988	no	yes	no
165	-	0.8	0.0	168	1249	yes	yes	yes
165	-	0.8	20	168	1163	yes	yes	yes

Strange stellar matter							
$B_0^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	M ($M_\odot$)	$R_{1.4}$ (km)	T_{max}	μ_{max}
148	-	0.0	0.0	1.85	10.31	118	866
165	-	0.0	0.0	1.50	8.58	128	962
148	0.4	0.3	0.0	2.19	10.93	118	915
148	0.4	0.3	60	1.96	10.67	118	908
165	0.4	0.8	0.0	2.04	9.78	128	1111
165	0.4	0.8	20	1.56	8.86	128	1055
148	1.0	0.3	0.0	2.36	11.15	118	938
148	1.0	0.3	60	1.99	10.73	118	924
165	1.0	0.8	0.0	2.31	10.20	128	1186
165	1.0	0.8	20	1.58	8.94	128	1091

Table 7 – Maximum temperatures T_{max} , maximum chemical potentials μ_{max} , main properties of quark stars and some restrictions for symmetric matter (top) and stellar strange matter (bottom) for different values of $B_0^{1/4}$, X_V , G_V and b_4 using the **temperature-dependent** Bag model formalism. Temperatures and chemical potentials are all given in MeV.

3.3 PROTO-QUARK STARS AND THE SPEED OF SOUND

As already discussed at the beginning of the section 3.1.3, a newly formed neutron star can reach temperatures of up to $T = 50$ MeV (around 500 billion Kelvin) and, while the time for it to cool takes a few seconds, the deconfinement and chemical equilibrium are reached in a time scale of millions and millions of times smaller. Therefore, the study of hot quark matter is important both for the study of macroscopic properties of quark stars and to understand how temperature affects the chemical potential at which the phase transition occurs.

Here we study how a temperature of 40 MeV affects the mass-radius relation of quark stars. Although some studies (MENEZES, Debora P; PROVIDÊNCIA, 2004; PONS et al., 1999; MALFATTI et al., 2019) indicate that an isentropic formulation² is more suitable, here we adopt an isothermal formulation³, which allows a direct comparison with the mass-radius diagram shown in Fig. 12. Furthermore, we do not use the Bag value with dependence on temperature since for $T = 40$ MeV there is little difference between B_0 and $B(T)$. In Fig. 15 we present the mass-radius relation for $B^{1/4} = 148$ MeV and $B^{1/4} = 165$ MeV with $X_V = 0.4$ e $X_V = 1.0$. Notice that the solid lines correspond to $T = 0$ and the dashed lines to $T = 40$ MeV.

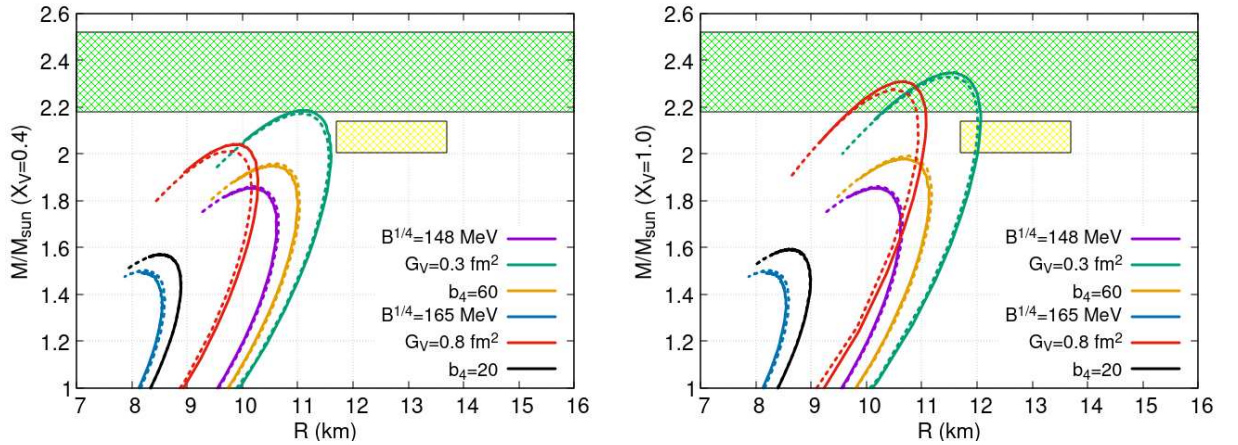


Figure 15 – Mass-radius relation for $T = 0$ (solid lines) and $T = 40$ MeV (dashed lines) for different values of $B^{1/4}$, G_V , b_4 and $X_V = 0.4$ (left) and $X_V = 1.0$ (right). The yellow area represents the uncertainties on both, the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021) and the green area refers to the mass of the PSR J0952-0607 (ROMANI et al., 2022) with its error bars.

We can see from Fig. 15 that, in all cases, for the same parameterization, the difference between $T = 0$ and $T = 40$ MeV is very small (lines of the same color, continuous and dashed, respectively). In the case where only the linear vector term is included (green and red lines) there is a decrease in the maximum masses when the

² where entropy is held constant

³ where the temperature is kept constant

temperature is finite. But when there is also the inclusion of the self-interacting term (yellow and black lines), the self-interaction term recovers the maximum masses for finite temperatures and leaves them practically equal to those for zero temperature. The decrease in maximum mass with the inclusion of the linear vector term is due to the reduction in central density with temperature. The results using the original MIT Bag model (purple and blue lines) are very similar to those when also the self-interacting term is included, but with the mass and radius dislocated to smaller values. The main contribution of temperature is the reduction of the chemical potential at which the transition from hadrons to quarks occurs.

As pointed out in (ANNALA et al., 2020), the speed of sound of quark matter is strongly related to the mass and radius of the quark nucleus in hybrid stars. The authors claim that, if the conformal limit ($v_s^2 < 1/3$) is not strongly violated, massive neutron stars should have sizable nuclei made up of quark matter.

The speed of sound squared is given by:

$$v_s^2 = \frac{\partial P}{\partial \epsilon}. \quad (44)$$

In Fig.16 we present v_s^2 as a function of the chemical potential for $B^{1/4} = 165$ MeV. We do not present the results for other Bag pressure values because v_s^2 does not depend on its value.

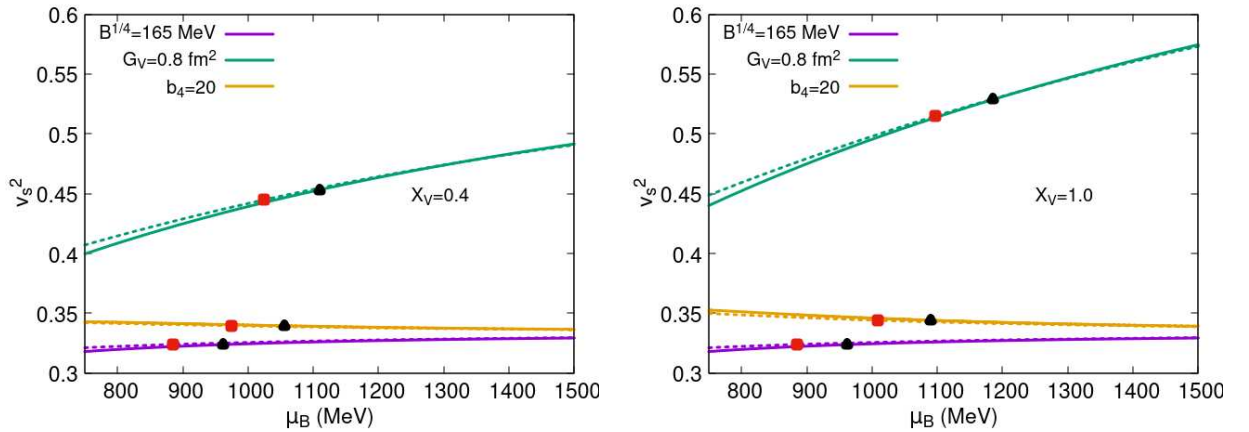


Figure 16 – Speed of sound of quark matter for $B^{1/4} = 165$ MeV with $X_V = 0.4$ (left) and $X_V = 1.0$ (right). The labeling notation is the same as that used in Fig. 15, where solid lines represent $T = 0$ and dashed lines $T = 40$ MeV. The black triangles represent the chemical potentials where the phase transition for $T = 0$ occurs, while the yellow squares represent the chemical potentials where the phase transition happens for $T = 40$ MeV.

Using the model with the inclusion of the vector field (green lines), we obtain a monotonic increase in the speed of sound with respect to the chemical potential. In addition, with this model we obtain the highest speed and highest chemical potential of phase transition. Both when we include the self-interaction term (yellow lines) and when

we use the model without any interaction (purple lines), we have an almost constant speed of sound. The original MIT Bag model produces the lowest v_s^2 and also the lowest phase transition chemical potential. When we compare $X_V = 0.4$ with $X_V = 1.0$ we see that only when the linear vector term is present, a significant change in the speed of sound is noticed. The main results are presented in the table 8.

$B^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	T (MeV)	μ_c (MeV)	M ($M_\odot$)	v_s^2 at μ_c
148	-	0.0	0.0	0	866	1.85	0.32
148	-	0.0	0.0	40	781	1.86	0.32
165	-	0.0	0.0	0	962	1.50	0.32
165	-	0.0	0.0	40	885	1.52	0.32
148	0.4	0.3	0.0	0	915	2.19	0.39
148	0.4	0.3	0.0	40	826	2.16	0.37
148	0.4	0.3	60	0	908	1.96	0.34
148	0.4	0.3	60	40	819	1.98	0.34
165	0.4	0.8	0.0	0	1111	2.04	0.46
165	0.4	0.8	0.0	40	1249	2.00	0.45
165	0.4	0.8	20	0	1055	1.56	0.34
165	0.4	0.8	20	40	974	1.56	0.34
148	1.0	0.3	0.0	0	938	2.36	0.42
148	1.0	0.3	0.0	40	847	2.33	0.40
148	1.0	0.3	60	0	924	1.99	0.35
148	1.0	0.3	60	40	836	2.00	0.35
165	1.0	0.8	0.0	0	1186	2.31	0.53
165	1.0	0.8	0.0	40	1096	2.26	0.51
165	1.0	0.8	20	0	1091	1.58	0.34
165	1.0	0.8	20	40	1009	1.59	0.34

Table 8 – Phase transition chemical potential (μ_c), maximum mass and speed of sound as a function of the various parameters discussed in the text.

3.4 CONCLUSIONS

In this chapter we advanced on the study of the modified versions of the MIT Bag model already discussed in the previous chapter. Here we introduced temperature, studied its effects and constructed QCD phase diagrams, as well as mass-radius diagrams for a temperature of $T = 40$ MeV and the speed of sound in those quark stars.

We started our study by considering symmetric matter consisting only of quarks u and d with $\mu_u = \mu_d$. We concluded that it is difficult to reconcile the restrictions imposed by the freeze-out and liquid-gas transition data with the existence of stable strange matter, as proposed by the Bodmer-Witten conjecture. A Bag pressure value of at least 205 MeV is necessary to satisfy those restraints, and this is a value far from the stability window established in the previous chapter, on section 2.1.1. We also observed that for this value of $B^{1/4}$ it is very hard to keep the chemical potential at $T \rightarrow 0$ between the values 1050 and 1400 MeV. $B^{1/4} = 148$ MeV and $B^{1/4} = 165$ MeV are more suitable for this region of the QCD phase diagram.

Based on the conclusion that we need a bigger Bag pressure value at high temperatures than at low, we introduced a temperature-dependent Bag model that increases with the temperature. Using this model with the parametrization where the Bag value at $T = 0$ is $B^{1/4}(T = 0) = B_0^{1/4} = 165$ MeV we were able to obtain a good description for the QCD phase diagram for both low and high temperatures.

We also constructed the QCD phase diagrams using stellar matter constituted by quarks u , d and s + leptons in chemical equilibrium and electrical charge neutrality for both constant and temperature-dependent Bag values. We observed a considerable decrease in the transition temperatures at $\mu \rightarrow 0$ (T_{max}) and in the chemical potentials at $T \rightarrow 0$ (μ_{max}).

Another conclusion drawn was that the T_{max} depends exclusively on the Bag pressure value and on the type of matter (symmetric or stellar) used, and that neither the value of the coupling constant G_V nor of the self-interacting term b_4 affects T_{max} . Furthermore, the bigger the value of $B^{1/4}$ the higher is T_{max} . As for the low-temperature region, all constants and also the type of matter play a role: bigger values of $B^{1/4}$ imply higher values for μ_{max} , bigger values for G_V also imply higher values for μ_{max} and bigger values for b_4 reduce the μ_{max} again, making its value lie somewhere between the first one (quark matter without interaction) and the last one (with only the linear vector field included).

For stellar matter we also checked the TOV solutions and confronted them against the same restraints used in the previous chapter. Unfortunately no parametrization used in the present chapter meets all the restraints imposed by the astrophysical data. This is, however, not a real problem since one parametrization, namely $B^{1/4} = 148$ MeV + $G_V = 0.3$ fm² and $X_V = 1.0$, which is slightly outside the stability window of strange matter, only fails to meet the restraints imposed by the NICER

results (MILLER et al., 2021), but, if the measurements of (CAPANO et al., 2020) are correct instead, this parametrization would meet all the restraints for compact stars. Unfortunately, it does not meet all the restraints for the QCD phase diagram. We should make a note here to comment on the fact that in the paper on which this chapter is based, (LOPES, L. L. et al., 2021), we only used the restraint for PSR J0740+6620 with now outdated measurements (CROMARTIE et al., 2020), which made the parametrization $B^{1/4} = 165 \text{ MeV} + G_V = 0.8 \text{ fm}^2$ suitable to describe at the same time compact stars and the QCD phase diagram. With the updated measurements we are no longer able to do so.

When we compared the TOV results for $T = 0$ with the ones for $T = 40 \text{ MeV}$ we concluded that no significant difference can be observed, being that there is a very small reduction in the maximum masses for finite temperature when only the linear vector term is included and almost no difference at all in the other cases considered.

At the very end of the chapter we also computed the speed of sound squared and saw that conformal limit ($v_s^2 < 1/3$) is always violated when we use any modified version of the MIT Bag model.

In the next chapters we will further explore the Modified MIT Bag models, but now we will join forces with another popular type of models, the QHD types to describe hadronic matter. In the next chapter we will continue working at finite temperature and build QCD phase diagrams by combining these two types of models.

4 QCD PHASE DIAGRAMS VIA QHD AND MIT BASED MODELS

Towards a more realistic description, in this chapter we want to obtain the QCD phase diagram using two effective models, one to describe each phase. To describe the quark phase we will use the MIT-based models already explored in previous chapters and to describe the hadron phase we will use the quantum hadrodynamics (QHD) model with non-linear terms and $\omega - \rho$ interaction (BOGUTA; BODMER, 1977; FATTOYEV, Farrukh J et al., 2010). In both models the relative strength of the vector channel is fixed by symmetry group arguments, which allows us to build a unified scheme for the strong interaction.

To reproduce reliable results, some restrictions are imposed to the models mentioned above. The MIT based models are used only with parameterizations that don't allow stable u-d matter; the QHD based models, in turn, are restricted to parameterizations that satisfy well known nuclear and astrophysical properties (DUTRA et al., 2014; DUTRA, Mariana; LOURENÇO; MENEZES, Débora P, 2016).

To make the connection between the two models and establish the phase transition regions, we will use the Gibbs conditions, which are the necessary conditions for chemical equilibrium.

First we obtain the phase diagrams for two-flavour symmetric matter, i.e., we take the hadron matter to be constituted only by nucleons with $\mu_n = \mu_p$ and the quark matter constituted only by the quarks u and d with $\mu_u = \mu_d$ and repeat what was done in the previous chapter, i.e., check whether we can get the freeze-out line and the phase transition of the liquid-gas type on the hadronic side of the diagram, as both processes are expected to be purely hadronic.

At $T = 0$ we also use the same restrictions used in the previous chapter, namely, we expect the phase transition to occur at chemical potential values between 1050 MeV and 1400 MeV.

We also obtain the phase diagram for stellar matter, i.e. β -stable and charge neutral matter, but here we compare two different prescriptions that affect the quark phase, one where we impose flavour conservation during the phase transition, so that the quark matter is totally determined from the hadronic matter (BOMBACI; LOGOTETA, 2017) and the other where the quark matter is also β -stable and charge neutral, the so called Maxwell prescription, usually taken to build hybrid stars (VOSKRESENSKY; YASUHIRA; TATSUMI, 2002; PAOLI, M.; MENEZES, 2010). The prescription of flavour conservation has already been used for $T=0$ (BOMBACI; LOGOTETA, 2017; MARQUEZ; MENEZES, Débora P, 2017), but for finite temperatures relevant to protoneutron star evolution, it is something completely new.

It is important to stress here again that, as in the previous chapter, the prescriptions we have just mentioned can only provide first order phase transitions since they

depend on relativistic models within mean field approximations (MFA) and quantum fluctuations are completely disregarded. And we should also remember the limitations imposed by extrapolations of stellar matter conditions to high temperatures. As in the previous chapter, for completeness we analyse the dependence of the results on the models used at both ends of the curve, i.e, high temperatures and vanishing chemical potentials and low temperatures and large chemical potentials, but our results at vanishing chemical potentials serve as a guide only and should be regarded with a grain of salt.

Finally, at the last section, we calculate the latent heat based on two different simple expressions to try to estimate the position (temperature and corresponding chemical potential) of the critical end point (CEP).

The discussions presented in this chapter are strongly based on the paper entitled "QCD Phase Diagrams via QHD and MIT-Based Models" published in the Brazilian Journal of Physics (BIESDORF, Carline; MENEZES, Debora P; LOPES, L. L., 2023) and also on the proceedings paper entitled "QCD phase diagram constructed with effective models" (BIESDORF; LOPES; MENEZES, 2022) resulted from the participation in the the XV International Workshop on Hadron Physics.

4.1 EFFECTIVE MODELS

4.1.1 Hadronic matter - Relativistic Mean Field QHD models

To describe hadronic matter we use the Walecka Model (SEROT, B.; WALECKA, 1986) with non-linear terms (BOGUTA; BODMER, 1977), whose first, linear, version was proposed during the 70s by J. D. Walecka. Afterwards, new non-linear terms were added to better describe nuclear matter. There are several versions of this model in the literature. Here we will consider a version where four types of mesons are included to describe the interactions between baryons: the scalar σ , the isoscalar-vector ω^μ , the isovector-vector $\vec{\rho}^\mu$ and the strange isoscalar-vector φ^μ meson fields. As in (CAVAGNOLI; MENEZES, Debora P; PROVIDENCIA, 2011; TODD-RUTEL; PIEKAREWICZ, 2005; FATTOYEV, F.; PIEKAREWICZ, 2010; FATTOYEV, Farrukh J et al., 2010) we also consider the ω - ρ meson coupling term as it influences the symmetry energy and its slope¹, resulting in Equations of State (EoSs) that can satisfy all important nuclear matter saturation properties and recent observational constraints. The inclusion of the φ meson does not affect the properties of the nuclear matter as it does not couple to the nucleons, but it stiffens the EoSs that include hyperons. The Lagrangian density reads:

¹ Symmetry energy is related to the difference between the binding energy of a symmetric system and the energy of pure neutron matter at the same (saturation) density. The slope tells us how the symmetry energy varies when we vary the density and it is associated with the pressure of a pure neutron matter.

$$\begin{aligned}
\mathcal{L}_{NLWM} = & \sum \bar{\psi}_B [\gamma_\mu (i\partial^\mu - g_{B\omega}\omega^\mu - g_{B\rho}\vec{\tau}_B\vec{\rho}^\mu - g_{B\varphi}\varphi^\mu) - m_B^*] \psi_B + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 \\
& - \frac{1}{3!}\kappa\sigma^3 - \frac{1}{4!}\lambda\sigma^4 + \frac{1}{2}m_\varphi^2\varphi_\mu\varphi^\mu - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4!}\xi g_{N\omega}^4(\omega_\mu\omega^\mu)^2 \\
& - \frac{1}{4}\Phi^{\mu\nu}\Phi_{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu + \Lambda_V g_{N\omega}^2 g_{N\rho}^2 \omega_\mu\omega^\mu \vec{\rho}_\mu\vec{\rho}^\mu, \tag{45}
\end{aligned}$$

where the Dirac spinor ψ_B represents the baryons with the effective mass $m_B^* = m_B - g_{B\sigma}\sigma$, $\vec{\tau}_B$ are the corresponding Pauli matrices, g_{Bi} are the coupling constants of the mesons $i = \sigma, \omega, \rho, \varphi$ with the baryon B , m_i is the mass of the meson i , $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{R}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g_\rho(\vec{\rho}_\mu \times \vec{\rho}_\nu)$ and $\Phi_{\mu\nu} = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu$. κ and λ are scalar self-interacting constants introduced by (BOGUTA; BODMER, 1977), ξ is the coupling of the quartic isoscalar-vector self interacting and Λ_V is the coupling constant of the mixed quartic isovector-vector interaction. The sum extends over the octet of the lightest baryons $\{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ or only over the nucleons, depending on the scenario considered (see section 4.3).

After applying the mean field approximation², the EoSs can be easily obtained from Eq. (45). Using the Euler–Lagrange equation the following mesonic fields are obtained:

$$\sigma_0 = \sum \frac{g_{B\sigma}n_S}{m_\sigma^2} - \frac{\kappa}{2} \frac{\sigma_0^2}{m_\sigma^2} - \frac{\lambda}{6} \frac{\sigma_0^3}{m_\sigma^2}, \tag{46}$$

$$\omega_0 = \sum \frac{g_{B\omega}n_H}{m_\omega^2} - \frac{\xi}{6} \frac{g_\omega^4\omega_0^3}{m_\omega^2} - \frac{2\Lambda_V g_\omega^2 g_\rho^2 \bar{\rho}_{0(3)}^2 \omega_0}{m_\omega^2}, \tag{47}$$

$$\bar{\rho}_{0(3)} = \sum \frac{g_{B\rho}l_3 n_H}{m_\rho^2} - \frac{2\Lambda_V g_\omega^2 g_\rho^2 \omega_0^2 \bar{\rho}_{0(3)}}{m_\rho^2} \tag{48}$$

and

$$\varphi_0 = \sum \frac{g_{B\varphi}n_H}{m_\varphi^2}, \tag{49}$$

where l_3 is the isospin³ of the baryon B and n_S and n_H are the total scalar and baryon densities and given, respectively, by:

$$n_S = \sum \frac{1}{\pi^2} \int dk_B \cdot k_B^2 \frac{m_B^*}{\sqrt{k_B^2 + m_B^{*2}}} \cdot (f_{B+} + f_{B-}) \tag{50}$$

² $\sigma \rightarrow \langle \sigma \rangle = \sigma_0$
 $\omega^\mu \rightarrow \langle \omega^\mu \rangle = \omega_0$
 $\vec{\rho}^\mu \rightarrow \langle \vec{\rho}^\mu \rangle = \bar{\rho}_{0(3)}$
 $\varphi^\mu \rightarrow \langle \varphi^\mu \rangle = \varphi_0$

³ Note that here we use the values $l_3 = \{-1/2, 1/2, 0, -1, 0, 1, -1/2, 1/2\}$ so that l_3 does not appear divided by 2 in the equations above, as often is the case in the literature.

and

$$n_H = \sum n_B = \sum \frac{1}{\pi^2} \int dk_B \cdot k_B^2 \cdot (f_{B+} - f_{B-}). \quad (51)$$

The energy density and pressure are given, respectively, by:

$$\begin{aligned} \mathcal{E} = & \sum \frac{1}{\pi^2} \int dk_B \cdot k_B^2 \sqrt{k_B^2 + m_B^{*2}} \cdot (f_{B+} + f_{B-}) \\ & + \frac{1}{2} (m_\sigma^2 \sigma_0^2 + m_\omega^2 \omega_0^2 + m_\rho^2 \bar{\rho}_{0(3)}^2 + m_\phi^2 \phi_0^2) \\ & + \frac{\kappa}{3!} \sigma_0^3 + \frac{\lambda}{4!} \sigma_0^4 + \frac{\xi}{8} g_\omega^4 \omega_0^4 + 3\Lambda_\nu g_\omega^2 g_\rho^2 \omega_0^2 \bar{\rho}_{0(3)}^2 \end{aligned} \quad (52)$$

$$\begin{aligned} P = & \sum \frac{1}{3\pi^2} \int dk_B \cdot \frac{k_B^4}{\sqrt{k_B^2 + m_B^{*2}}} (f_{B+} + f_{B-}) \\ & - \frac{1}{2} (m_\sigma^2 \sigma_0^2 - m_\omega^2 \omega_0^2 - m_\rho^2 \bar{\rho}_{0(3)}^2 - m_\phi^2 \phi_0^2) \\ & - \frac{\kappa}{3!} \sigma_0^3 - \frac{\lambda}{4!} \sigma_0^4 + \frac{\xi}{4!} g_\omega^4 \omega_0^4 + \Lambda_\nu g_\omega^2 g_\rho^2 \omega_0^2 \bar{\rho}_{0(3)}^2 \end{aligned} \quad (53)$$

where $f_{B\pm}$ are the Fermi-Dirac distribution functions of baryons and anti-baryons, respectively, given by:

$$f_{B\pm} = \frac{1}{1 + \exp[(E_B^* \mp \mu_B^*)/T]}, \quad (54)$$

where $\mu_B^* = \mu_B - g_{B\omega}\omega_0 - g_{B\rho}\bar{\rho}_{0(3)} - g_{B\phi}\phi_0$ and $E_B^* = \sqrt{k_B^2 + m_B^{*2}}$.

4.1.1.1 Parametrizations

As it is well known, it is easy to find in the literature several parametrizations for this model, but not all of them satisfy the constraints imposed by experimental results for nuclear matter, as was shown in (DUTRA et al., 2014). Besides, it is recommendable, as already mentioned in Chapter 2, that the EoSs reproduce neutron star masses that are larger than $2 M_\odot$, as imposed by the NICER results for PSR J0740 + 6620 ($2.08 \pm 0.07 M_\odot$) (MILLER et al., 2021) and the recently detected PSR J0952-0607, with a mass of $2.35 \pm 0.17 M_\odot$ (ROMANI et al., 2022).

In this chapter we choose two parametrizations: eL3 $\omega\rho$ and NL3* $\omega\rho$. The eL3 $\omega\rho$ is the parametrization proposed in (LOPES, L. L., 2021) with a slightly modification to adjust the symmetry energy and its slope accordingly to (ESSICK et al., 2021) and the NL3* $\omega\rho$ is the NL3* parametrization proposed in (LALAZISSIS et al., 2009) with the addition of the $\omega\rho$ -channel as done in (LOPES, L. L.; MENEZES, Debora P, 2022). All of them satisfy the symmetric nuclear matter properties at the saturation density and also reproduce maximum star masses above $2 M_\odot$ even when hyperons are included.

The main parameters of these parametrizations are presented in Table 9 and the main nuclear properties are presented in Table 10.

Model	m_σ	m_ω	m_ρ	$g_{N\sigma}$	$g_{N\omega}$
eL3 $\omega\rho$	512.000	783.000	770.000	9.0286	10.5970
NL3* $\omega\rho$	502.574	782.600	763.000	10.0944	12.8065
-	$g_{N\rho}$	κ	λ	ξ	Λ_ν
eL3 $\omega\rho$	9.4381	$0.008280 \cdot g_\sigma^3$	$-0.023400 \cdot g_\sigma^4$	0.000	0.0283
NL3* $\omega\rho$	14.4410	$0.004417 \cdot g_\sigma^3$	$-0.017422 \cdot g_\sigma^4$	0.000	0.045

Table 9 – Parameter sets for the two models discussed in the text. The meson masses m_σ , m_ω , and m_ρ are all given in MeV. The nucleon and the φ meson masses were fixed at $M = 939$ MeV and $m_\varphi = 1020$ MeV, respectively, in both models.

Model	eL3 $\omega\rho$	NL3* $\omega\rho$	Constraints
n_0 (fm $^{-3}$)	0.156	0.150	0.148 - 0.170
E/A (MeV)	16.2	16.3	15.8 - 16.5
K (MeV)	256	258	220 - 260
M*/M	0.69	0.59	0.6 - 0.8
E_{sym} (MeV)	32.1	30.7	28.6 - 34.4
L (MeV)	66	42	36 - 86.8

Table 10 – Properties at saturation density of the models eL3 $\omega\rho$ and NL3* $\omega\rho$. We present the saturation density (n_0), energy per particle (E/A), compressibility (K), and effective nucleon mass (M^*/M) in symmetric nuclear matter, and also the symmetry energy (E_{sym}) and slope of the symmetry energy (L) at n_0 . The phenomenological constraints are taken from (DUTRA et al., 2014) and (OERTEL et al., 2017).

We consider the hyperon masses to be $m_\Lambda = 1116$ MeV, $m_\Sigma = 1193$ MeV and $m_\Xi = 1318$ MeV. The couplings of the hyperons to the vector mesons are related to the nucleon couplings, $g_{N\omega}$ and $g_{N\rho}$, by assuming SU(6)-flavour symmetry, according to the ratios (DOVER; GAL, 1984; SCHAFFNER; MISHUSTIN, 1996; BANIK; HEMPEL; BANDYOPADHYAY, 2014; TOLOS; CENTELLES; RAMOS, 2017; LOPES, L. L.; MENEZES, Debora P, 2014):

$$\begin{aligned}
g_{\Lambda\omega} : g_{\Sigma\omega} : g_{\Xi\omega} : g_{N\omega} &= \frac{2}{3} : \frac{2}{3} : \frac{1}{3} : 1, \\
g_{\Lambda\rho} : g_{\Sigma\rho} : g_{\Xi\rho} : g_{N\rho} &= 0 : 2 : 1 : 1, \\
g_{\Lambda\varphi} : g_{\Sigma\varphi} : g_{\Xi\varphi} : g_{N\omega} &= -\frac{\sqrt{2}}{3} : -\frac{\sqrt{2}}{3} : -\frac{2\sqrt{2}}{3} : 1,
\end{aligned} \tag{55}$$

noting that $g_{N\varphi} = 0$. The coupling of each hyperon to the σ field is adjusted to reproduce the hyperon potential in strange nuclear matter (SNM) derived from hypernuclear

observables. We fix this potentials as $U_\Lambda(n_0) = -28$ MeV, $U_\Sigma(n_0) = +30$ MeV and $U_\Xi(n_0) = -4$ MeV and obtain the coupling constants presented in Table 11.

Model	$g_{\Lambda\sigma}/g_{N\sigma}$	$g_{\Sigma\sigma}/g_{N\sigma}$	$g_{\Xi\sigma}/g_{N\sigma}$
eL3 $\omega\rho$	0.610	0.406	0.269
NL3* $\omega\rho$	0.613	0.461	0.279

Table 11 – Hyperon- σ coupling constants adjusted to reproduce the hyperon potential in SNM derived from hypernuclear observables.

4.1.2 Quark matter - MIT Bag based models

To describe quark matter we use the MIT Bag models discussed in the previous chapters, but for finite temperature. Therefore, we will work with the equations presented in section 3.1 of last chapter, where we present all the equations for finite temperature. As we are also going to analyze data obtained using Bag pressure value with temperature dependence, we will also use (43) presented in section 3.2. Here, however, we opted not to include the self-interacting vector field. As the addition of this parameter presents nothing new and all its effects have already been extensively discussed in the previous chapters, we chose not to include it in the present chapter so as to avoid over-saturating the discussion and will only mention some of its effects.

Here, as in the previous chapters, we choose the quark masses as being $m_U = m_D = 4$ MeV and $m_S = 95$ MeV. The meson mass m_V is the same as the ω meson mass m_ω considered in the QHD models.

As for the parameterizations, we chose $B^{1/4} = 148$ MeV, which is the smallest value within the stability window that satisfies the Bodmer-Witten conjecture, and $B^{1/4} = 165$ MeV, which is outside the stability window, but which allows us to satisfy other constraints, as will become clear later on. These are also the same values used in Chapter 3 and so we can make a direct comparison with the results of that chapter. When using the temperature-dependent Bag model we just change $B^{1/4}$ to $B_0^{1/4}$ and use $T_0 = 131$ MeV for $B_0^{1/4} = 148$ MeV and $T_0 = 155$ MeV for $B_0^{1/4} = 165$ MeV, also the same parametrizations already used in Chapter 3.

Like we have done in the hadronic phase and also in Chapter 3, for the relation between the coupling constants we opt to use the ones obtained via symmetry group, which allows us to build an unified scheme for the strong interaction. In such approach we have

$$g_{ssV} = \frac{2}{5}g_{uuV} = \frac{2}{5}g_{ddV}.$$

This means that here we will be using $X_V = 0.4$, X_V being defined by Eq. (28). We also define $G_V = (g_{uuV}/m_V)^2$ and choose here $G_V = 0.05$ fm², $G_V = 0.1$ fm² and $G_V = 0.3$ fm².

4.2 THE GIBBS' CONDITIONS

Two states of matter which can exist simultaneously in equilibrium with each other and in contact are described as different phases. The equilibrium conditions can be used to determine when the transition from one phase to another occurs. For the two phases to be in equilibrium it is necessary that, first of all, their temperatures are the same. Also, as the forces exerted by the two phases on each other at their surface of contact must be equal and opposite, the pressures have also to be equal. And, finally, the chemical potentials have to be identical (LANDAU; LIFSHITZ, 2013).

As in our case the two phases considered are the hadronic phase (H) and the quark phase (Q), we can write:

$$\begin{aligned} T^H &= T^Q = T, \\ P^H &= P^Q = P_0 \\ \mu^H(P_0, T) &= \mu^Q(P_0, T) = \mu_0, \end{aligned} \tag{56}$$

These are the necessary conditions for thermodynamic equilibrium of the hadronic and quark phases, also called Gibbs' conditions. The total baryonic chemical potential for the hadron (μ^H) and the quark (μ^Q) phases are given in terms of its individual constituents:

$$\begin{aligned} \mu_B^H &= \frac{(\sum_H n_H \mu_H + \sum_l n_l \mu_l)}{\sum_H n_H}, \\ \mu_B^Q &= \frac{3(\sum_Q n_Q \mu_Q + \sum_l n_l \mu_l)}{\sum_Q n_Q}, \end{aligned} \tag{57}$$

where H , l and Q stand for hadrons, leptons and quarks, respectively. It is also worth emphasizing that due to the model limitations and the prescriptions we use, Eq. (56) gives rise to a first order phase transition, while a more realistic approach should reproduce a smooth crossover at low chemical potentials (AOKI et al., 2006; BELLWIED et al., 2015). In the last section of this chapter, we analyse possible ways to limit the temperature and chemical potentials at which the CEP is reached and thereafter, define the limits of our calculation.

In Fig. 17 we show examples where the Gibbs' conditions are met for various temperatures. In this case we use symmetric matter for both phases, with $\mu_n = \mu_p$ and $\mu_u = \mu_d$, but later on we explore more scenarios. The red dots are the points where the Gibbs conditions are satisfied and, therefore, where there is a first-order phase transition from the hadronic to the quark phase.

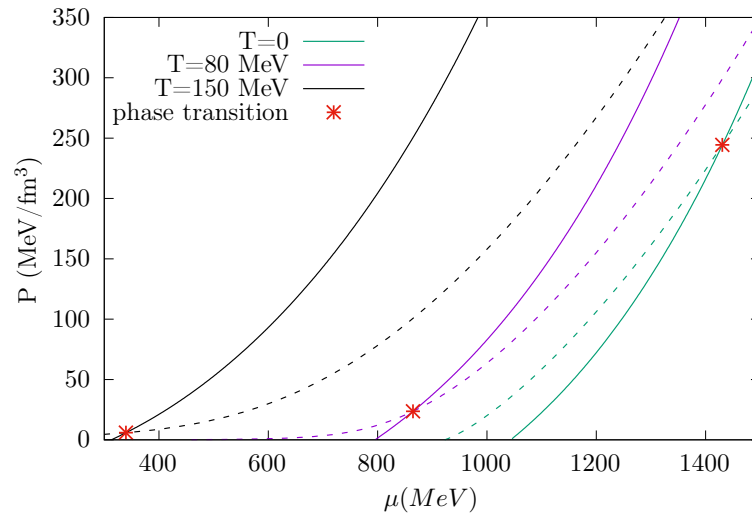


Figure 17 – Relation between pressure and chemical potential for the hadron (dashed lines) and quark (solid lines) phases, respectively, described by the $eL3\omega\rho$ parametrization and $B_0^{1/4} = 165$ MeV considering three different temperatures. Both phases are of symmetric matter. The red dots are the points where the Gibbs' conditions are satisfied.

4.3 DIFFERENT MATTER HYPOTHESES

Here we discuss the conditions that we impose to the hadronic matter before it goes through a first order phase transition and also the sort of conservation that we impose when the transition occurs.

4.3.1 Based on symmetric matter

The first scenario we consider is a two-flavour symmetric matter where the nucleons have equal chemical potentials ($\mu_n = \mu_p$) as well as the quarks ($\mu_u = \mu_d$).

The main reason to consider this scenario is to adjust our parametrizations in order to satisfy the constraints imposed by the chemical freeze-out line (CLEYMANS et al., 2006), which, as being a pure hadronic process, needs to be in the hadronic phase.

4.3.2 Based on stellar matter

As have already been discussed at sections 3.1.3 and 3.3, stable neutron and quark stars are usually described using zero temperature matter, but temperatures up to $T = 50$ MeV can be important in the early moments of the star. For higher temperatures we do not expect β -stable matter (GUPTA, V. et al., 2003; CAVAGNOLI; MENEZES, Debora P, 2008), nevertheless here, as done in the previous chapter, we extend our study beyond the temperature of $T = 50$ MeV for educational purposes.

For a description of neutral and stable stellar matter we include a non-interacting

lepton gas to both the hadronic matter and the quark matter. For that, we add the Lagrangian density as already given by Eq. (30) to the Lagrangian densities for both models used in this chapter.

Beyond that, as neutron stars present internal densities that can be up to 10 times higher than the nuclear saturation density ($n_0 \approx 0.16 \text{ fm}^{-3}$), making the onset of hyperons energetically favorable (MENEZES, Débora Peres, 2021), here we add the six lightest hyperons to the hadron matter so that the sum in Eq. (45) extends over the baryon octet. Despite that, in section 4.4.3 we also study the case where only nucleons are considered.

We also impose β -equilibrium and electric charge neutrality to the hadronic matter:

$$\begin{aligned} \mu_B &= \mu_n - q_B \mu_e \quad \text{and} \quad \mu_e = \mu_{\mu^-}, \\ n_p + n_{\Sigma^+} &= n_{e^-} + n_{\mu^-} + n_{\Sigma^-} + n_{\Xi^-}. \end{aligned} \quad (58)$$

As for the quark matter, we discuss in the following two scenarios that affect its constitution. The first of flavour conservation and the second of the Maxwell construction.

4.3.2.1 Flavour conservation

In this first scenario we impose flavour conservation at the point of the phase transition, so that the quark phase is completely determined from the initial hadronic matter through the bond:

$$y_q = \frac{1}{3} \sum_i n_{qi} y_i, \quad (59)$$

where $i = n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0$, $y_i = n_i/n_B$, being n_i the baryon density of baryon i and n_B the total baryon density, and n_{qi} the number of quarks with flavour q that constitute baryon i (OLESEN; MADSEN, 1994). Because of this bond, the quark matter will not be in β -equilibrium (MARQUEZ; MENEZES, Débora P, 2017; BOMBACI; LOGOTETA, 2017).

This prescription was proposed by (BOMBACI; LOGOTETA, 2017) based on the fact that the phase transition must occur in the strong interaction (which is responsible for the deconfinement phase transition) time scale, which is many orders of magnitude smaller than the weak interaction time scale, leaving, no time, for the β -equilibrium to be reestablished. Soon afterwards a critical size drop of quark matter is formed, the weak interactions will have enough time to act, and the chemical equilibrium will then be reestablished through the Urca process until it reaches the lowest energy state in the form of deconfined β -stable quark matter. In (BOMBACI; LOGOTETA, 2017) and

also in (MARQUEZ; MENEZES, Débora P, 2017) this prescription was applied to study the hadron-quark phase transition in compact stars for matter at $T = 0$. Here we extend this study to finite temperatures.

As for the charge neutrality, which depends on the lepton contribution to the quark phase, we also briefly analyze two sub-scenarios. One where the lepton fraction, defined as $y_l = n_l^H/n_B^H$ in the hadron phase and $y_l = n_l^q/n_B^q$ in the quark phase, where n_l^H and n_l^q are the lepton density, n_B^H and n_B^q are the baryon densities of the hadron phase and the quarks phase, respectively, is also conserved at the point of the phase transition, which means that the quark phase is also charge neutral. And the other where we have the same lepton density, and consequently same lepton energy density and lepton pressure, for both phases at the point of the hadron-quark phase transition. For this last sub-scenario the charge neutrality is not preserved.

4.3.2.2 Maxwell construction

We also consider a second scenario, where we do not impose flavour conservation, but charge neutrality and chemical equilibrium to both phases. So, besides the impositions to the hadron matter given by Eqs. (58), we have the equivalent impositions to the quark matter, this ones already presented in the previous chapters and given by Eqs. (31). This means that, in this scenario, the EoSs for the quark matter are exactly the same as the ones used in Chapter 3 for stellar matter.

This second scenario is the so called Maxwell construction (or prescription), most commonly used to construct the EoSs to describe hybrid stars, as done, for instance, in (LOPES, L. L.; BIESDORF, Carline; MENEZES, Debora P, 2022) and will be explored in more details on the next chapter.

4.4 RESULTS

4.4.1 Symmetric Matter

In Fig. 18 we present the phase diagrams obtained considering two-flavoured symmetric matter for both phases, with $\mu_n = \mu_p$ and $\mu_d = \mu_u$. In each graph, we present three curves, one for the NL3 $\omega\rho$ (blue line) and another for the eL3 $\omega\rho$ (purple line) parametrization in combination with the MIT Bag model and we also present the curve obtained using only the MIT Bag model (red line), i.e., instead of constructing the phase diagram with two models, quark matter pressure is simply set to be zero (as done in the previous chapter). The graphs at the top are obtained using a Bag pressure value (without temperature dependence) of $B^{1/4} = 148$ MeV and for the ones at the bottom, we use $B^{1/4} = 165$ MeV. From left to right we vary the value of the vector channel of the MIT Bag model where we use $G_V = 0.0$, $G_V = 0.05$ fm², and $G_V = 0.1$ fm².

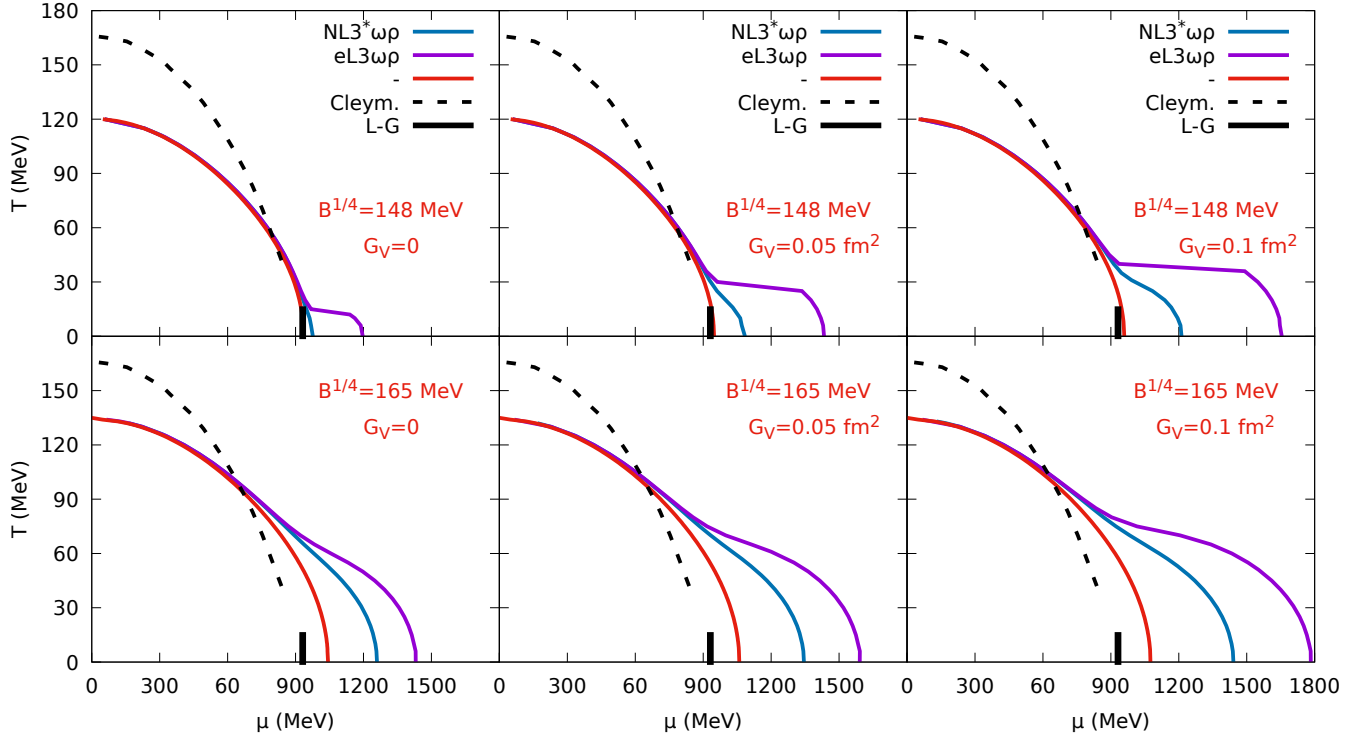


Figure 18 – Phase diagrams for two-flavour symmetric matter considering the $NL3^*\omega\rho$ and $eL3\omega\rho$ parametrizations for the hadronic matter and different MIT Bag based models for two Bag pressure values B for the quark matter. The Cleyman. line is the experimental freeze-out (CLEYMANS et al., 2006) and L-G is the region where we expect a liquid-gas phase transition (FINN et al., 1982). The red lines correspond to the results obtained with the MIT based Bag models only, where the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in Chapter 3.

We first take a look at the region of low chemical potentials. In this case, the maximum critical temperature obtained depends solely on the value of the $B^{1/4}$, i.e., for a fixed $B^{1/4}$ value, different parametrizations for the hadronic phase or even the inclusion of a vector field to the MIT Bag model do not change the value of the maximum temperature (by maximum temperature T_{max} we mean merely the highest temperature where we are able to satisfy the phase transition conditions at $\mu \rightarrow 0$). Hence we draw the same conclusion as in chapter 3: the bigger the $B^{1/4}$ value the higher the maximum temperature. It is also interesting to note that the results at this region coincide even with the ones obtained using only the quark model. That is because the Gibbs' conditions at this region are met at very low pressures ($P_0 \leq 5 \text{ MeV/fm}^3$), as can be seen in Fig. 17, and so, as the condition for phase transition when using only the MIT Bag model is $P = 0$, it comes with no surprise that these results are very much alike.

As a consequence, however, we are, once again, not able to satisfy the restraints at this region, i.e., we are not able to obtain maximum temperatures that satisfy the constraints imposed by LQCD (BAZAVOV et al., 2017) nor the freeze-out results (CLEYMANS et al., 2006) and so we are not able to fit the Cleymans line entirely inside the

confined (hadron) phase. We will come back to that later. Now let's first analyze the region of high chemical potentials, i.e., $T \rightarrow 0$.

At the high chemical potential region we see that the results are different for each combination parametrizations. Let's first analyze the effect of the different MIT Bag based model parametrizations. The most evident effect is that a higher Bag pressure value results in a higher chemical potential. This follows what happens when using only the MIT model, a case easy to understand: in the equation for the pressure (40), the value of the Bag subtracts from the value of the pressure of the quarks (likewise, the meson ω when $G_V \neq 0$) and so, the higher the Bag value the higher must the contribution for the pressure coming from the quarks be in order to meet the condition for phase transition ($P = 0$) and, consequently, the chemical potential is also higher. When we use two models the increase in the chemical potential is even greater because the Gibbs' conditions are met at pressures higher than zero ($P_0 > 0$) so that the pressure realized by the quarks and ω meson is even higher, which also increases the chemical potential at the transition point μ_0 .

The inclusion and increase of the G_V value also follows what happens when we use only one model, the higher its value, the higher the chemical potential. As already stated in chapter 3, the vector field causes an additional repulsion between the quarks, increasing the chemical potential of the phase transition. And again, as $P_0 > 0$, the effect when using two models is increased, so much so that here we use smaller values for G_V than used in the previous chapter.

As for the influence of the different parametrizations of the QHD based model, we can see that the eL3 $\omega\rho$ parametrization always results in higher chemical potentials, and, as the NL3* $\omega\rho$ is a stiffer EoS, we conclude that a stiffer hadron EoS results in a lower chemical potential.

The behavior of the diagrams at low temperatures is not so smooth, as we can observe an abrupt increase of the chemical potential, especially when using a Bag pressure value of 148 MeV. This happens because the lower the temperature, the smaller the change in the graphs of $\mu \times P$ for different temperatures. And more so, at lower temperatures the curves for the hadron and the quark phases are very close to each other. As a result, a small change in the temperature makes the crossing point of the lines take a big leap resulting in an abrupt change in the chemical potential on the phase diagram.

In Table 12 we present the values of the maximum chemical potential obtained for each combination of parametrizations used in Fig. 18. By maximum chemical potential μ_{max} we mean the chemical potential where a phase transition at $T = 0$ takes place. Only a few combinations of parametrizations result in a chemical potential within the range $1050 \leq \mu \leq 1400$ MeV, namely: NL3* $\omega\rho$ in combination with $B^{1/4} = 148$ MeV + $G_V = 0.05$ fm² and $G_V = 0.1$ fm² and $B^{1/4} = 165$ MeV + $G_V = 0.0$ and $G_V = 0.05$ fm²

and $eL3\omega\rho$ in combination with $B^{1/4} = 148$ MeV. When using only the MIT based model the results for $B^{1/4} = 165$ MeV + $G_V = 0.05$ fm² and $G_V = 0.1$ fm² are also within this range.

Models	$B^{1/4} = 148$ MeV	$B^{1/4} = 148$ MeV + $G_V = 0.05$ fm ²	$B^{1/4} = 148$ MeV + $G_V = 0.1$ fm ²
-	$\mu_{max} = 936$	$\mu_{max} = 948$	$\mu_{max} = 959$
NL3* $\omega\rho$	$\mu_{max} = 976$	$\mu_{max} = 1084$	$\mu_{max} = 1212$
eL3 $\omega\rho$	$\mu_{max} = 1197$	$\mu_{max} = 1434$	$\mu_{max} = 1655$
Models	$B^{1/4} = 165$ MeV	$B^{1/4} = 165$ MeV + $G_V = 0.05$ fm ²	$B^{1/4} = 165$ MeV + $G_V = 0.1$ fm ²
-	$\mu_{max} = 1043$	$\mu_{max} = 1060$	$\mu_{max} = 1075$
NL3* $\omega\rho$	$\mu_{max} = 1260$	$\mu_{max} = 1345$	$\mu_{max} = 1441$
eL3 $\omega\rho$	$\mu_{max} = 1431$	$\mu_{max} = 1592$	$\mu_{max} = 1783$

Table 12 – Chemical potentials (μ_{max}) at $T = 0$ for all combinations of parametrizations considering **symmetric** matter. The first line corresponds to the results obtained with the MIT based Bag models only, where the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in chapter 3. All values are given in MeV.

Summarizing the results for symmetric matter so far: for high temperatures (and low chemical potential) none of the parametrizations chosen are able to fulfill the restraints and we need higher T_{max} s; but at low temperatures (and high chemical potentials) many of the parametrizations satisfy the imposed restraints. We could have also explored higher values of the Bag constant, as done in the previous chapter where we used also $B^{1/4} = 205$ MeV, but the conclusions here would be more or less the same, i.e., at high temperatures the restraints are satisfied, but at low temperatures, the chemical potentials would be too high (even when using the simple Bag model (without any interaction) and the NL3* $\omega\rho$ parametrizations, which produces the smallest chemical potentials, we would already have $\mu_{max} = 1563$ MeV). So, in order to maintain the results for low chemical potentials but get better results for high temperatures, we again make use of the temperature-dependent Bag model and substitute B by $B(T)$ given by Eq. (43). In Fig. 19 we present those results.

As can be seen in Fig. 19, all curves converge more or less to the same point at $\mu \rightarrow 0$, so that the maximum temperatures obtained are above the freeze-out line. This happens because we choose the values of T_0 exactly to satisfy the restraint imposed by the LQCD and freeze-out results, which states that the temperatures at very low chemical potentials should be around $T = 168$ MeV. Because $B(T)^{1/4}$ has more or less the same value, namely, around 205 MeV, at the maximum temperature for both parametrizations, it still holds that the maximum temperature obtained depends only on the Bag pressure value.

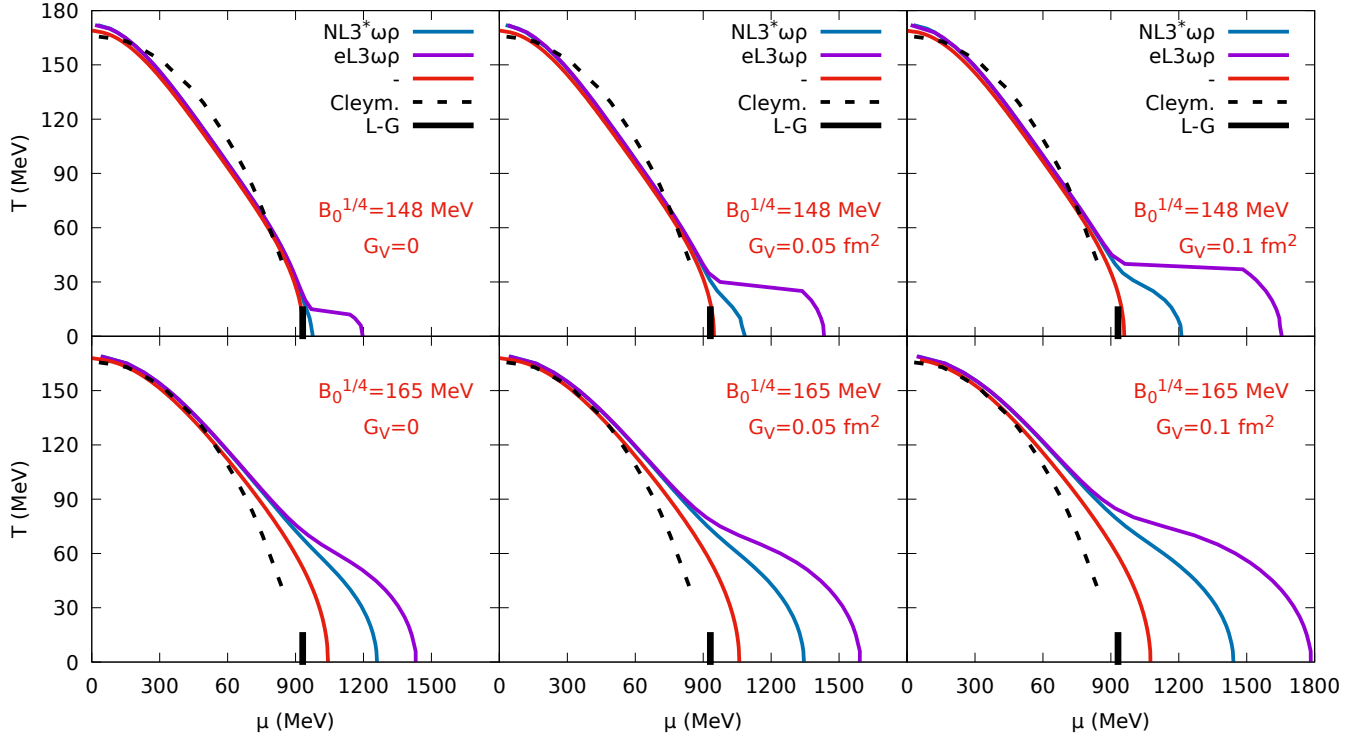


Figure 19 – Phase diagrams for two-flavour symmetric matter considering the NL3* $\omega\rho$ and eL3 $\omega\rho$ parametrizations for the hadronic matter and different MIT Bag based models for two temperature dependent Bag $B(T)$ values for the quark matter. The Cleym. line is the experimental freeze-out (CLEYMANS et al., 2006) and L-G is the region where we expect a liquid-gas phase transition (FINN et al., 1982). The red lines correspond to the results obtained with the MIT based Bag models only, where the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in Chapter 3.

The results and conclusions for the region of low temperatures are the same drawn from Fig. 18 and so the table 12 also holds for when we use $B(T)$.

As we chose T_0 in order to obtain higher temperatures at low chemical potentials, we are able to obtain a maximum temperature around $T = 168$ MeV in all combinations of parametrizations, but we are only able to fit the freeze-out line entirely inside the confined (hadron) phase for the combinations that include a $B_0^{1/4} = 165$ MeV and a QHD based model. This means that we were able to obtain only two combinations of parametrizations that satisfy both constraints for low and high temperatures: NL3* $\omega\rho$ + $B_0^{1/4} = 165$ MeV and NL3* $\omega\rho$ + $B_0^{1/4} = 165$ MeV + $G_V = 0.05$ fm².

As we are only able to fulfil the restraints at high temperatures when we use the temperature-dependent Bag model, from now on we will present only the results where this model $B(T)$ is used.

4.4.2 Stellar Matter

Here we present the results for charge neutral matter in β -equilibrium for two scenarios, one considering flavour conservation, which results in quark matter that is not in β -equilibrium, and another with no flavour conservation where both phases are in β -equilibrium, a typical Maxwell prescription used to construct hybrid stars, as we will explore in the next chapter. For the hadron phase we allow the presence of the whole baryon octet and for the quark phase the quarks u , d and s .

4.4.2.1 Sub-scenario within the scenario of flavour conservation

We first analyze the results for a sub-scenario within the scenario of flavour conservation: different prescriptions for the lepton matter during the phase transition are investigated. In the first one, the lepton fraction, defined as the fraction between lepton density and the baryon density of each phase is preserved at the point of the phase transition. In the second one, the lepton density is taken to be the same, and consequently, the same lepton energy density and lepton pressure are obtained, for both phases at the point of the hadron-quark phase transition. In Fig. 20 we compare the two prescriptions.

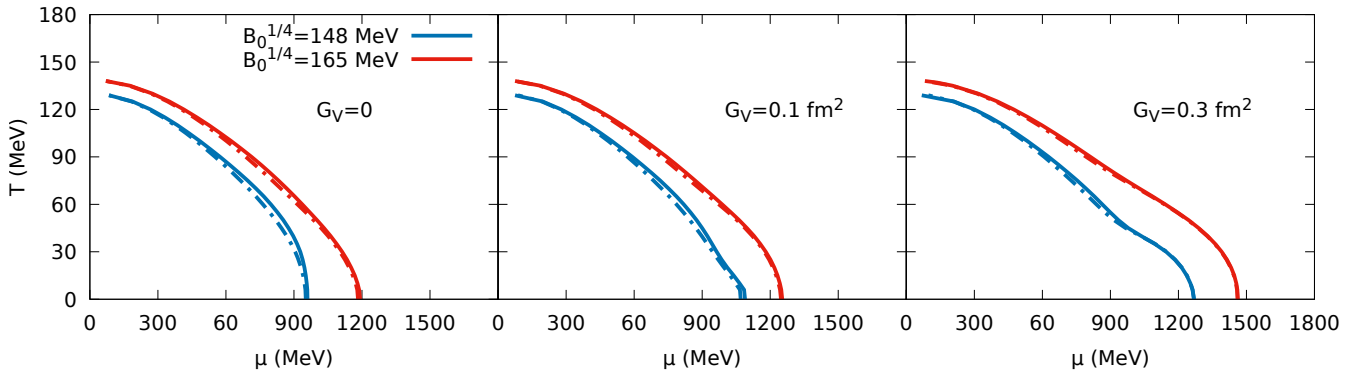


Figure 20 – Comparison between two different prescriptions for the lepton matter in the quark phase for strange stellar matter considering flavour conservation. Here we present the results for the NL3* $\omega\rho$ and different parametrizations for the MIT-based model. The solid lines represent the prescription of lepton fraction conservation and the dashed lines represent the prescription of lepton density conservation.

As the different prescriptions only affect the quark phase, we choose here to present the results from only one of the QHD-based model, namely, the NL3* $\omega\rho$, and vary the MIT-based model parametrizations. As can be seen, the differences are very small. They are greater for the Bag pressure value of $B_0^{1/4} = 148$ MeV (blue lines) than for the $B_0^{1/4} = 165$ MeV (red lines) and also decrease with the increase of the G_V constant. But for all parameterizations, the prescription considering the lepton fraction equal at both phases at the transition point (solid lines) is the one where the chemical

potentials are always slightly higher. That happens because of the combination of the imposition of flavour conservation and the Gibbs' conditions as criteria for the phase transition, which has as a consequence an increase of the baryon density at the point of the phase transition. Hence, in the sub-scenario where we impose the lepton fraction conservation this means also an increase, in the same proportion, of the lepton density, which, in turn, means an increase in the chemical potential. As the contribution to the chemical potential from the leptons is very small, these differences are also very small.

From this sub-scenario from now on we choose to use only the one where there is a conservation of the lepton fraction at the point of the phase transition because, in this case, the charge neutrality is also preserved. This approach is contrary to what was done in (PELICER; MENEZES, Débora P, 2022). As there is an increase of the baryon density at the phase transition there may be a decrease in the volume occupied by the quark phase in comparison to the one occupied by the hadron phase and, as the leptons accompany the quarks to this new phase, they may also be in this smaller volume. To summarize, in this sub-scenario that we choose, at the quark phase we have the same quarks that were (confined) in the hadron phase and also the same leptons, changing only the volume from one phase to another, and, as a consequence, we also have charge neutrality preserved.

4.4.2.2 Flavour conservation and Maxwell prescription

In the sequence we present in Fig. 21 the phase diagrams for stellar matter and compare the results considering flavour conservation (top) and the Maxwell prescription (bottom). The blue lines are used for the NL3* $\omega\rho$ parametrization, the purple ones are for the eL3 $\omega\rho$ parametrization, and the red ones for the MIT-based models only (in this case, the quark matter is always β -stable). In addition, the solid lines stand for the results using a Bag pressure value $B_0^{1/4} = 148$ MeV and the dashed lines for $B_0^{1/4} = 165$ MeV. The value of G_V increases from 0 to 0.3 fm^2 from left to right.

As expected, the general behavior is the same as observed in Fig. 19 for symmetric matter. For both prescriptions, only the Bag pressure value, as far as parametrizations are concerned, dictates the maximum temperature obtained for $\mu \rightarrow 0$. At low temperatures, the maximum chemical potential depends largely on the Bag pressure value, but also on the inclusion or not of the vector field to the Bag model and the parametrization chosen for the QHD model.

As done in the subsection 4.4.1, we analyse first the region of low chemical potentials. At this region we have $T_{max} = 129$ MeV for $B_0^{1/4} = 148$ MeV and $T_{max} = 138$ MeV for $B_0^{1/4} = 165$ MeV no matter the QHD parametrization, if we add the vector channel to the MIT model or not or even if we use the prescription of Maxwell or of flavour conservation. This is different from what happened in Fig. 19, where the results for T_{max} are more or less the same, i.e. around $T_{max} = 168$ MeV, no matter the value

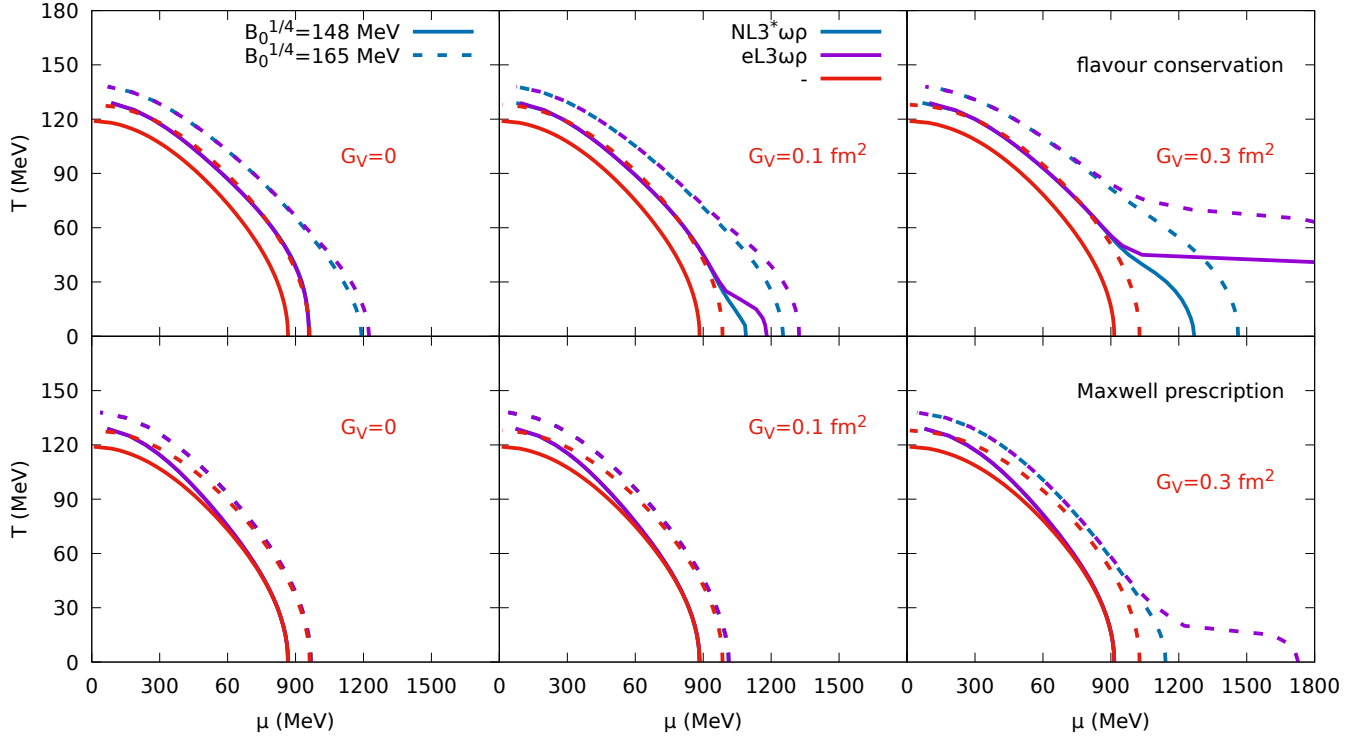


Figure 21 – Phase diagrams for strange stellar matter considering the NL3* $\omega\rho$ and eL3 $\omega\rho$ parametrizations for the hadronic matter and different MIT Bag based models for two temperature dependent Bag $B(T)$ values for the quark matter. At the top we use the prescription of the flavour conservation and at the bottom the of Maxwell.

of B_0 , but that is because we used the symmetric matter to choose the values of T_0 in Eq. (43) exactly so that the maximum temperatures were around this value. Here, as we are not able to reach the same T_{max} , the maximum values of $B(T)$ are different for each value of B_0 , which, in turn, means that the maximum temperatures here are also different and depend on the value of B_0 . Also differently from what happened with the symmetric matter, the results for T_{max} do not match the results obtained via the MIT-based model alone. This is so, for both prescriptions, because the crossing of the curves $\mu \times P$ occurs at pressures that are not so close to zero as in the symmetric matter case. At the same time, these pressures are low enough so that they fall in a region where the hadronic EoSs for stellar matter are stiffer than the ones for symmetric matter, and so we have again that a softer hadronic EoS favors the hadron phase.

When using a Bag value with no dependence on the temperature we obtain the maximum temperatures $T_{max} = 109$ MeV for $B^{1/4} = 148$ MeV and $T_{max} = 122$ MeV for $B^{1/4} = 165$ MeV.

Now we take a better look at the region of low temperatures. Here, as for the symmetric matter, the results are different for each combination of parametrizations and they also change depending on the prescription used, either flavour conservation or Maxwell construction. The maximum chemical potentials obtained for each combination

of parametrizations are presented in Table 13.

Models	NL3* $\omega\rho$	eL3 $\omega\rho$	-
flavour conservation			
$B^{1/4} = 148 \text{ MeV}$	$\mu_{max} = 960$	$\mu_{max} = 959$	$\mu_{max} = 867$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$\mu_{max} = 1089$	$\mu_{max} = 1181$	$\mu_{max} = 884$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$\mu_{max} = 1268$	no crossing	$\mu_{max} = 915$
$B^{1/4} = 165 \text{ MeV}$	$\mu_{max} = 1192$	$\mu_{max} = 1225$	$\mu_{max} = 962$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$\mu_{max} = 1253$	$\mu_{max} = 1324$	$\mu_{max} = 986$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$\mu_{max} = 1462$	no crossing	$\mu_{max} = 1027$
Maxwell prescription			
$B^{1/4} = 148 \text{ MeV}$	$\mu_{max} = 867$	$\mu_{max} = 867$	$\mu_{max} = 867$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$\mu_{max} = 884$	$\mu_{max} = 884$	$\mu_{max} = 884$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$\mu_{max} = 915$	$\mu_{max} = 915$	$\mu_{max} = 915$
$B^{1/4} = 165 \text{ MeV}$	$\mu_{max} = 967$	$\mu_{max} = 967$	$\mu_{max} = 962$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$\mu_{max} = 1012$	$\mu_{max} = 1013$	$\mu_{max} = 986$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$\mu_{max} = 1142$	$\mu_{max} = 1727$	$\mu_{max} = 1027$

Table 13 – Chemical potentials (μ_{max}) at $T = 0$ for all combinations of parametrizations considering strange **stellar** matter. The last column corresponds to the results using the MIT based Bag models only, where the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in chapter 3. All values are given in MeV.

First we will analyze the effect of the different MIT Bag based model parametrizations at this region of low temperatures. We notice that the effect of G_V here, in stellar matter, causes smaller changes in the μ_{max} , which allows us to increase the values up to 0.3 fm^2 . Nevertheless the qualitative results are the same: the higher the values for G_V the higher are the results for μ_{max} . We can still compare the results between symmetric matter and stellar matter when using the same parametrizations, namely, when $G_V = 0$ and $G_V = 0.1 \text{ fm}^2$. In doing so we see very clearly that there is a reduction in the values of chemical potentials at $T = 0$ here, being this reduction even greater when we use the Maxwell prescription. This reduction also grows with the value of G_V and $B_0^{1/4}$ and is also greater when using the eL3 $\omega\rho$ than when using the NL3* $\omega\rho$ parametrization.

Being that the quark EoSs for symmetric matter are always stiffer than the ones for stellar matter and that the ones for stellar matter resulting from the flavour conservation prescription are stiffer than the β -stable and charge neutral ones, in general, stiffer quark EoSs favor the hadron phase. The same is true when we compare the results for different values of G_V , as higher values of G_V result in higher values for μ_{max} and also increase the stiffness of the EoS, the same behaviour found in symmetric matter. The increase of the Bag pressure value $B_0^{1/4}$ also results in higher values of μ_{max} , but

it softens the EoS. So, what actually happens is that, stiffer quark EoSs always favor the hadron phase, unless we change the value of the Bag constant $B_0^{1/4}$ and then the opposite is true, a softer quark EoS favors the hadron phase.

An interesting aspect to analyse when comparing the results at low temperatures from the two different prescriptions is their behavior in relation to the results obtained via MIT Bag based model alone. As it is very clear from Fig. 21 and Table 13, when we use the prescription of flavour conservation the results are quite different, whereas when using the Maxwell prescription the diagrams generally converge to the same μ_{max} s. First we have to point out that the diagrams obtained via MIT Bag based model alone (red lines) presented at the top and at the bottom of Fig. 21 are the same, i.e., of β -stable and charge neutral quark and lepton matter. Now, when we use a QHD+MIT models, with the flavour conservation the hadronic phase completely determines the quark phase through the bond given by Eq. (59). The quark matter at the phase transition point is not yet β -stable, and, adding to that, in this case, we have P_0 s that are not so close to zero, so, it is not surprising that the results for this prescription are different from the ones obtain only with the MIT based model. When using the Maxwell prescription, however, we have a quark matter at the phase transition point that is β -stable, and, when the the Gibbs' conditions at this region are met at very low pressures ($P_0 \leq 5 \text{ MeV/fm}^3$), we get μ_{max} s very similar to the ones obtained via MIT alone. When the P_0 s start to get higher, also the μ_{max} s grow.

As for the effects of the different QHD model parametrizations at this region of low temperatures, we can see that they are not so preponderant here as for the symmetric matter and are the smallest when using the Maxwell prescription. But, whenever there is a difference, the μ_{max} s for the eL3 $\omega\rho$ are the highest and in some cases, we were not able to find crossings at $T = 0$, as for example, the case with $G_V = 0.3 \text{ fm}^2$ and this QHD parametrization.

To finalize the analysis of the results in this region of low temperatures we point out that we obtain results for μ_{max} within a range of 100 MeV from the $\mu = 1200 \text{ MeV}$, as indicated in (AYRIYAN et al., 2018) for β -stable matter, for various combinations of parametrizations when using the prescription of flavour conservation, namely, for NL3* $\omega\rho$ in combination with $B_0^{1/4} = 148 + G_V = 0.3 \text{ fm}^2$, $B_0^{1/4} = 165 + G_V = 0$ and $B_0^{1/4} = 165 + G_V = 0.1 \text{ fm}^2$ and for eL3 $\omega\rho$ in combination with $B_0^{1/4} = 148 + G_V = 0.1 \text{ fm}^2$ and $B_0^{1/4} = 165 + G_V = 0$, but only for one combination of parametrizations when using the Maxwell prescription, namely, for NL3* $\omega\rho$ in combination with $B_0^{1/4} = 165 + G_V = 0.3 \text{ fm}^2$. So, considering the MIT parametrizations, only two, the ones with $B_0^{1/4} = 148$, are within the stability window and are able to describe stable strange matter, as we saw in chapter 2. However, in order to construct hybrid stars, parametrizations outside the stability window are preferable to avoid that once the phase transition starts, the entire star converts to a strange quark star. In the next chapter we will construct

hybrid stars using the original $L3\omega\rho$ parametrization (a slight different parametrization than the one used here) for the QHD model and the MIT based model.

We could improve the results at this region by adding also a quartic term to the MIT vector model, as done in chapters 2 and 3. Using the very small value of $b_4 = 2$ we are able to obtain crossings at $T=0$ for the two cases pointed out in table 13 where we did not obtain results without this term, although the values for the μ_{max} , and also the pressure, would still be very high. Choosing higher values for b_4 we can adjust the values of μ_{max} without altering the values of T_{max} and, the higher the value of G_V the higher is the affect of this term, as already shown in 3.

4.4.3 Testing different hypotheses

In this section we take a better look at the differences occurring due to the different matter hypothesis and prescriptions imposed along this chapter. To do so we choose only two different combinations of parametrizations and plot all results in Fig. 22. We also include the results for stellar matter with no strange matter, i.e., with no hyperons and no strange quarks (dashed lines).

As already pointed out, the maximum temperature for strange stellar matter depends only on the Bag pressure value and it is the same for both prescriptions, the one with flavour conservation (solid red line) and the one with Maxwell construction (solid blue line). It is also more evident here that at low temperatures the prescription of flavour conservation favors the hadronic phase. What is completely new here, though, are the diagrams for stellar matter with no hyperons and no strange quarks. We chose to present those results for only these two combinations of parametrizations because they are enough to point out the differences between the different matter hypotheses. The effect of different parametrizations was already stressed above.

As expected, the results for the diagrams for stellar matter without strangeness are very close to the ones for two-flavour symmetric matter. This is due to the fact that the contribution of the leptons to the chemical potential, only present in the stellar matter, is very small, leaving only the contributions from the nucleons, which is the same for both matter hypotheses, as we consider the same mass for both nucleons.

At very low temperatures we have obtained an interesting feature for the flavour conservation prescription. For the $NL3^*\omega\rho + B_0^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ parametrization, the phase transition for the case where the strange matter is allowed to appear occurs at chemical potentials where the hyperon onset has not yet taken place. Hence, the results for stellar matter with (solid red line) and without (dashed red lines) strange matter are the same. At higher temperatures, where the hyperons are already present at the point of the phase transition, the two lines start to differentiate and the line without strange matter approximates the line of two-flavour symmetric matter. For the other parametrization, i.e., $NL3^*\omega\rho + B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$, however, the phase

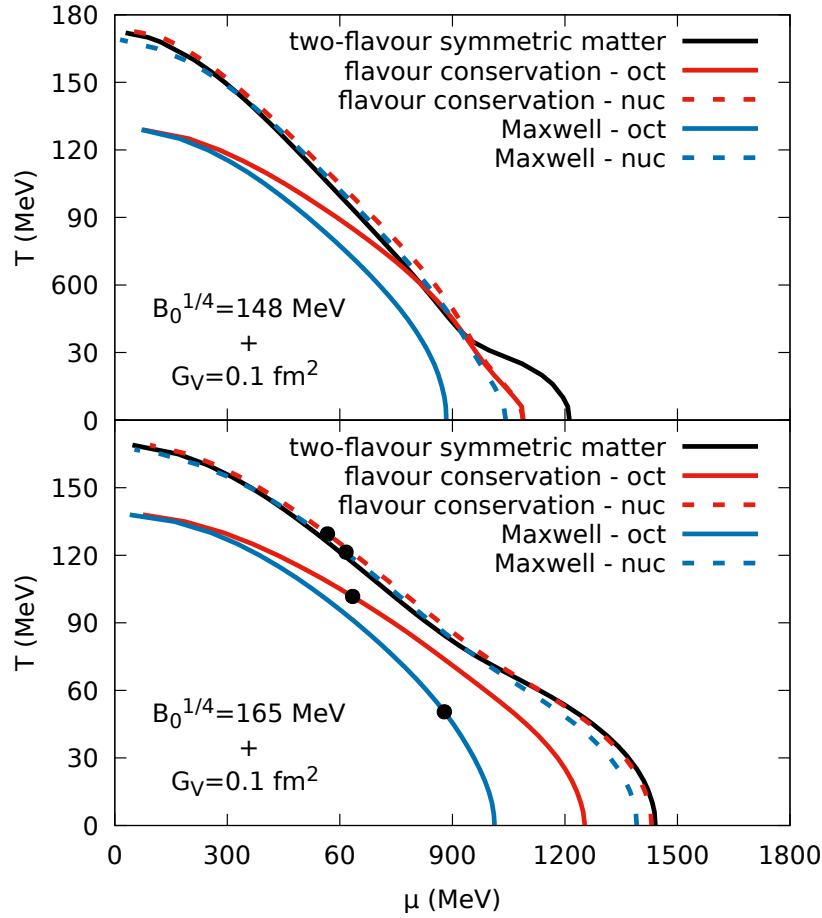


Figure 22 – Phase diagrams for all different matter hypothesis and prescriptions for phase transition used in this chapter considering the $NL3^*\omega\rho$ parametrization for the hadronic phase and $B_0^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ (top) and $B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ (bottom) for the quark phase. We also include the results for stellar matter with no strange matter (dashed lines). The points presented on the bottom diagrams correspond to the ones where the latent heat $L|_S$ goes to zero (see section 4.5) and may correspond to the critical end points.

transition for strange matter occurs at chemical potentials where there are already hyperons present and so, the results for stellar matter with and without strange matter are different throughout the entire phase diagram. As for the Maxwell prescription, despite the phase transition for strange matter at low temperatures for both parametrizations occurs at chemical potentials where there are no hyperons, the results for strange matter (solid blue lines) and no strange matter (dashed blue lines) are always very different. This happens because for strange quark stellar matter the strange quarks are always present.

In Table 14 we present the results for the maximum chemical potentials at the phase transition for stellar matter without strange matter. We also include the results

obtained with the MIT-based model only and, as can be noted, its results are only similar to the ones that combine models with $B_0^{1/4} = 148$ MeV and $G_V = 0$. In these cases $P_0 \rightarrow 0$ and for all the other parametrizations the phase transition happens at increasing pressures. Concerning the results for stellar matter without strange matter using only the MIT-based model, its T_{max} s reach around only $T=140$ MeV, i.e., very below the ones obtained for the same matter but combining models. That happens because, when combining models, the crossing of the $\mu \times P$ occurs at pressures considerably high, and higher than for strange matter.

Models	NL3* $\omega\rho$	eL3 $\omega\rho$	-
flavour conservation			
$B_0^{1/4} = 148$ MeV	$\mu_{max} = 960$	$\mu_{max} = 959$	$\mu_{max} = 953$
$B_0^{1/4} = 148$ MeV + $G_V = 0.1$ fm ²	$\mu_{max} = 1089$	$\mu_{max} = 1568$	$\mu_{max} = 976$
$B_0^{1/4} = 148$ MeV + $G_V = 0.3$ fm ²	$\mu_{max} = 1750$	no crossing	$\mu_{max} = 1017$
$B_0^{1/4} = 165$ MeV	$\mu_{max} = 1259$	$\mu_{max} = 1405$	$\mu_{max} = 1063$
$B_0^{1/4} = 165$ MeV + $G_V = 0.1$ fm ²	$\mu_{max} = 1431$	$\mu_{max} = 1751$	$\mu_{max} = 1094$
$B_0^{1/4} = 165$ MeV + $G_V = 0.3$ fm ²	$\mu_{max} = 1947$	no crossing	$\mu_{max} = 1149$
Maxwell prescription			
$B_0^{1/4} = 148$ MeV	$\mu_{max} = 955$	$\mu_{max} = 957$	$\mu_{max} = 953$
$B_0^{1/4} = 148$ MeV + $G_V = 0.1$ fm ²	$\mu_{max} = 1042$	$\mu_{max} = 1450$	$\mu_{max} = 976$
$B_0^{1/4} = 148$ MeV + $G_V = 0.3$ fm ²	$\mu_{max} = 1671$	no crossing	$\mu_{max} = 1017$
$B_0^{1/4} = 165$ MeV	$\mu_{max} = 1229$	$\mu_{max} = 1350$	$\mu_{max} = 1063$
$B_0^{1/4} = 165$ MeV + $G_V = 0.1$ fm ²	$\mu_{max} = 1391$	$\mu_{max} = 1681$	$\mu_{max} = 1094$
$B_0^{1/4} = 165$ MeV + $G_V = 0.3$ fm ²	$\mu_{max} = 1886$	no crossing	$\mu_{max} = 1149$

Table 14 – Chemical potentials (μ_{max}) at $T = 0$ for all combinations of parametrizations considering stellar matter without strange quarks. All values are given in MeV.

4.5 LATENT HEAT AND LATENT ENERGY

As we are considering first order phase transitions, we can also study the latent heat associated to those transitions. This quantity can be understood as the hidden energy which is supplied or extracted to change the state of a substance without changing neither its temperature nor its pressure. Hence, the latent heat is an important quantity in the study of phase transitions and we investigate two different expressions found in the literature.

We first take a look at the latent heat $L|_\varepsilon$, as given in (AGASIAN; FEDOROV, 2008; LOPE-OTER; LLANES, et al., 2022) by equation:

$$L|_\varepsilon = P H \frac{\varepsilon^Q - \varepsilon^H}{\varepsilon^Q \varepsilon^H}, \quad (60)$$

where ε^Q and ε^H are the energy densities at the point of the phase transition for the quark and hadronic matter, respectively. We call it latent energy next, since it yields results that are not zero at zero temperature. Notice that this quantity has no dimension. According to (AGASIAN; FEDOROV, 2008), when the latent energy becomes zero, the critical endpoint, where the first-order phase transition turns into a crossover, is reached. Notice in Fig. 23 that this never happens for eq. (60).

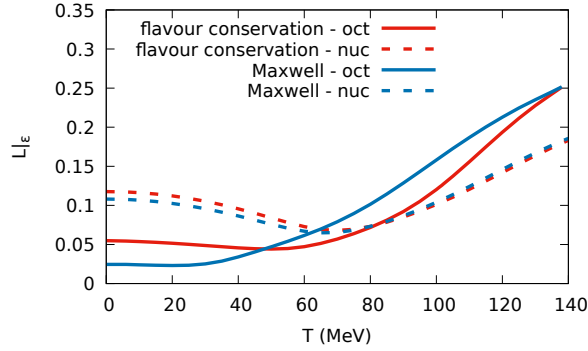


Figure 23 – Results for latent energy for different matter hypothesis and prescriptions for phase transition used in this work considering the NL3* $\omega\rho$ parametrization for the hadronic phase and $B_0^{1/4} = 165$ MeV + $G_V = 0.1$ fm² for the quark phase.

Additionally, we analyse the usual expression found in textbooks and investigated in (ROARK et al., 2019) as well:

$$L|_S = (S^Q - S^H) T, \quad (61)$$

where S^Q and S^H are the entropy densities, defined in Eq. (62), at the point of the phase transition for the quark and hadronic matter, respectively.

$$S = -\gamma \sum_i \int \frac{d^3k}{(2\pi)^3} \left[f_{i+} \ln \left(\frac{f_{i+}}{1 - f_{i+}} \right) + \ln(1 - f_{i+}) + f_{i-} \ln \left(\frac{f_{i-}}{1 - f_{i-}} \right) + \ln(1 - f_{i-}) \right], \quad (62)$$

where γ is degeneracy of spin (and color in the case of the quarks) and $f_{i\pm}$ are the Fermi-Dirac distribution functions of the particles and anti-particles, respectively.

In Figs. 23 and 24 we present the results obtained with the latent energy and latent heat, respectively, as a function of the temperature at the point of the hadron-quark phase transition. We choose only one parametrization since they yield qualitatively similar results. One can see that our results for the latent energy at zero temperature are of the same order of magnitude as the ones obtained in (PELICER; MENEZES, Débora P, 2022), but never reach the values shown in (LOPE-OTER; LLANES, et al.,

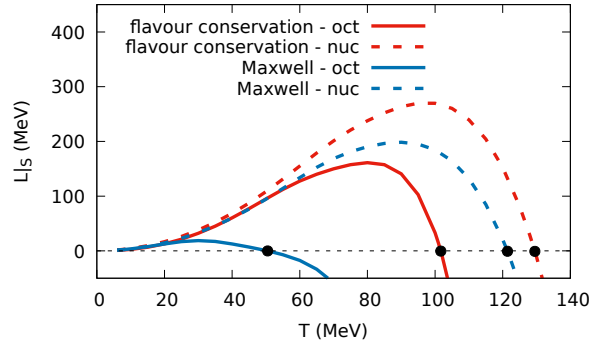


Figure 24 – Results for latent heat for different matter hypothesis and prescriptions for phase transition used in this work considering the $NL3^*\omega\rho$ parametrization for the hadronic phase and $B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ for the quark phase.

2022) and are always positive. As far as the latent heat is concerned, the behaviour we find is more similar to the result shown in (ROARK et al., 2019) and we do find a maximum point, but at much lower temperatures. Our curves then cross the zero value, which, according to (AGASIAN; FEDOROV, 2008) is an indication of the end of the first-order phase transition. Assuming this interpretation is correct, our curves depicted in the QCD phase diagrams are only valid until the temperatures where $L|_S$ is zero. The critical points shown in Fig. 24 correspond to the values shown in Fig.22 (bottom).

In table 15 we also show all the critical values of temperature and chemical potential corresponding to the points where $L|_S = 0$ for all parametrizations and matter hypothesis assumed in this chapter for the stellar matter. We do not include results for symmetric matter as in this case the latent heat is always positive and increases with the temperature. Note that this also happens for some cases of stellar matter, which in the table are shown with $L|_S > 0$. In some cases, for stellar matter, we also have the situation where the latent heat is always negative, which we indicate with $L|_S < 0$ in the table 15. If (AGASIAN; FEDOROV, 2008) is correct, then all phase diagrams to which we get $L|_S < 0$ yield invalid results and the opposite for $L|_S > 0$, in this case all results should be valid, but without indication of a CEP, which, in turn, contradicts the LQCD results.

In (BORSANYI et al., 2020; DING et al., 2019) the authors use more sophisticated methods to determine the location of the CEP and argue that it should be located at $\mu_B \gtrsim 300 \text{ MeV}$ and $T_c \lesssim 132 \text{ MeV}$. Observing the results in table 15, we see that only one result, obtained with the parametrization $eL3\omega\rho + B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ for flavour conservation without strange matter fails to satisfy this hypothesis.

Models	NL3* $\omega\rho$	NL3* $\omega\rho$	eL3 $\omega\rho$	eL3 $\omega\rho$
flavour conservation - with strange matter				
$B^{1/4} = 148 \text{ MeV}$	$T_c = 10$	$\mu_c = 956$	$L _s < 0$	-
$B^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 67$	$\mu_c = 773$	$T_c = 68$	$\mu_c = 765$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$T_c = 82$	$\mu_c = 697$	$T_c = 84$	$\mu_c = 680$
$B^{1/4} = 165 \text{ MeV}$	$T_c = 93$	$\mu_c = 687$	$T_c = 96$	$\mu_c = 661$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 102$	$\mu_c = 631$	$T_c = 105$	$\mu_c = 602$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$T_c = 116$	$\mu_c = 526$	$T_c = 118$	$\mu_c = 502$
Maxwell prescription - with strange matter				
$B^{1/4} = 148 \text{ MeV}$	$L _s < 0$	-	$L _s < 0$	-
$B^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$L _s < 0$	-	$L _s < 0$	-
$B^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$L _s < 0$	-	$L _s < 0$	-
$B^{1/4} = 165 \text{ MeV}$	$T_c = 19$	$\mu_c = 950$	$L _s < 0$	-
$B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 51$	$\mu_c = 876$	$T_c = 59$	$\mu_c = 838$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$T_c = 82$	$\mu_c = 742$	$T_c = 86$	$\mu_c = 715$
flavour conservation - without strange matter				
$B^{1/4} = 148 \text{ MeV}$	$T_c = 19$	$\mu_c = 948$	$L _s < 0$	-
$B^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 85$	$\mu_c = 719$	$T_c = 89$	$\mu_c = 695$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$T_c = 112$	$\mu_c = 603$	$T_c = 123$	$\mu_c = 536$
$B^{1/4} = 165 \text{ MeV}$	$T_c = 116$	$\mu_c = 637$	$T_c = 122$	$\mu_c = 594$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 129$	$\mu_c = 571$	$T_c = 138$	$\mu_c = 498$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$L _s > 0$	-	$L _s > 0$	-
Maxwell prescription - without strange matter				
$B^{1/4} = 148 \text{ MeV}$	$T_c = 4$	$\mu_c = 954$	$L _s < 0$	-
$B^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 61$	$\mu_c = 828$	$T_c = 70$	$\mu_c = 783$
$B^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$T_c = 99$	$\mu_c = 668$	$T_c = 108$	$\mu_c = 616$
$B^{1/4} = 165 \text{ MeV}$	$L _s > 0$	-	$L _s > 0$	-
$B^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$T_c = 121$	$\mu_c = 619$	$T_c = 130$	$\mu_c = 553$
$B^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$L _s > 0$	-	$L _s > 0$	-

Table 15 – Temperatures (T_c) and chemical potentials (μ_c) at the point where the latent heat ($L|_s$) becomes negative for all combinations of parametrizations considering **stellar** matter. These points may correspond to the CEP of the QCD phase diagram. In some cases $L|_s$ is always positive or always negative. All values are given in MeV.

4.6 CONCLUSIONS

In this chapter we reproduced the QCD phase diagram using two effective models. For the description of hadronic matter, a model of quantum hadrodynamics, the Walecka Model with non-linear terms, was used. To describe the quark matter we made use again of the MIT-based models already discussed in the previous chapters.

The Gibbs' conditions were used to establish the crossing points of the pressure as a function of the chemical potentials obtained in both phases.

In order to make a direct comparison with the results of the previous chapter, the same parametrizations for the MIT-based models were also used here yielding qualitatively the same results but with an increased effect caused by the vector field associated with the constant G_V . So, here we used the Bag pressure value with and without dependence on the temperature with one set of constants determined by the stability window, i.e., $B_0^{1/4} = 148$ MeV and another set outside this window, i.e., $B_0^{1/4} = 165$ MeV, both sets with G_V varying from zero to $G_V = 0.3$ fm². As for the QHD-based model, we used the NL3* $\omega\rho$ and eL3 $\omega\rho$ parametrizations, which satisfy several nuclear and astrophysical properties.

We first obtained the phase diagrams for two-flavour symmetric matter. As the parametrizations for the $B(T)$ were adjusted so that the T_{max} would satisfy the restraint imposed by the LQCD and freeze-out results, the T_{max} s are all around 168 MeV, but only the results for $B_0^{1/4} = 165$ MeV fit the Cleymans line entirely inside the confined (hadron) phase. The results for low temperatures depend on all parametrizations. As for the MIT-based models, we observed again that, the bigger the Bag pressure value, the higher the chemical potentials obtained. The same happens when we include the vector field, but in this case, the results are more sensible to the different values of G_V s. Only a few combinations of parametrizations resulted in maximum chemical potentials μ_{max} within the range $1050 \leq \mu \leq 1400$ MeV. Ultimately only NL3* $\omega\rho + B_0^{1/4} = 165$ MeV and NL3* $\omega\rho + B_0^{1/4} = 165$ MeV + $G_V = 0.05$ fm² satisfy all restraints we imposed.

Furthermore, we calculated the phase diagrams for stellar matter using two different prescriptions, one where we impose flavour conservation at the point of the phase transition, a prescription never applied to finite temperatures before, and the other where both phases are β -stable and charge neutral, being this last one the Maxwell prescription. Between the two prescriptions, the flavour conservation results in higher chemical potentials at low temperatures. Comparing these results to the symmetric matter, we conclude that the stellar matter favors the quark phase, and T_{max} s and μ_{max} s are higher for symmetric matter.

In all cases, a higher Bag pressure and G_V value increase the value of the chemical potentials at low temperatures, but this increase is lower for the stellar matter in the flavour conservation prescription and even lower in the Maxwell prescription than in the two-flavour symmetric matter. As for the QHD model parametrization, the eL3 $\omega\rho$ always favors the hadron phase as compared to the NL3* $\omega\rho$ one.

We also discussed the results for stellar matter without strange matter and concluded that those results are very similar to the ones for two-flavour symmetric matter.

In the last section we discussed the results for latent energy $L|_\epsilon$ and showed that they are always positive. Using another definition and calling it latent heat $L|_S$, we searched for the points where its value becomes negative, which may correspond to the CEP of the QCD phase diagram. If that is true, all the phase diagrams obtained in

this chapter are valid only up to this point.

The modified vector MIT Bag model used in combination with the QHD parametrizations consistent with recent astrophysical observations are new to this kind of approach. Furthermore, the use of these two models allows us to apply the same technique to both phases, once both models are relativistic and are dealt with at the mean-field level. Although our results are model-dependent, we do believe that the general trends would be the same had we chosen different models, even though the temperature and chemical potential values in all scenarios are not expected to be the same.

Although we made a lot of comparisons between the results of this chapter and the previous one, we did not explore the TOV equations solutions. That we will do in the next chapter.

5 HYBRID STARS

As the quantum chromodynamics (QCD) phase diagram became widely accepted, the possibility that neutron stars could, in fact, contain both a hadronic and a quark phases, started to be considered. In the same way that asymptotic freedom allows matter to become deconfined when the density increases at low temperatures, a similar behavior can take place in the interior of neutron stars. As the density increases towards the core of the star, quarks can become more energetically favorable than baryons, and, ultimately, the core of a neutron star may be composed of deconfined quarks. If the entire star does not convert itself into a quark star, as suggested by the Bodmer-Witten conjecture (see section 2.1.1 and Appendix A for more details), the final composition is a quark core surrounded by a hadronic layer. This is what is generally called a hybrid star (LUKÁCS; ZIMÁNYI; BALAZS, 1987; GLENDENNING, 1992).

In this chapter we aim to explore even further the MIT-based models already explored in the previous chapters, as well as the QHD-based models, whereas we construct hybrid stars within the Maxwell criteria of hadron-quark phase transition using those models. For the hadronic model we use the parametrization NL3* $\omega\rho$ already used in chapter 4, which fulfills most of the experimental constraints at the saturation density and also produces massive neutron stars, even when hyperons are present, fulfilling the constraints of the PSR J0740+6620. For the quark model we use parametrizations that reproduce unstable strange matter, as we don't want the entire star to convert into a quark star.

We use the Maxwell prescription to define the hadron-quark phase transition and construct the hybrid EoSs. After we use these EoSs to solve the TOV equations to generate the macroscopic properties of the hybrid stars.

Even though various studies (LOPES, L. L.; MENEZES, Débora P, 2021; GERSTUNG; KAISER; WEISE, 2020; CHATTERJEE; VIDANA, 2016; TOLOS; FABBIETTI, 2020) show that hyperons seem inevitable, based on the fact that the hyperon onset in the stellar matter is still an open issue, we dedicate a section to study hybrid stars without hyperons.

To finish this chapter, we analyze two important physical quantities, the dimensionless tidal polarizability (HINDERER et al., 2010) and the speed of sound. We compare the results for the tidal parameter to the recent constraints presented in the literature coming from LIGO/VIRGO gravitational wave observatories, more specifically, the GW170817 event (ABBOTT, B. P. et al., 2018). As for the speed of sound, we obtain its squared value and study its relation with the size and mass of the quark core in hybrid stars as proposed in (ANNALA et al., 2020).

The discussions presented in this chapter are based on the paper entitled "Hypermassive quark cores" published in the Monthly Notices of the Royal Astronomical

Society in 2022 (LOPES, L. L.; BIESDORF, Carline; MENEZES, Debora P, 2022). The parametrizations used in this chapter are slightly different than the ones used in the paper yielding also slightly different results.

5.1 EFFECTIVE MODELS

We will present the effective models that will be used to construct the hybrid stars and discuss their parametrizations. We will also discuss the criteria of the hadron-quark phase transition.

5.1.1 Hadronic EoS

To describe hadronic matter we use again the Walecka Model (SEROT, B.; WALECKA, 1986) with non-linear terms (BOGUTA; BODMER, 1977), which was already used in the previous chapter. So, the hadronic EoS is derived again from the lagrangian density given in Eq. (45). However, next we take $T = 0$, which makes the Fermi-Dirac distribution functions of anti-baryons and baryons equal to zero and $\Theta(\mu_B - E_B)$, respectively, which, in turn, defines the limits of all the integrals from zero to k_{fB} , which is the Fermi momentum of the baryon B .

As for the parametrization, we choose to use only the NL3 $\omega\rho$, which was also used in the previous chapter and its parameters can be found in Table 9 and the main nuclear properties are presented in Table 10. We also use the same hyperon masses and for the hyperon couplings we use again the SU(6)-flavour symmetry argument so that the ratios are given by Eq.(55) and the hyperon couplings are the ones presented in Table 11. We should point out that in the paper on which this chapter is based we used another parametrization, namely, the one proposed in (LOPES, L. L., 2021) and lately adjusted to become the eL3 $\omega\rho$ already used in the last chapter. Also, the hyperon couplings are adjusted differently in this chapter. In the paper (LOPES, L. L.; BIESDORF, Carline; MENEZES, Debora P, 2022) we used the SU(3) symmetry group to adjust the ratios of the constants. We find that the parametrizations used here reproduce massive stars and agrees with the most recent measurements.

And, as we are working with stellar matter, we once again include the leptons e and μ as a free Fermi gas and impose the conditions of β -equilibrium and charge neutrality given by Eqs. (58).

5.1.2 Quark EoS

To describe quark matter we use again the MIT Bag models, discussed in previous chapters, for $T = 0$. Therefore, we will work with Eqs. (15), (34),(35) and (36). We use the same quark masses as before. We will compare three variations of the MIT Bag model. The original MIT Bag model, where we just take $G_V = b_4 = 0$, and two

variations, with a vector channel, where we have $G_V \neq 0$ and with the vector channel altogether with the Dirac sea contribution, where both G_V and b_4 have finite values.

And once again, as we are dealing with stellar matter here, we need to add the leptons e and μ as a free Fermi gas and, as we are using the Maxwell criteria as phase transition (see next section), impose β -equilibrium and charge neutrality, as given by Eqs. (31).

As in this chapter we want to construct hybrid stars, we need unstable $u - d - s$ matter. If the $u - d - s$ matter is stable, as soon as the core of the star converts to the quark phase, the entire star may convert into a quark star in a finite amount of time (OLINTO, 1987). This means that we need to use parametrizations outside the stability window with Bag pressure values higher than the maximum allowed by this window (see section 2.1.1 and Appendix A for more details). Therefore, we use values at least $B^{1/4} > 159$ MeV when working with the original MIT Bag model. Also, in the vector MIT Bag model, the stability window depends on the value of the G_V , the higher the value of G_V , the lower the maximum value of B that satisfies the Bodmer-Witten conjecture, as we have seen in chapter 2. Indeed, for $G_V = 0.3 \text{ fm}^2$, the stability window lies between $139 \text{ MeV} < B^{1/4} < 150 \text{ MeV}$. So, when working with the vector MIT Bag model, we will use $B^{1/4} = 158$ MeV; this value allows us to explore more G_V values. As in the previous chapter, here we also use $X_V = g_{ssV}/g_{uuV} = g_{ssV}/g_{ddV} = 0.4$, a value fixed based on SU(3) symmetry group arguments.

5.2 MAXWELL CONSTRUCTION AND SOME RESTRAINTS ON THE CHEMICAL POTENTIALS

Another important quantity is the baryon chemical potential at the interface, μ_0 , which is the chemical potential value where the quark phase becomes energetically favorable, and a hadron-quark phase transition occurs. Its value is strongly model dependent, and it is defined at the point where the quark chemical potential equals the hadron chemical potential at the same pressure, which are the so-called Gibbs' conditions already used in the previous chapter (see section 4.2 for more details).

The use of this criteria to define hadron-quark phase transition without the imposition of flavour conservation is called Maxwell construction. In this construction, the hadron and quark phases are spatially separated and there is no mixed-phase. In the previous chapter we also studied Gibbs' conditions associated with the imposition of flavour conservation, which completely determines the composition of the quark phase from the hadron phase at the point of the phase transition, but in this prescription, the quark matter is not yet stable, but will quickly migrate to a β -stable matter. Another possibility is the so-called Gibbs construction, where there is a mixed phase and the quarks and hadrons can coexist (HUSAIN; THOMAS, 2021).

Some authors (MARUYAMA et al., 2007; PAOLI, P.; MENEZES, D., 2010) per-

formed studies on hybrid stars with both, Maxwell and Gibbs constructions and concluded that there is no significant difference on the macroscopic properties of the hybrid stars. On the other hand, some studies (MASLOV et al., 2019; YASUTAKE et al., 2014) point out that the presence of a mixed phase can affect some neutron star properties, especially those with intermediate masses, although the value of the maximum mass is hardly affected. Moreover, (LUGONES; GRUNFELD, 2021) suggests that the Maxwell construction is favoured if the surface tension between quarks and hadrons is high enough, therefore, we use this construction.

There is no experimental evidence about the value of the baryon chemical potential at the interface at zero temperature. Thus, we follow what we did in Chapter 3 and only analyze models where the baryon chemical potential at the interface lies between $1050 \text{ MeV} < \mu_0 < 1400 \text{ MeV}$.

5.3 CONSTRAINTS ON THE MASS AND RADIUS AND CORE OF THE COMPACT STARS

As already mentioned in the introduction of this thesis and also in the previous chapters, our EoSs need to be able to reproduce compact stars with masses that agree with the astrophysical findings. So, we will use some of the same restraints used in the previous chapters.

We will check if we are able to obtain masses in the range of $2.072^{+0.067}_{-0.066} M_{\odot}$ and equivalent radius between 11.41 km and 13.70 km as this is the range of uncertainties of PSR J0740+6620 according to NICER (RILEY, T. E. et al., 2021). We will also check if we are able to fulfill the restraints imposed by PSR J0952-0607, with a mass of $2.35 \pm 0.17 M_{\odot}$ (ROMANI et al., 2022), bearing in mind that these measurements are still not confirmed. NICER telescope has also constrained the radius of the canonical $1.4 M_{\odot}$ star. For instance, in (RILEY, T. et al., 2019) the authors constrain the radius from 11.52 km to 13.85, while in (MILLER et al., 2019) the radius is constrained from 11.96 km to 14.24 km. We will also check if we are able to fulfill these constraints.

A new constraint we will check concerns the nature of the core of the compact stars. In a study about the nature of massive stars (ANNALA et al., 2020), the authors suggest that massive neutron stars have sizable quark core. And more than that, they argue that hybrid stars are not only possible, but also probable. They found that for a two solar masses star with a radius of 12 km, the quark core can reach $0.8 M_{\odot}$ (40%) and 7 km (58%) of the total composition of the star. We will confront our results with these findings.

5.4 RESULTS

5.4.1 Original MIT Bag model

Here we analyze the results using the original MIT Bag model to construct the quark EoSs, i.e., we make $G_V = b_4 = 0$ in the equations mentioned above.

In Fig. 25 we show a very similar plot to the one shown in Fig. 17, but for $T = 0$ and considering only one hadronic EoS. We plot the pressure as a function of the chemical potential for the hadron matter (QHD) and for different Bag pressure values in the original MIT Bag model. When the QHD curve crosses the MIT one, we have the Maxwell criteria satisfied and a hadron-quark phase transition occurs. The baryonic chemical potential at the interface, as well as the pressure and the energy densities (in both phases) are displayed in Table 16. As can be seen, for $B^{1/4} = 165$ MeV, the baryonic chemical potential at the interface is below 1050 MeV, therefore, this result will not be analyzed any further. Moreover, μ_0 grows with the Bag pressure value, as already stated in the previous chapter.

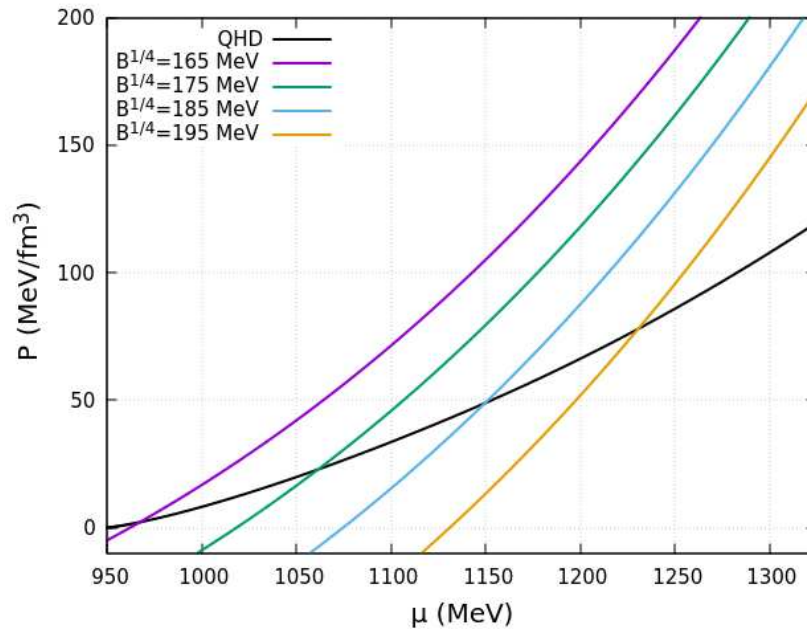


Figure 25 – The pressure as a function of the chemical potential for different values of the Bag. The crossing points indicate the baryon chemical potential at the interface, where the quark phase becomes energetically favorable.

Now we construct hybrid EoSs with different values of the Bag for the original MIT model. The EoS will be hadronic for values below the baryon chemical potential at the interface μ_0 , and of quarks for values above it. Then, we use these EoSs as input to solve the TOV equations (66) to obtain the macroscopic properties of the hybrid stars. For the inner and outer crust EoS we use (NEGELE; VAUTHERIN, 1973)

$B^{1/4}$ (MeV)	μ_0 ($\mu^H = \mu^Q$)	P_0 ($P^H = P^Q$)	ε_H	ε_Q
165	967 MeV	2	147	407
175	1062 MeV	23	259	576
185	1150 MeV	49	324	779
195	1231 MeV	78	411	1013

Table 16 – Baryon chemical potential and pressure (in MeV/fm³) at the interface of the phase transition from hadron (H) to quark (Q) for different values of the Bag. We also show the energy density (in MeV/fm³) at both phases at the interface.

and (RÜSTER; HEMPEL; SCHAFFNER-BIELICH, Jürgen, 2006), respectively. We also define the quantity M_{min} as the minimum mass of a star that has a quark core; stars with masses below M_{min} are purely hadronic. The results are presented in Fig. 26, and summarized in Table 17.

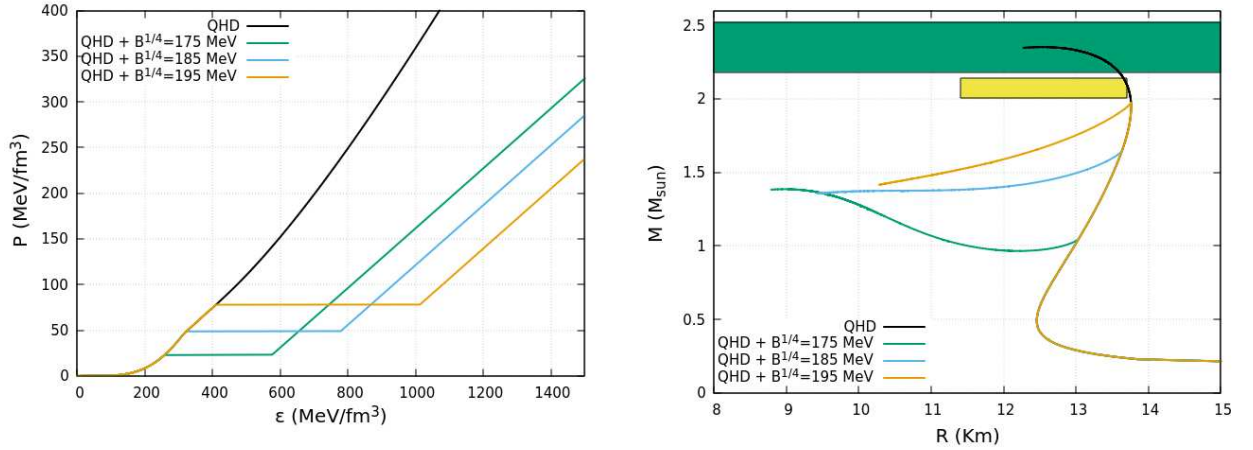


Figure 26 – EoS (left) and the TOV solutions (right) for different values of the Bag. The yellow area represents the uncertainty about both, the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021) and the green area the uncertainty about the mass of the PSR J0952–0607 (ROMANI et al., 2022). For the original Bag model no dynamically stable hybrid star reaches either the masses of the PSR J0740+6620 or the of PSR J0952–0607.

Bag ^{1/4} (MeV)	$M_{max}/M_{\odot}$	R (km)	ε_c	$M_{min}/M_{\odot}$	$R_{1.4}$ (km)
175	1.40	9.00	2199	1.05	9.00
185	1.65	13.63	324	1.65	13.42
195	1.97	13.76	411	1.97	13.42
QHD	2.35	12.54	965	-	13.42

Table 17 – Hybrid star properties for different values of the Bag. Only $B^{1/4}=175$ MeV produces dynamically stable hybrid stars.

The maximum masses appearing in Fig. 26 grow with the Bag pressure value. But we need to look at these results more deeply once we are interested in hybrid stars.

For values of $B^{1/4} > 175$ MeV the most massive star is completely hadronic, as we can deduce by comparing the central energy density ε_C of the maximum mass star shown in table 17 with the values of the energy densities at the hadron-quark interface shown in table 16. As can be seen, for $B^{1/4} > 175$ MeV the ε_C corresponds to the value of the energy density of the hadron matter at the interface. We could have also come to the same conclusion by looking at the mass-radius diagram in Fig. 26. For $B^{1/4} = 185$ MeV and $B^{1/4} = 195$ MeV the star with the maximum mass is located at the hadron branch and all the other results, which are dislocated from this branch, are unstable ones¹. For $B^{1/4} = 175$ MeV we not only have stable hybrid stars, but the most massive one is also a hybrid one. And beyond that, with this parametrization we also get twin-stars, i.e., stars with the same mass, but different radius².

Therefore, only with $B^{1/4} = 175$ MeV, we have a dynamically stable hybrid star, i.e., a star family that satisfies $\partial M / \partial \varepsilon_0 > 0$. Unfortunately the maximum mass is of only $1.40 M_\odot$, which is way below the observational limit of the PSR J0740+6620 and PSR J0952–0607. Therefore, with the chosen QHD parametrization we cannot explain massive pulsars as a hybrid star within the original MIT Bag model.

5.4.2 Vector MIT Bag model

If we introduce the vector channel to the MIT Bag model we may overcome the issue encountered in last section. So now we attribute values to G_V , but still maintain $b_4 = 0$, in the Eqs. (15), (34), (35) and (36). As for the Bag pressure value, here we use $B^{1/4} = 158$ MeV, ensuring an unstable $u - d - s$ matter ($E/B > 930$ MeV).

We plot the pressure as function of the chemical potential for the vector MIT Bag model within different values of G_V in Fig. 27. Unlike Fig. 25 for the original MIT, here the crossing of the hadron-quark curves are not so easily identifiable. This is because the slopes of the curves in both phases are very similar. To help identify the baryon chemical potential at the interface, we plot dots in Fig. 27. The baryon chemical potential at the interface, the pressure, and the energy density for different values of G_V are displayed in Table 18. As can be seen, the higher the value of G_V , the higher the μ_0 , as we already concluded in chapter 4. We also see that G_V must lie between 0.36 fm^2 and 0.56 fm^2 so that the baryon chemical potential at the interface falls between $1050 \text{ MeV} < \mu_0 < 1400 \text{ MeV}$.

¹ Here we include the unstable results in the mass-radius diagrams, as done in the previous chapters, and only call the attention to it when strictly needed. In Appendix B we discuss the stability of the stars in more detail

² Although we have some unstable results after the maximum mass at the hadron branch is reached, once the masses start to increase again, the stability is recovered and some stable twin-stars appear. More about twin-stars can be found in (CHRISTIAN; ZACCHI; SCHAFFNER-BIELICH, Jürgen, 2018) and references therein.

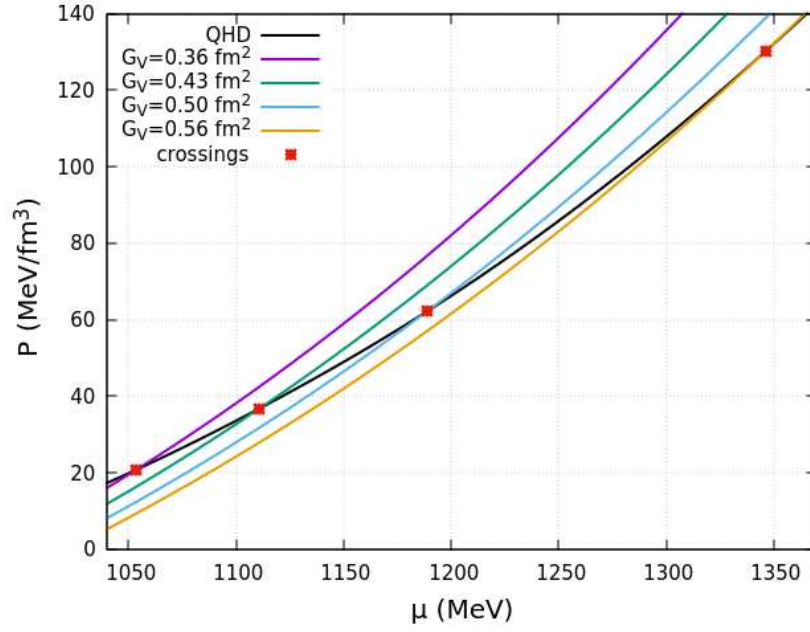


Figure 27 – The pressure as a function of the chemical potential for $B^{1/4} = 158$ MeV and different values of G_V . The dots indicate where the hadron-quark phase transition happens.

G_V (fm ²)	μ_0	P_0	ε_H	ε_Q
0.35	1047	19	246	349
0.36	1054	21	252	353
0.38	1068	25	264	361
0.39	1076	27	270	365
0.40	1084	29	276	370
0.43	1111	37	295	385
0.45	1130	43	308	398
0.50	1189	62	363	439
0.56	1346	130	549	583
0.57	1416	167	635	662

Table 18 – Chemical potential and pressure (in MeV/fm³) at phase transition for hadron (H) to quark (Q) for different values of G_V . We also show the energy density (in MeV/fm³) at both phases at the interface

We plot in Fig. 28 the EoS and the corresponding TOV solutions for the hybrid stars within the vector MIT Bag model. In Table 19 those results are summarized. Compared with the original MIT Bag model, we see that the EoSs are much stiffer within the vector channel. We also see that the gap in the energy density is significantly smaller here than in the original one. Another point is the fact that varying G_V from 0.36 to 0.56 fm², although it causes a large effect on the value of μ_0 , does not affect too much the stiffness of the EoSs. All these facts are reflected in the mass-radius relation. For instance, the maximum mass of a hybrid star varies from $2.04M_\odot$ to $2.30M_\odot$, a difference around 13%, but the M_{min} , the minimum mass value for which the hybrid

star is formed, varies from $0.98M_{\odot}$ to $2.20M_{\odot}$, a difference greater than 120%. As the stiffness of the EoS controls the maximum mass achieved and the value of μ_0 controls when the quarks start to appear, those results make sense.

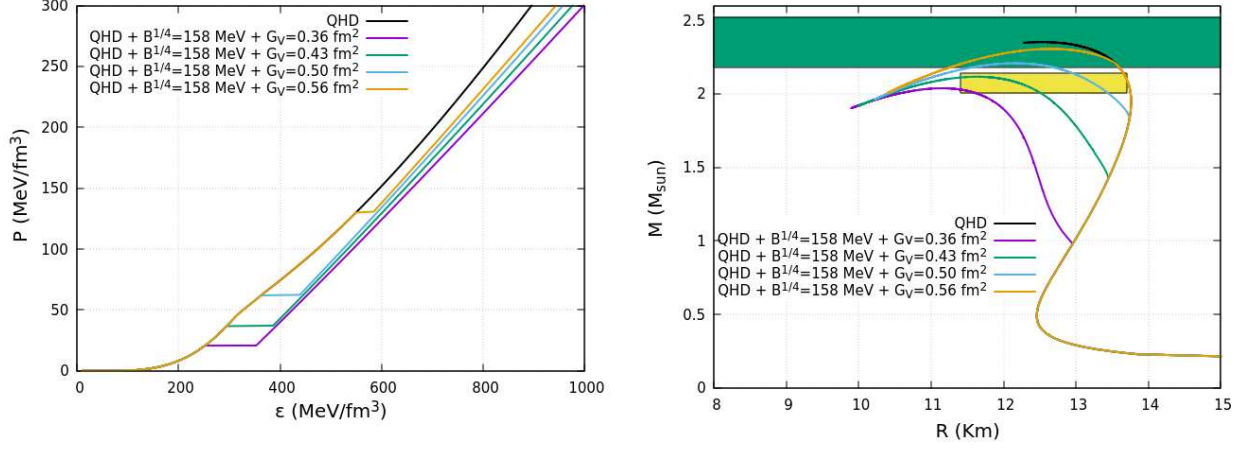


Figure 28 – EoS (left) and TOV solutions (right) for different values of G_V . All the curves cross the yellow area representing the uncertain about the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021). The green area represents the uncertain about the mass of the PSR J0952–0607 (ROMANI et al., 2022)

G_V (fm ²)	$M_{max}/M_{\odot}$	R (km)	ε_c	$M_{min}/M_{\odot}$	$R_{1.4}$ (km)
0.36	2.04	11.15	1274	0.98	12.50
0.43	2.11	11.66	1157	1.42	13.42
0.50	2.21	12.21	1034	1.83	13.42
0.56	2.30	12.72	908	2.20	13.42
QHD	2.35	12.54	965	-	13.42

Table 19 – Hybrid star properties for different values of the G_V .

We can also see that all models discussed in this section fulfill the mass constraint of the PSR J0740+6620, $2.072^{+0.067}_{-0.062} M_{\odot}$, and also the radius uncertainty (RILEY, T. E. et al., 2021), as all curves cross the yellow box in Fig. 28. And for G_V values equal or greater than 0.50 fm^2 we are also able to obtain maximum masses that satisfy the constraint of the PSR J0952-0607, with a mass of $2.35 \pm 0.17 M_{\odot}$ (ROMANI et al., 2022).

As for the constraints for the radius of the canonical stars, we can see from the results shown in Table 19, that they agree with the NICER results for both, hadronic and hybrid stars, as they are all within the limits suggested by (RILEY, T. et al., 2019) and (MILLER et al., 2019).

For G_V up to values of 0.45 fm^2 , the hadron-quark phase transition takes place below the hyperon threshold. The first hyperon onset happens around $n = 0.31 \text{ fm}^{-3}$, and at a neutron chemical potential of $\mu_n = 1136 \text{ MeV}$. A baryon chemical potential at the

interface below 1136 MeV could, therefore, be faced as a possible solution to the so-called hyperon puzzle, i.e., the difficulty to reconcile the measured masses of compact stars with the presence of hyperons in their interiors.

Now we take a look at the nature of the core of our hybrid stars. We estimate the mass and size of the quark core in the most massive hybrid star of each model presented in Table 19. To accomplish that we solve the TOV equations for the quark EoS from the energy density corresponding to the pressure where the phase transitions happens, as displayed in Table 18, up to the energy density at the maximum mass shown in Table 19. The results are presented in absolute and relative values in Table 20.

G_V (fm ²)	0.36	0.43	0.50	0.56
$M_Q/M_\odot$	1.71	1.53	1.22	0.57
R_Q (km)	9.23	8.83	8.09	6.02
% M_Q/M_{max}	84%	73%	55%	25%
% R_Q/R_{total}	82%	76%	66%	47%

Table 20 – Masses and radii of the quark core and their proportional contribution for the maximally massive star within different hybrid stars models.

As we can see in Table 20, within the vector MIT Bag model, the quark core can be even more significant than within the pQCD used in (ANNALA et al., 2020). Indeed, for $G_V=0.36$ fm², we are able to produce hybrid stars with a hypermassive quark core of $1.71M_\odot$ and a radius larger than 9 km. In relative values, this corresponds to more than 80% of both the total mass and radius of the hybrid star. Also for $G_V=0.43$ fm² we produce hybrid stars with a hypermassive quark core of $1.53M_\odot$ and a radius of almost 9 km, which, in turn, corresponds to more than 70% of both the total mass and radius of the hybrid star. It is also worthy to point out that such hybrid stars with the hypermassive quark cores still fulfill the constraints of the PSR J0740+6620. For $G_V=0.50$ fm² our quark core is still a bit bigger than the ones from (ANNALA et al., 2020) and in this case we are able to fulfill not only the constraints of the PSR J0740+6620 but also the ones from the PSR J0952-0607.

We compare our results with other hybrid star studies found in the literature. For instance, our results are close to those presented in (GOMES, R.; CHAR; SCHRAMM, 2019), although both the hadronic and the quark models are different. Here, we use an extended version of the QHD, with the NL3* $\omega\rho$, while (GOMES, R.; CHAR; SCHRAMM, 2019) uses a many-body model with a derivative coupling. We obtain more massive stars, up to $2.35M_\odot$ whereas (GOMES, R.; CHAR; SCHRAMM, 2019) only reaches $2.15M_\odot$. There is also a difference in the radius of the canonical star: 13.42 km here vs 14.44 km in (GOMES, R.; CHAR; SCHRAMM, 2019). Both studies use the vector MIT Bag model, although in (GOMES, R.; CHAR; SCHRAMM, 2019) the mass term,

important to keep the thermodynamic consistency, is missing. Within the set with the larger quark core, in Maxwell construction, we found a maximum mass of $2.04M_{\odot}$ with a $M_{min}=0.98M_{\odot}$, while (GOMES, R.; CHAR; SCHRAMM, 2019) found a maximum mass of $1.99M_{\odot}$ with a $M_{min} = 1.03 M_{\odot}$.

On (KLÄHN; FISCHER, 2015), the authors use the so-called tdBag, which is, ultimately, based on the NJL models for the quark phase, and the TM1 parametrization of the QHD for the hadron phase. They found that only with two flavor $u-d$ quark matter the phase transition is possible. And, unlike in the present work, as well as in (GOMES, R.; CHAR; SCHRAMM, 2019), the hybrid star has a very similar radius with respect to the pure hadronic one.

5.4.3 Self-Interacting Vector Field contribution

Now we attribute values not only to G_V , but also to b_4 , in the Eqs. (15), (34),(35) and (36). As pointed out before, the quartic term softens the EoS at high density, which allows us to use higher values of G_V without crossing the limit $\mu = 1400$ MeV. Indeed, in this section we use $G_V=0.8 \text{ fm}^2$ and keep $B^{1/4}=158$ MeV, as in the previous one.

In Fig. 29 we plot the pressure as a function of the chemical potential for three different values of b_4 . The dots are also plotted to help identify the baryon chemical potential at the interface. The results are also shown in Table 21.

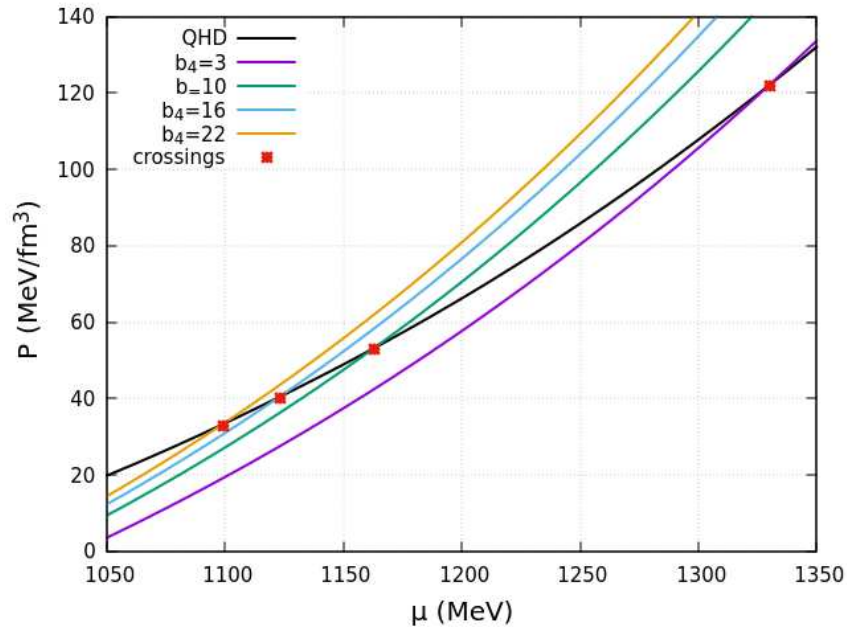


Figure 29 – The pressure as a function of the chemical potential for different values of b_4 with $G_V=0.8 \text{ fm}^2$ and $B^{1/4}=158$ MeV. The dots indicate where the hadron-quark phase transition happens.

As can be seen, there is the already expected relation between the μ_0 and the value of b_4 . The higher the b_4 value, the softer is the quark EoS and therefore, the

b_4	μ_0	P_0	ε_H	ε_Q
3	1330 MeV	122	530	631
10	1163 MeV	53	337	466
16	1123 MeV	40	303	436
22	1099 MeV	33	286	419

Table 21 – Chemical potential and pressure (in MeV/fm³) at phase transition for hadron (H) to quark (Q) for different values of b_4 , with $G_V=0.8$ fm² and $B^{1/4}=158$ MeV. We also show the energy density (in MeV/fm³) at both phases at the hadron-quark interface.

lower is the μ_0 . Even with the high value of $G_V=0.8$ fm², we are able to choose values of b_4 that produce baryon chemical potential at the interface in the range between 1050 MeV < μ_0 < 1400 MeV.

Following, in Fig. 30 we plot the EoSs and the TOV solutions for different values of b_4 , maintaining $B^{1/4}=158$ MeV and $G_V=0.8$ fm². The main results are also displayed in Table 22.

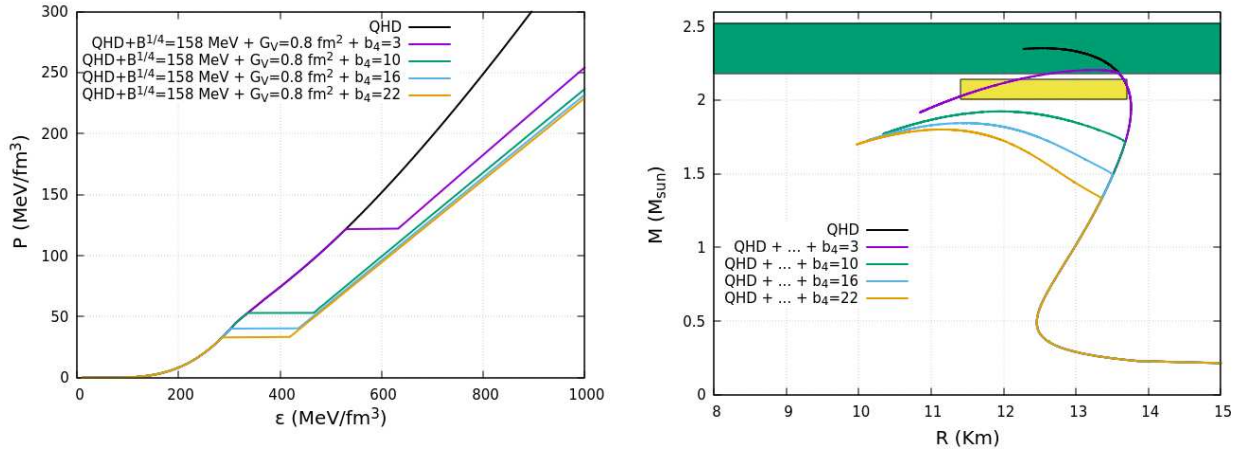


Figure 30 – EoS (left) and TOV solutions (right) for different values of b_4 . Only for $b_4 = 3$ we have stable hybrid stars that reach the mass of the PSR J0952–0607 (ROMANI et al., 2022) (green area). The yellow area represents the uncertain about the radius and mass of the PSR J0740+6620 (RI-LEY, T. E. et al., 2021).

b_4	$M_{max}/M_\odot$	R (km)	ε_c	$M_{min}/M_\odot$	$R_{1.4}$ (km)
3	2.20	13.30	760	2.18	13.42
10	1.92	12.02	1065	1.71	13.42
16	1.84	11.45	1238	1.48	13.42
22	1.80	11.23	1255	1.33	13.12
QHD	2.35	12.54	965	-	13.42

Table 22 – Hybrid star properties for different values of the b_4 with $G_V = 0.8$ fm² and $B^{1/4}=158$ MeV.

As we can see, the Dirac sea contribution makes the gap in the energy density larger than in the previous section. Also, we can see that the gap depends on the value of b_4 , the higher the value, the bigger the gap. With respect to the TOV solutions, we see that, despite the high value of G_V , with the quartic term in the quark phase, only when we use the very small $b_4 = 3$ we are able to obtain masses that reach the ones of PSR J0740+6620 (yellow area), but these masses are smaller than its M_{min} , which means that these stars are not hybrid. For this same parametrization, we also obtain some results that reach the masses of PSR J0952-0607 (green area) and that are hybrid³. Due to this drawback, we do not calculate the relative values of the quark core. It is, nevertheless, worth pointing out that, for all utilised values of b_4 , we are able to produce a dynamically stable hybrid branch. Also, unlike the vector case, the maximum mass strongly depends on the new term added here, the value of b_4 . The maximum mass can vary from $1.80M_\odot$ to $2.20M_\odot$, a difference of about 22%. Furthermore, within the Dirac sea approximation, only for $b_4 = 22$ we obtain a canonical star that has a quark phase. All the radii of the canonical stars agree with the NICER results.

5.5 NO HYPERONS

One of the open issues in nuclear astrophysics is the content of the inner core of massive neutron stars. Because of the Pauli principle, as the number density increases, the Fermi energy of the nucleons exceeds the mass of the heaviest baryons, and the conversion of some nucleons into hyperons becomes energetically favorable. It is also well known that the hyperon onset softens the EoS. In some cases, the softening of the EoS pushes the maximum mass below the observational limit of massive neutron stars. This possible conflict between theory and observation is called hyperon puzzle. Some extensive studies, as, for example, (LOPES, L. L.; MENEZES, Débora P, 2021; ĐAPO; SCHAEFER; WAMBACH, 2010; CHATTERJEE; VIDANA, 2016; TOLOS; FABBETTI, 2020), discuss the hyperon puzzle and show that hyperons are, ultimately, inevitable. This is the reason why, up to now, we have always considered the EoS with hyperons.

Nevertheless, the hyperon onset in stellar matter is still an open issue. Moreover, a recent study on non-relativistic models shows that the use of strongly repulsive three-body forces can suppress the hyperon threshold (GERSTUNG; KAISER; WEISE, 2020). Therefore we dedicate this section to study hybrid stars without hyperons. But we have to keep in mind that, for the QHD model we use, the hyperon suppression is obtained simply by not taking them into account. We also don't allow the strange quark to appear in the quark phase.

As the vector MIT Bag model without the Dirac sea contribution predicts the hybrid stars with the highest maximum mass, as well as the biggest quark core, we only

³ Remembering that, in this configuration, the maximum mass also corresponds to the last stable result. So, all results of hybrid stars that cross the yellow box are dynamically unstable results

use this set in this section. We plot in Fig. 31 the pressure as a function of the chemical potential. The results are also presented in Table 23.

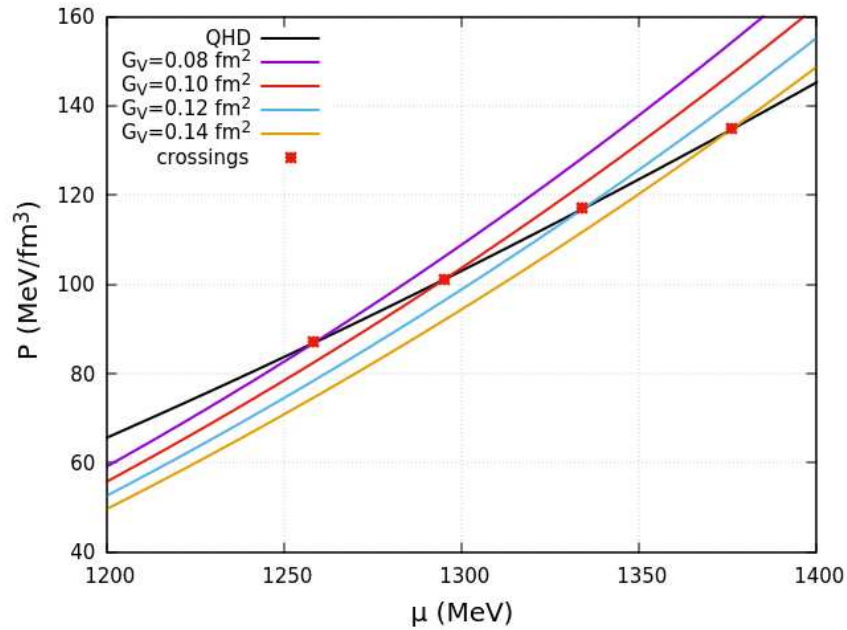


Figure 31 – The pressure as a function of the chemical potential for different values of G_V and $B^{1/4}=158$ MeV without hyperons and quarks s . The dots indicate where the hadron-quark phase transition happens.

G_V (fm ²)	μ_0	P_0	ε_H	ε_Q
0.08	1258 MeV	87	389	549
0.10	1295 MeV	101	412	579
0.12	1334 MeV	117	436	612
0.14	1376 MeV	135	462	647

Table 23 – Chemical potential and pressure (in MeV/fm³) at phase transition from hadron (H) to quark (Q) for different values of G_V without strangeness in both phases. We also show the energy density (in MeV/fm³) at both phases at the hadron-quark interface.

We can see that, without hyperons and strange quarks, we need to use smaller values of G_V to obtain the transition chemical potential in the range $1050 \text{ MeV} < \mu_0 < 1400 \text{ MeV}$. This was already established in section 4.4.3 of the last chapter, where we obtained much higher values for the maximum chemical potential using the same parametrizations as used when the strange matter was allowed to appear.

In Fig. 32 we display the EoSs and the mass-radius relations obtained for the present configuration. The results are also presented in Table 24.

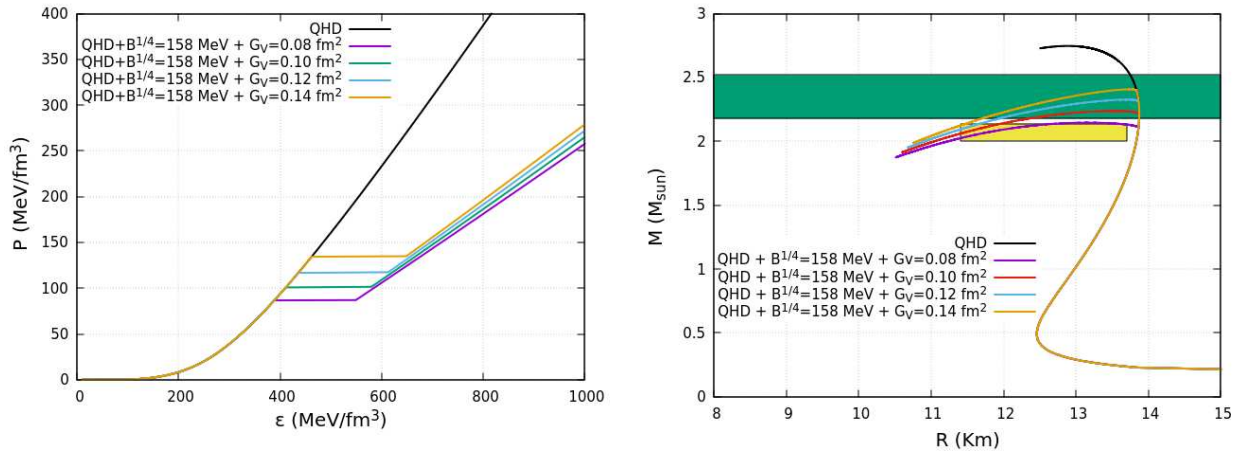


Figure 32 – EoS (left) and TOV solution (right) for different values of G_V without hyperons and strange quarks. The yellow area represents the uncertain about both, the radius and mass of the PSR J0740+6620 (RILEY, T. E. et al., 2021) and the green area the uncertain about the mass of the PSR J0952–0607 (ROMANI et al., 2022). A hybrid star with maximum mass of $2.41M_\odot$ is possible.

G_V (fm ²)	$M_{max}/M_\odot$	R (km)	ε_c	$M_{min}/M_\odot$	$R_{1.4}$ (km)
0.08	2.14	13.48	690	2.10	13.43
0.10	2.24	13.46	738	2.21	13.43
0.12	2.33	13.69	696	2.31	13.43
0.14	2.41	13.73	708	2.40	13.43
QHD(N)	2.75	12.89	896	-	13.43

Table 24 – Hybrid star properties for different values of the G_V without hyperons and strange quarks.

We can observe that the EoSs without strange matter are stiffer than the ones with strange matter and this is reflected in the TOV solutions so that we obtain bigger masses. We see also that the energy gap here is bigger than in the hyperonic case. Also, the gap grows with the increase of G_V . We also see that only for $G_V = 0.08$ fm² we have some stable results that reproduce the PSR J0740+6620 constraint (yellow box); the other parameterizations meet these constraints, but they are not dynamically stable results. Also for these other parametrizations, namely, for $G_V = 0.10$ fm², $G_V = 0.12$ fm² and $G_V = 0.14$ fm², we have some stable results that meet the constraints of PSR J0952-0607 (green area). The minimum masses that produce the hybrid branch are bigger than when the strange matter is present.

Now we estimate the size and mass of the quark core in hybrid stars without strangeness. The results are presented in Table 25.

The quark cores here are not so significant and do not agree with the ones presented in (ANNALA et al., 2020), which say that the quark core should represent

G_V (fm ²)	0.08	0.10	0.12	0.14
$M_Q/M_\odot$	0.27	0.28	0.12	0.07
R_Q (km)	4.91	4.88	3.68	3.02
% M_Q/M_{max}	13%	13%	5%	3%
% R_Q/R_{total}	36%	36%	27%	22%

Table 25 – Masses and radii of the quark core and their proportional contribution for the maximally massive star within different hybrid stars models without hyperons and strange quarks.

around 40% of the total mass.

5.6 THE DIMENSIONLESS TIDAL PARAMETER AND THE SPEED OF SOUND

Once again, before we finish the analyses of this chapter, we take a look, as already done in chapter 2, at the dimensionless tidal parameter Λ , as given by Eq. (6). We also analyse the squared speed of sound, v_s^2 , as already done in chapter 3, and its relation with the size and mass of the quark core in hybrid stars as presented in (ANNALA et al., 2020). As the models that produce higher maximum masses, as well as larger values of quark core masses and radii, are the models within the vector MIT bag model without the Dirac sea contribution, we restrict our analyzes here to the parameters presented in Table 19.

Once more we check if we are able to satisfy the restrictions $70 < \Lambda_{1.4} < 580$, as imposed by the LIGO/VIRGO gravitational wave observatories related to the GW170817 event (ABBOTT, B. P. et al., 2018). In Fig. 33 we plot the dimensionless tidal parameter for different values of G_V , as well as for the pure hadronic model with hyperons (QHD). As can be seen, only for $G_V = 0.36$ fm² we satisfy the restraint for $\Lambda_{1.4}$.

In table 26 we show the value of the dimensionless tidal parameter for the canonical mass ($\Lambda_{1.4}$) as well as the squared speed of sound of quark matter at the hadron-quark interface.

We can see that, for $G_V=0.36$ fm², the tidal parameter is significantly smaller, as it also has a smaller radius for the canonical star. However, this is not the main reason, as pointed out in (LOPES, L. L.; BIESDORF, Carline; MENEZES, Débora P, 2021; HINDERER, 2008), the tidal parameter is, in fact, more dependent of the microscopic k_2 (see Eq. (7)) than of the macroscopic compactness. For $G_V=0.43$ fm² and above, the $\Lambda_{1.4}$ is already equal to a pure hadronic EoS, as the minimum mass that corresponds to a hybrid star with this parametrization is already above $1.40M_\odot$.

As for the speed of sound, we see that the higher its value, the smaller the quark core in the hybrid star. However, unlike (ANNALA et al., 2020), the results here are not so drastic. For instance, for all the values of G_V we have a speed of sound that violates

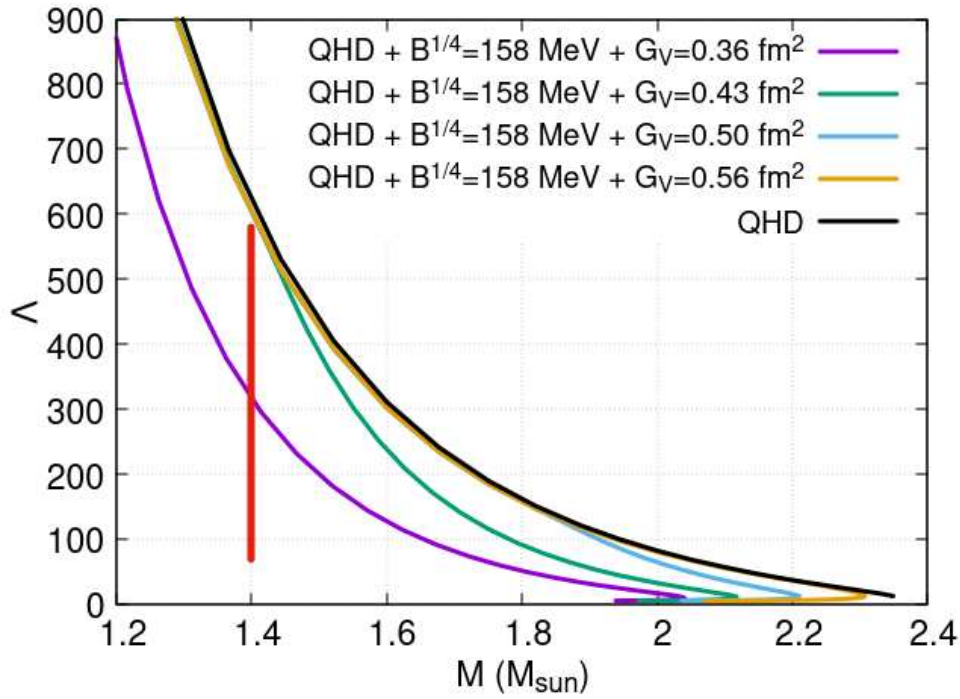


Figure 33 – Dimensionless tidal parameter as a function of the star mass for different values of G_V as presented in Table 19. The red line is the LIGO/VIRGO constraint $70 < \lambda_{1.4} < 580$ for the $1.4M_\odot$ star.

G_V (fm ²)	0.36	0.43	0.50	0.56	QHD
$\lambda_{1.4}$	317	604	604	604	624
v_s^2	0.41	0.42	0.44	0.46	-

Table 26 – Tidal dimensionless parameter for the canonical $1.4M_\odot$ star and the speed of sound of quark matter at the hadron-quark interface.

the conformal limit. Moreover, while the differences in the speed of sound are around 10%, the quark core mass differences reach over 100%. Nevertheless, we agree with (ANNALA et al., 2020) about the fact that a quark core in massive neutron star is not only possible, but probable.

5.7 CONCLUSIONS

In this chapter we combined again a QHD model with the Modified MIT bag models, as already done in the previous chapter, but here we went back to zero temperature in order to study hybrid stars. As criteria to build the hybrid EoSs from the two models we used the Maxwell construction.

In the first section of results, we showed that the original MIT Bag model, without interaction between quarks, is not enough to explain hybrid stars that satisfy the restraints imposed by either PSR J0740+6620 (RILEY, T. E. et al., 2021) or PSR J0952-0607 (ROMANI et al., 2022). Only with the QHD model alone, i.e., for purely hadronic

stars, we were able to satisfy those restraints. Besides that, even though with smaller masses, we were only able to obtain stable hybrid configurations for a Bag pressure value of $B^{1/4} = 175$ MeV.

When we introduced the vector channel and varied G_V from 0.36 fm^2 to 0.56 fm^2 we were able to reproduce hybrid stars within the limits imposed by the astrophysical observations considered in this chapter. For G_V from 0.36 fm^2 to 0.50 fm^2 the restraints for PSR J0740+6620 were met, for G_V from 0.50 fm^2 to 0.56 fm^2 the restraints for PSR J0952-0607 were met, but only $G_V = 0.50 \text{ fm}^2$ meet both restraints. Higher values for G_V would violet the imposition of the chemical potential at the interface being less than 1400 MeV as well as result in more massive hybrid stars that do not explain PSR J0740+6620 as a hybrid star. All $1.4M_\odot$ obtained within the vector MIT Bag model have a radius within the limits of the astrophysical observations.

Still within the vector MIT Bag model we did obtain hypermassive quark cores, corresponding from 25% for $G_V = 0.56 \text{ fm}^2$ up to 84% of the total mass of the star for $G_V = 0.36 \text{ fm}^2$.

When we also introduced the self-interacting term proportional to the b_4 constant we verified that, if we keep the same Bag pressure value, we need to significantly increase the value of G_V to obtain chemical potentials at the hadron-quark interface larger than 1050 MeV. We were able to vary b_4 from 3 to 22 using $G_V = 0.8 \text{ fm}^2$, but were not able to reproduce hybrid stars within the astrophysical restraints.

We also checked if our models are able to describe possible hybrid stars without any strange matter, i.e., without hyperons and strange quarks, and concluded that, within the vector MIT Bag model, but with very small G_V s, we are, although we were never able to fulfill at the same time the mass restraints imposed by PSR J0740+6620 and PSR J0952-0607. Also in this scenario, the quark core is much smaller than predicted for strange hybrid stars, corresponding to 13% of the total mass.

To finish, we analyzed the dimensionless tidal deformability (Λ) for the configurations where we obtained the most promising results, i.e., for strange hybrid stars within the vector MIT Bag model, and concluded that only for the smallest value of $G_V = 0.36 \text{ fm}^2$ we meet the restraints for $\Lambda_{1.4}$ imposed by the LIGO/VIRGO gravitational wave observatories. In this last section we also took a look at the squared speed of sound and observed an inverse relation between its value and the size of the quark core, the higher its value, the smaller is the quark core. The conformal limit ($v_s^2 < 1/3$) was violated in all cases.

To sum it up, the most promising way to describe hybrid stars within the chosen models is to consider strange matter and only a vector interaction within the MIT Bag model, i.e., without the self-interacting term. We are never able to meet all the restraints at the same time, considering the restraints in mass, radius, and tidal deformability discussed in the present chapter.

In the next chapter, we will add dark matter to the stellar matter.

6 MASQUERADING HYBRID STARS WITH DARK MATTER

In this last chapter we analyse some aspects of dark matter admixed hybrid star (DMAHS). We consider two species of matter, ordinary matter, or normal matter (NM), and dark matter (DM), which interacts only gravitationally, so we will use two-fluid TOV equations to obtain the mass-radius relations.

For the NM we consider hybrid matter very similar to the one studied in the last chapter. In fact, we use the same models to describe each matter as before, a QHD-based model to describe the hadronic matter and the modified MIT Bag model to describe the quark matter. We also use again the Maxwell prescription to determine the conditions of hadron-quark phase transition. As for the parametrizations, for the QHD-based model, we use the $NL3^*\omega\rho$ as before, but for the MIT-based model, we chose here a different one, namely, $B^{1/4} = 155$ MeV, $G_V = 1.0$ and $b_4 = 1.2$. This specific combination of parametrizations produces only unstable hybrid stars in a one-fluid system and, with the addition of DM we will see that stable ones are also possible.

For DM we consider weak and strongly interacting DM with particle masses $m_D = 5$ GeV and $m_D = 100$ GeV.

We first analyse the general effect this DM has on the mass-radius relation of neutron stars (NSs) and white dwarfs (WDs) and confirm what was already discussed in (DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2022; TOLOS; SCHAFFNER-BIELICH, Jürgen, 2015; DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2021; DELIYERGIYEV et al., 2019), among other things, that when one of the central pressures is much larger compared to the other, the fluid with the largest central pressure dominates, and the configuration behaves as a single fluid. Then we concentrate on the NS branch and conclude that the DM can trigger the appearance of deconfined quark matter (QM) in the core of NSs. In the last section, we take a quick look at what we will call here dark oysters, COs with a big DM halo.

6.1 MICROSCOPIC MODELS

6.1.1 NM Equations of State

For the normal matter, we once again build a hybrid EoS using the Maxwell construction as criteria to define hadron-quark phase transition as done in the previous chapter. And we also use the same models as used in the previous chapter, i.e., to describe hadronic matter we use the Walecka Model (SEROT, B.; WALECKA, 1986) with non-linear terms (BOGUTA; BODMER, 1977) so that our hadronic EoS is derived again from the lagrangian density given by Eq. (45), and to describe the quark matter (QM) we use the Modified MIT Bag model with vector field and self-interacting terms, so that our quark EoS is derived again from the lagrangian density given by Eq. (33).

P_0 (MeV/fm ³)	μ_0 (MeV)	ε_H (MeV/fm ³)	ε_Q (MeV/fm ³)	$M_{max}/M_\odot$	ε_c (MeV/fm ³)
361	1698	1003	1280	2.35	1003

Table 27 – Main thermodynamic values at the hadron-quark phase transition and macroscopic values considering only the NM and the neutron star branch. The first four quantities will remain the same throughout the entire paper.

As for the parametrizations, for the hadronic model, we use the same one used in the previous chapter, i.e., the NL3* $\omega\rho$ parametrization to which the parameters can be found in Table 9 and the main nuclear properties in Table 10. We also use the same hyperon masses and as for the hyperons couplings we use again the SU(6)-flavour symmetry argument so that the ratios are given by Eq.(55) and the hyperon couplings are the ones presented in Table 11.

As for the parametrization for the QM we choose here $B^{1/4} = 155$ MeV, $G_V = 1.0$ fm² and $b_4 = 1.2$, with $g_{ssV} = \frac{2}{5}g_{uuV} = \frac{2}{5}g_{ddV}$. This parametrization describes unstable strange matter, which allows the existence of a quark core that will not convert the whole star into a strange.

In order to produce β stable matter, with zero net electric charge, we also add leptons as a free Fermi gas to both hadronic and quark matter and impose the conditions of β -equilibrium and charge neutrality as given by Eqs. (58) and (31) for the hadronic and quark matter, respectively.

As for the inner and outer crust of the stars, we use again (NEGELE; VAUTHERIN, 1973) and (RÜSTER; HEMPEL; SCHAFFNER-BIELICH, Jürgen, 2006), respectively and for $\varepsilon < 3.3 \times 10^3$ g/cm³ we use the Harrison-Wheeler EoS (HARRISON et al., 1965).

This specific choice of parametrizations results in baryon chemical potential at the hadron-quark interface μ_0 larger than the limit adopted in the previous chapters, as can be seen in table 27. Based on the fact that there is no real evidence of an upper limit for this value, we relaxed this condition here and prefer to attend to other conditions. As can be seen in Fig. 34, where we plot the EoS for NM (left) and the mass-radius relation for the neutron star branch (right), and also in table 27, no stable hybrid star results from the chosen parametrizations. We want to see if this changes when we add DM.

We also impose an upper limit on the hybrid EoS and allow pressures only up to 1000 MeV/fm³, which corresponds to a density around $12n_s$, where n_s is the nuclear saturation density. So we stay far below the density at which the perturbative QCD (pQCD) becomes relevant, which is said to be around $40n_s$ (KOMOLTSEV; KURKELA, 2022). Higher pressures impact the effects of the DM, as we will comment later on, but as we don't take into consideration pQCD effects, we consider this limit is appropriate for the studies carried out in the present chapter.

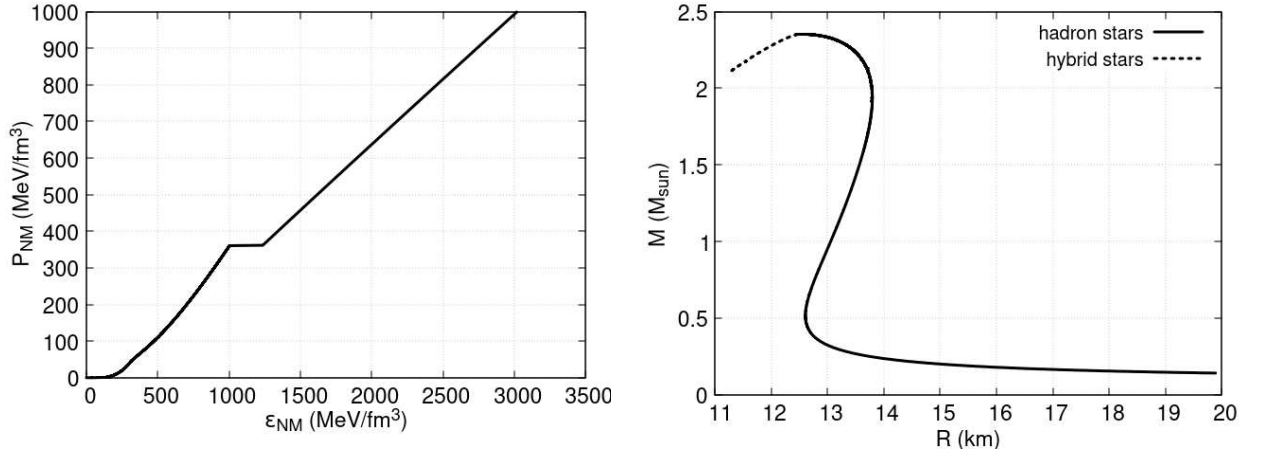


Figure 34 – EoS (left) and TOV solution (right) considering only the NM and the neutron star branch. No stable hybrid star is possible within these parametrizations.

Because we want to study the effect of the DM on the NM and on the hadron-quark phase transition, here we fix the above discussed parametrizations for the NM and use only one hybrid EoS and vary some aspects of the DM. So, quantities like P_0 , μ_0 , ϵ_H and ϵ_Q will remain the same throughout the entire chapter.

6.1.2 DM Equations of State

For the dark matter the EoS is taken from (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006) for a non-annihilating massive DM particle. The dimensionless energy density, pressure and number density are given by, respectively:

$$\begin{aligned} \epsilon'_{DM} &= \frac{\epsilon_{DM}}{m_D^4} = \frac{1}{\pi^2} \int_0^z x^2 \sqrt{1+x^2} dx + \left(\frac{1}{3\pi^2} \right)^2 y^2 z^6 \\ &= \frac{1}{8\pi^2} \left[(2z^3 + z)(1+z^2)^{1/2} - \sinh^{-1}(z) \right] \left(\frac{1}{3\pi^2} \right)^2 y^2 z^6, \end{aligned} \quad (63)$$

$$\begin{aligned} P'_{DM} &= \frac{P_{DM}}{m_D^4} = \frac{1}{3\pi^2} \int_0^z \frac{x^4}{\sqrt{1+x^2}} dx + \left(\frac{1}{3\pi^2} \right)^2 y^2 z^6 \\ &= \frac{1}{24\pi^2} \left[(2z^3 - 3z)(1+z^2)^{1/2} + 3\sinh^{-1}(z) \right] \left(\frac{1}{3\pi^2} \right)^2 y^2 z^6, \end{aligned} \quad (64)$$

$$n'_{DM} = \frac{n_{DM}}{m_D^3} = \frac{z^3}{3\pi^2} \quad (65)$$

where z is the dimensionless Fermi momentum and $y = m_D/m_l$ the interaction strength, with m_l being the energy scale of the interaction between fermions. For strong interactions $m_l \sim 100$ MeV (exchange of vector mesons) and for weak interactions $m_l \sim 300$

GeV (exchange of W and Z bosons). Note that for $y \ll 1$ the EoS will be of an ideal Fermi gas. Here we will explore the cases of y varying from weakly interacting $y = 10^{-1}$ to strongly interacting $y = 10^3$ DM.

As for the DM particle, we also explore two cases: $m_D = 5$ GeV and $m_D = 100$ GeV.

Fig 35 we plot the DM EoS for weakly ($y = 10^{-1}$) and strongly ($y = 10^3$) interacting matter for the two masses considered in this work.

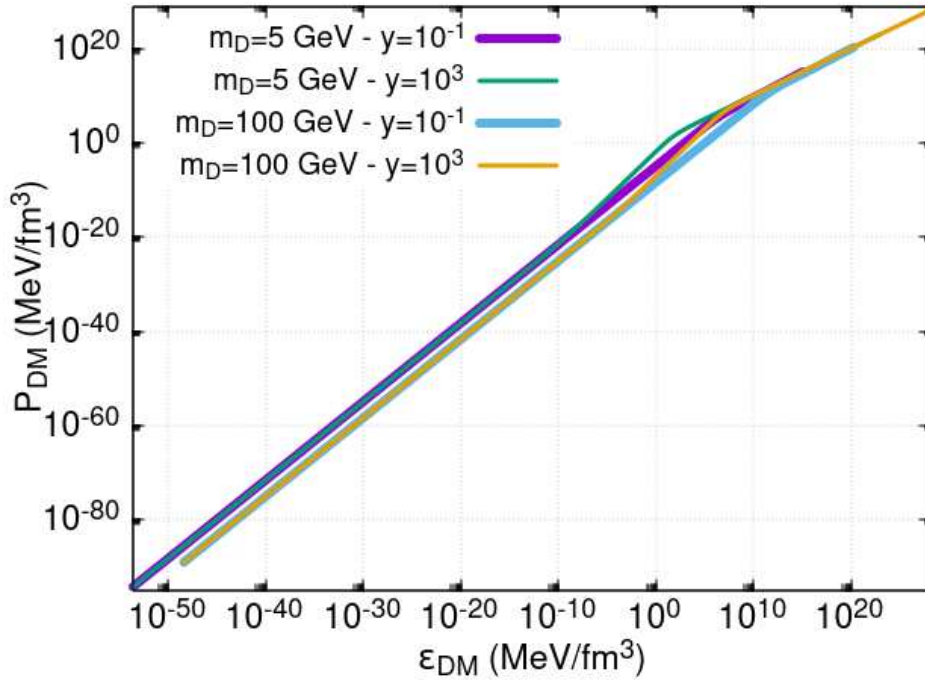


Figure 35 – Dark matter EoS for weakly ($y = 10^{-1}$) and strongly ($y = 10^3$) interacting matter).

6.2 STELLAR STRUCTURE EQUATIONS

There are two possible approaches one can follow to obtain the mass-radius diagrams for compact stars admixed DM. One is to assume that the DM interacts with the NM through the exchange of a Higgs boson. In this case the TOV equations used are the same we used in the previous chapters. The other approach is to consider that the two species of matter only interact gravitationally. This is what we consider here. And, in this last case, we need to use the two-fluid TOV equations, which, in its dimensionless form are given by (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006):

$$\begin{aligned}
\frac{dP'_{NM}}{dr} &= -(P'_{NM} + \varepsilon'_{NM}) \frac{dv}{dr}, \\
\frac{dm_{NM}}{dr} &= 4\pi r^2 \varepsilon'_{NM}, \\
\frac{dP'_{DM}}{dr} &= -(P'_{DM} + \varepsilon'_{DM}) \frac{dv}{dr}, \\
\frac{dm_{DM}}{dr} &= 4\pi r^2 \varepsilon'_{DM}, \\
\frac{dv}{dr} &= \frac{(m_{NM} + m_{DM}) + 4\pi r^3 (P'_{NM} + P'_{DM})}{r(r - 2(m_{NM} + m_{DM}))}
\end{aligned} \tag{66}$$

where $P' = P/m_D^4$ and $\varepsilon' = \varepsilon/m_D^4$ are the dimensionless pressure and energy density, respectively, being m_D the DM particle mass¹.

6.3 RESULTS

6.3.1 The effect of DM on the mass-radius diagram

Here we show the effect of the DM on the whole mass-radius diagram, i.e., we do not only take a look at the neutron star branch, which is the main focus of this paper, but also at the lower densities region, at the so-called white dwarf branch. In Fig. 36 we show the results for the total mass (NM mass plus DM mass) in function of the observable radius (NM radius) where we vary the ratio (P_{DM}/P_{NM}) from 10^{-1} to 10^{10} and the interaction strength of the DM from a weak $y = 10^{-1}$ to a strong $y = 10^3$ interaction for the masses of $m_D = 5$ GeV and $m_D = 100$ GeV for the DM particle. Note that only stable results are shown.

As already discussed in (DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2022), when one of the central pressures is much larger compared to the other one, the fluid with the largest central pressure dominates, and the configuration behaves as a single fluid. In Fig. 36 we can see that for very small DM pressures the NM configuration dominates, being that all results for $P_{DM}/P_{NM} = 10^{-1}$ are very similar and correspond to the one we would obtain by solving the single fluid TOV equations for the NM EoS. And, as the ratio increases, the DM starts to dominate and the configurations change until they reach the one corresponding to a single fluid made up of the DM. The rate at which the configuration changes depends on the strength of the interaction y and on the mass of the DM particle m_D .

As expected, the more interacting the DM is, the quicker the configuration changes, but it changes much faster for the lower m_D . That is, we need to add much

¹ In order to obtain the dimensionless quantities for each specie of matter we could also divide the physical quantities by a NM particle, as the neutron mass, for instance, the important thing being that we divide all physical quantities by the same mass.

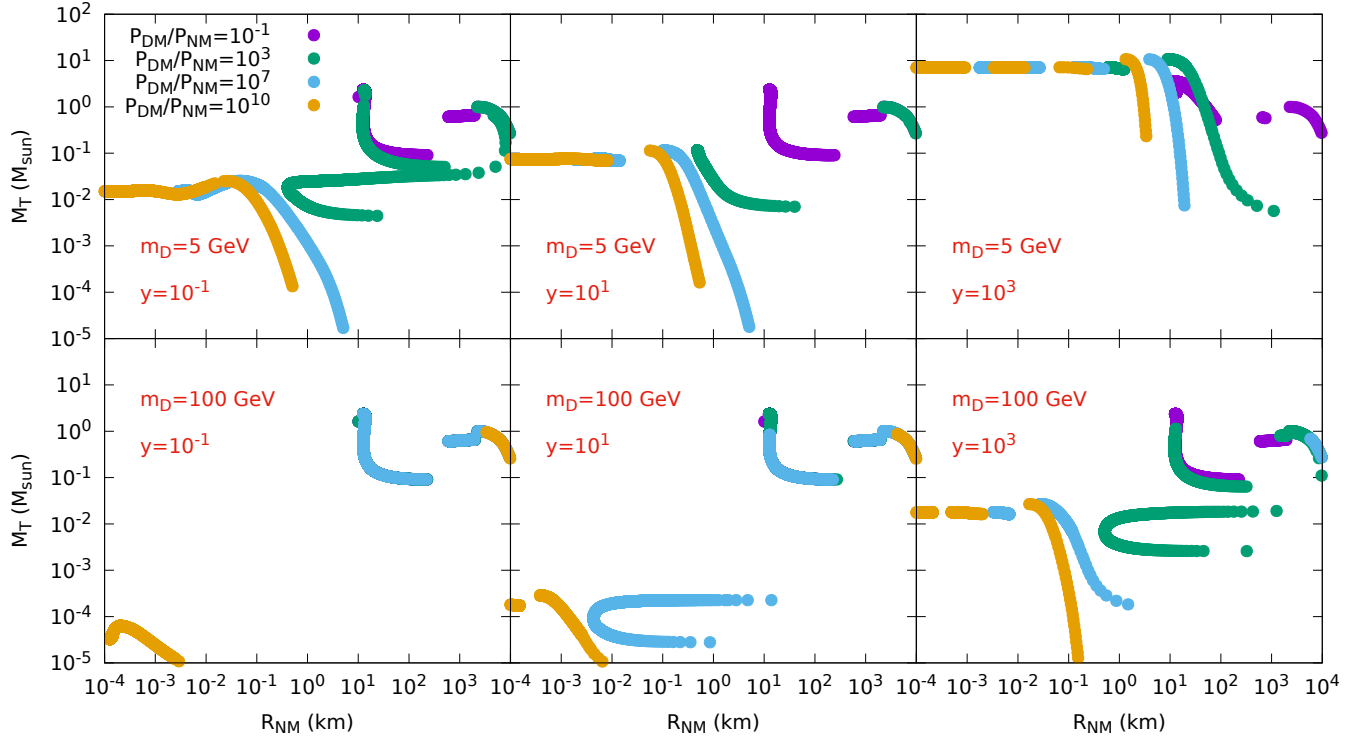


Figure 36 – Total mass in function of the observable radius of neutron star and white dwarf branches for different values of the interacting strength y and $m_D = 5$ GeV (top) and $m_D = 100$ GeV (bottom) for different pressure ratios.

more weakly interacting DM to the mixture than strongly interacting DM to achieve the DM single fluid configuration, and also more for a more massive DM particle than for a lighter one.

As for the configurations we achieve for very high DM ratios, we can also see from Fig. 36 that the masses and the radii depend on the mass of DM particle and the interaction strength. As discussed in (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006), for the same value of y , the total mass, which is then dominated by the DM, scales with the inverse of the square of the dark matter particle mass ($M_T \propto m_D^{-2}$), so we shift to much higher masses for $m_D = 5$ GeV, whereas for $m_D = 100$ GeV the masses are always lower than the ones of normal neutron stars. When we get results with lower masses and radius than the ones of the usual neutron stars, we get the so called dark compact planets (DCPs), as described in (TOLOS; SCHAFFNER-BIELICH, Jürgen, 2015; DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2021). These DCPs have Earth-like masses ($M_T = 10^{-4} - 10^{-6} M_\odot$ or lower), but radii around one meter for $y = 10^{-1}$ or Jupiter-like masses ($M_T = 10^{-2} - 10^{-4} M_\odot$ or lower) and radii of around 1 km for $y = 10^3$.

In Fig. 37 we plot the equivalent to Fig. 36, but instead of the observable radius, we show the radius of DM. Note that for $P_{DM}/P_{NM} = 10^{10}$ the radii of the DM and the masses increase with the increase of y . For weakly interacting DM, the slope of the mass-radius curve for masses below the maximum mass is proportional to R_{DM}^3 , and

for strongly interacting DM, the mass-radius relation is constant for a given R_{DM} , as was already shown in (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006).

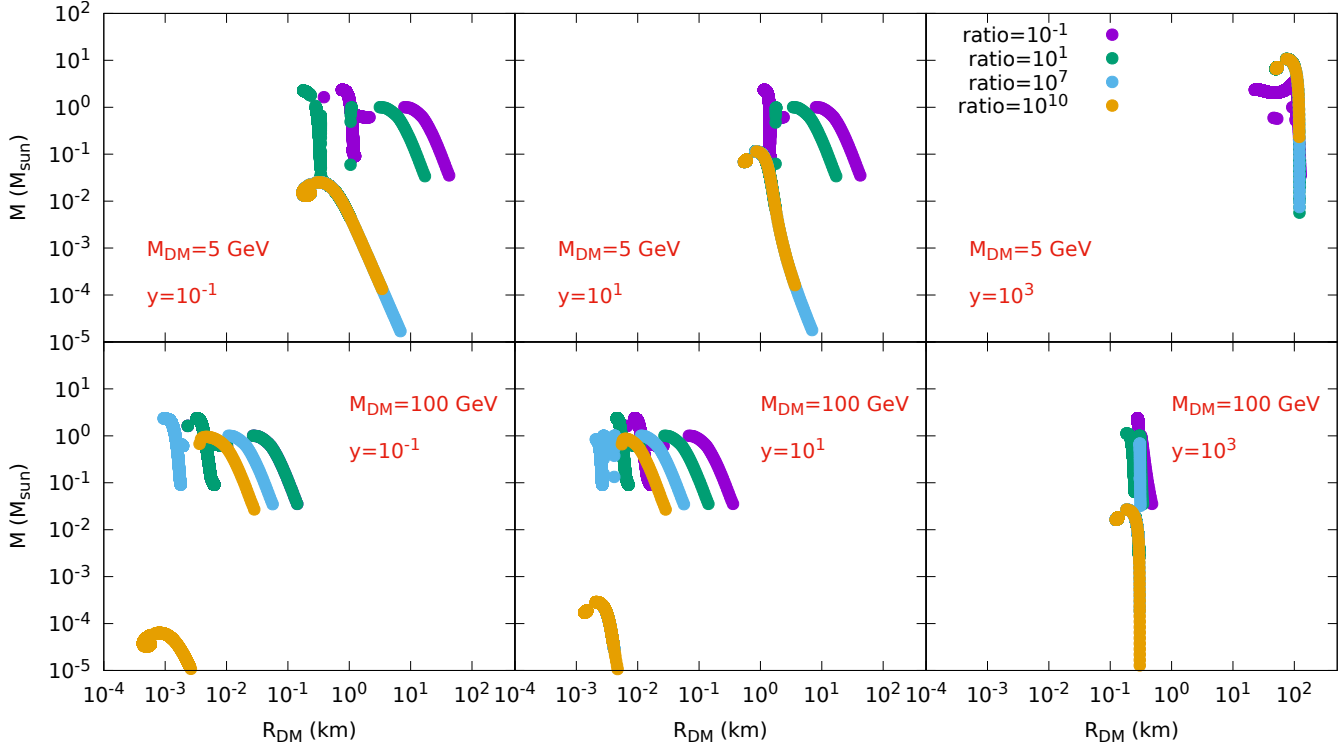


Figure 37 – Total mass in function of the DM radius of neutron star and white dwarf branches for different values of the interacting strength y and $m_D = 5$ GeV (top) and $m_D = 100$ GeV (bottom) for different pressure ratios.

If we compare the two radii, of NM and DM, we can see that, for the same y and P_{DM}/P_{NM} , in the case of $m_D = 100$ GeV the R_{DM} is always smaller. So, in this case, the DM is in the core of the star and is mixed up with the NM until the value of R_{DM} . But if we compare the results shown in the upper right in both figures, we can see that, in the case of $m_D = 5$ GeV and $y = 10^3$ the radii of the DM are much larger. In this case, NM is in the core of the star and is mixed with the DM up to the radius corresponding to the value of R_{NM} and the DM forms a halo around the core, what we call dark oysters. We look at these specific objects in more detail in section 6.3.4.

6.3.2 NS branch

Now we turn our attention only to the NS branch. Here we investigate the effect the DM has on compact stars like hybrid stars. More precisely, we want to know how the DM affects the hadron-quark phase transition in those objects. We define as critical mass M_{crit} the mass of the star with NM central pressure equal to the pressure where the hadron-quark phase transition happens, i.e., $P_0 = 361 \text{ MeV/fm}^3$, so that all the stars with higher NM central pressure are hybrid stars, i.e, have deconfined QM in their

core².

In Fig. 38 we show the total mass versus the observable radius (NM radius) for different configurations of DM, but keeping the same NM configuration. We remember that, without DM, we have only purely hadronic stable stars. However, once we add DM, stable hybrid stars start to appear. The round black dots show the location of the first stable hybrid star to appear, i.e., the first stable M_{crit} . The remaining dots show the location of the hybrid star with the smallest mass, i.e., M_{crit} , for the corresponding ratios shown in each diagram. As can be seen, the results corresponding to strongly interacting DM $\gamma = 10^3$ and $m_D = 5$ GeV differ substantially from the others. In this graph the black squared dots merely show the location of M_{crit} for each configuration of DM showed in the plot. Notice that for the no DM case this star is unstable.

The quantity of DM necessary for the existence of deconfined quarks depends on the interacting strength γ and on the mass of the DM particle m_D . For weakly interacting DM one needs to add more DM to obtain the first stable hybrid star than for strongly interacting DM. Also, for the same γ one needs to add more DM when the DM particle mass is bigger. To try to understand this better, we take a look at how the central pressure of a star is affected by the increase of DM in its composition. In Fig. 39 we plot the central pressure of the ordinary matter as a function of the ratio of pressures P_{DM}/P_{NM} for a $1.4M_\odot$ star. As for $\gamma = 10^3$ and $m_D = 5$ GeV we get a $1.4M_\odot$ star only for a few values of P_{DM}/P_{NM} and it is never a hybrid one, we choose, instead, to show the results for intermediary values of γ in order to better understand the effect of γ for the same m_D . We also include the line (yellow dotted) representing the pressure at which the hadron-quark phase transition takes place.

The first thing to notice is that the central pressure increases with the increase of the DM ratio. That, ultimately, is what enables us to get stable hybrid stars. As the deconfined quarks start to appear only at pressures equal to $P_{NM} = 361$ MeV/fm³, a star with central pressure lower than this value is still completely hadronic, or a hadronic one with DM, but with no deconfined QM in its core. And, after a certain quantity of DM is added, the central pressure reaches the critical value of 361 MeV/fm³ and the star converts into a hybrid CO with a quark core. In the next section, we will discuss the size of this core.

What happens when we add DM to the NM is that, in general, the former compresses the latter so that the already compact object gets even more compact in the sense that one needs more NM to get an object with the same mass and approximately the same size. If we look again at Fig. 38, we see that, except for the bottom right plots, we get a different $1.4M_\odot$ star for each different value of P_{DM}/P_{NM} , for the top left

² What we call critical mass M_{crit} here is equivalent to what we called minimum mass M_{min} in the last chapter, i.e., the minimum mass at which we got a hybrid star, so that stars with masses below M_{min} were purely hadronic. We changed the name here because, as we will see, in some configurations here, we will have purely hadronic stars (but mixed with DM) with higher masses than M_{crit}

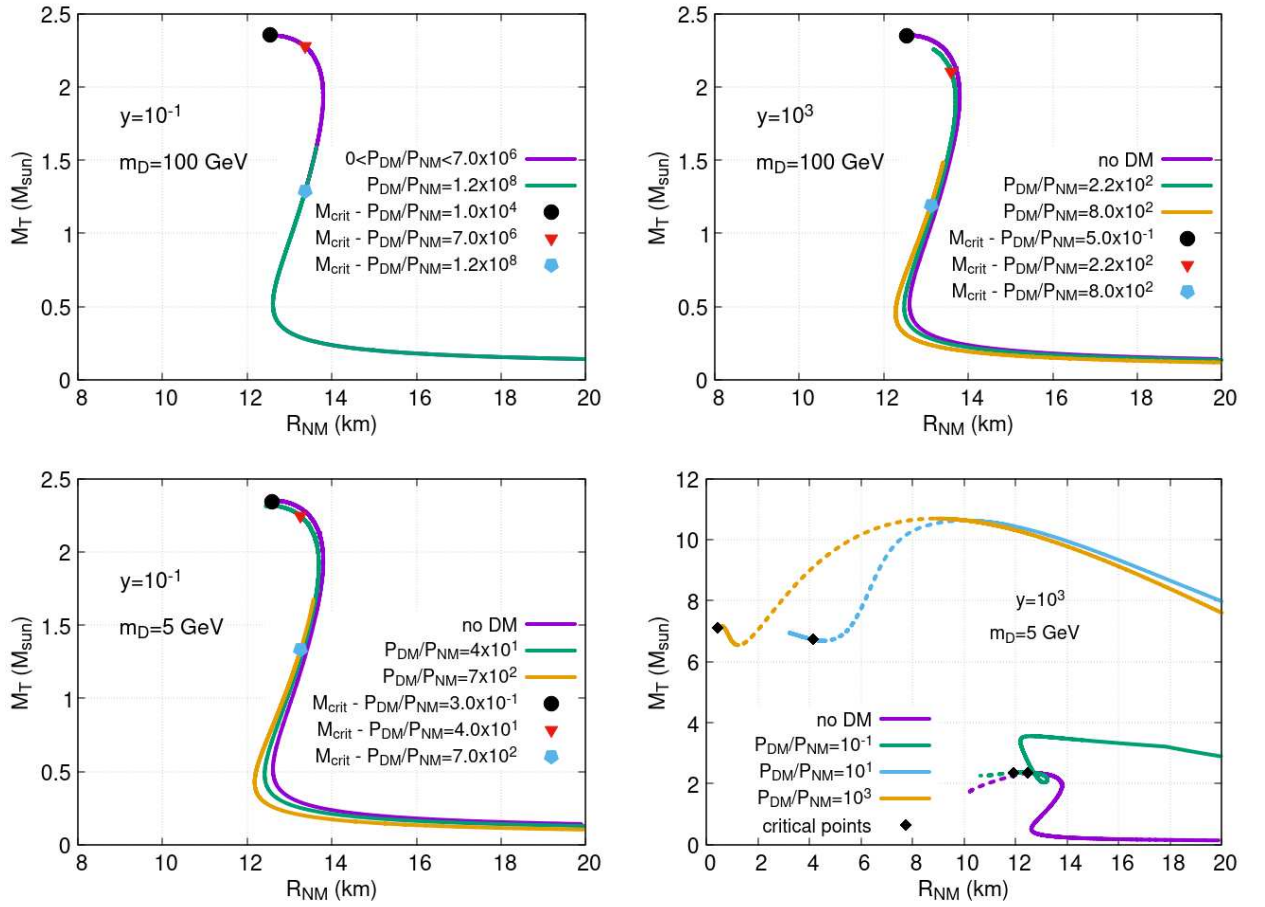


Figure 38 – Neutron Star branches for weakly ($y = 10^{-1}$) and strongly ($y = 10^3$) interacting DM for $m_D = 100$ GeV (top) and $m_D = 5$ GeV (bottom) for different pressure ratios. The dots indicate the critical mass M_{crit} at which hybrid stars start to appear for each configuration. The black round dots indicate the minimum amount of DM needed to get stable hybrid stars. In the last figure we also include the unstable results (dashed lines).

even the radius is of the same size, the only difference between those stars being the quantity of DM and, consequently, of NM, with different values of central pressures.

If the central pressure increases faster with the ratio of DM, it means that the nature of the DM allows it dominate over the NM faster, and so we come back to the previous section: lower m_D and a higher y mean a faster dominance over the NM and, as a consequence, stable hybrid stars will appear already at lower values of the P_{DM}/P_{NM} .

In Fig. 38 we can already see that, in general, the value of M_{crit} decreases with the increase of DM. Now let us take a closer look at this value. In Fig. 40 we plot the critical mass in function of the DM pressure ratios for different values of the interacting strength of the DM. We only show results that lead to stable hybrid stars. We notice that the behavior for $y = 10^3$ and $m_D = 5$ GeV is again much different from the others. As we will dedicate a whole section just for this configuration, here we restrain our discussions

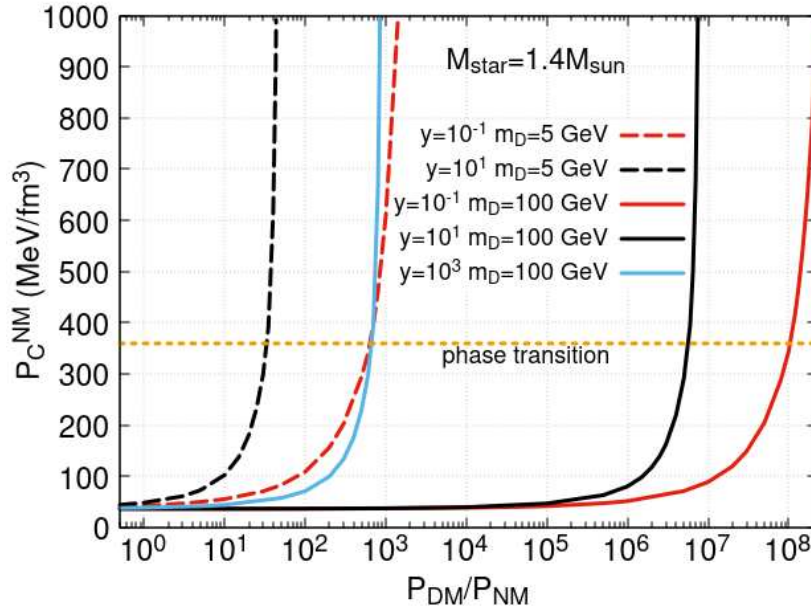


Figure 39 – Central pressure of the normal matter (P_C^{NM}) in function of the ratio of DM pressures (P_{DM}/P_{NM}) for a $1.4M_\odot$ star for $m_D = 5$ GeV (dashed lines) and $m_D = 100$ GeV (solid lines) for different values of the interacting strength of the DM (y). The yellow dotted line shows the pressure where the hadron-quark phase transitions takes place.

to all the other configurations.

Fig. 40 confirms our previous findings. As, in general, the DM compresses the OM, the more DM we add, the more the central pressure of the star increases, allowing, eventually, the appearance of QM. In other words, without DM a central pressure corresponding to the hadron-quark phase transition pressure, i.e., $P_0 = P_C = 361$ MeV/fm³ produces a star of around $2.35M_\odot$, but, once enough DM is added, this mass diminishes so that the same central pressure now produces a star with less mass. And, the more DM we add, the less mass this central pressure produces and, so, the M_{crit} decreases.

It is interesting to note that the same critical masses happen almost at the same pressure ratios for $y = 10^3$ with $m_D = 100$ GeV and for $y = 10^{-1}$ with $m_D = 5$ GeV.

It is not that the DM takes the place of the NM, but that DM allows for more matter to be compressed inside a star, eventually allowing the deconfined QM to appear. The mass and radius of a star are no longer defined exclusively by one central pressure, but by two. It should be clear from Fig. 39 that various combinations of the two central pressures result in a star with the same mass ($1.4M_\odot$ in that case) and that the addition of DM results in an increase of the central pressure of the star. From Fig. 38 we see that in most cases even the radius does not change much. So, by only looking at the mass and the radius of the star it is impossible to know its central pressure and if this central pressure allows the appearance of QM. The DM is, so to say, masquerading

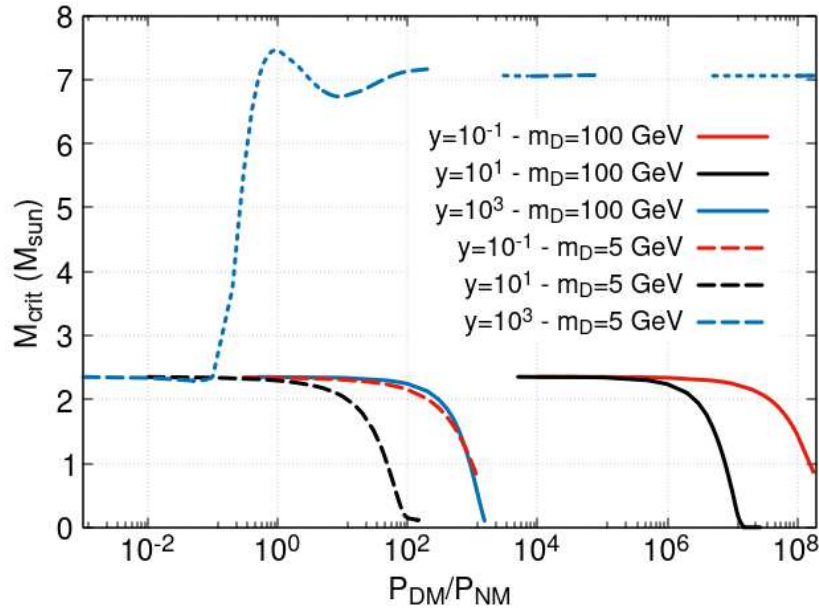


Figure 40 – Critical mass (M_{crit}) in function of the ratio of DM pressures (P_{DM}/P_{NM}) for $m_D = 5$ GeV (dashed lines) and $m_D = 100$ GeV (solid lines) for different values of the interacting strength of the DM (y).

hybrid stars.

6.3.3 Quantity of each type of matter

Now we take a look at how much of each type of matter is in fact mixed in a dark matter admixed hybrid star. In order to do so we analyze again $1.4M_{\odot}$ hybrid stars with different ratios of DM.

In Fig. 41 we plot the total mass produced by the deconfined QM (top left) and the corresponding radius (top right) as well as the total mass produced by the DM (bottom left) and the corresponding radius of the DM (bottom right) in function of the DM pressure ratios. We once more do not include the results for $y = 10^2$ and $y = 10^3$ with $m_D = 5$ GeV because we only get a few stable results for the former and none at all for the latter. To better follow what is happening in the bottom plots we show in table 28 the minimum and maximum pressure ratios that result in stable $1.4M_{\odot}$ with its corresponding mass and radius.

To obtain the contribution only of the quark matter we first identify the central pressure of the NM that results in a $1.4M_{\odot}$ (for it to be a hybrid one, this pressure needs to be larger than the hadron-quark phase transition pressure, i.e., $P_C^{NM} > P_0 = 361 \text{ MeV/fm}^3$) and then we solve the TOV equations from this central pressure up to the pressure where the phase transition happens and take the results for the NM. To obtain the contribution to the total mass coming from the DM we stop the integration when the DM pressure is zero and take the results for the DM.³ So, the QM may form

³ Remembering that the mass and the radius of one star are obtained by integrating the TOV equations

a core inside the star, whereas the DM is always mixed with the NM, not forming a separated core as the QM.

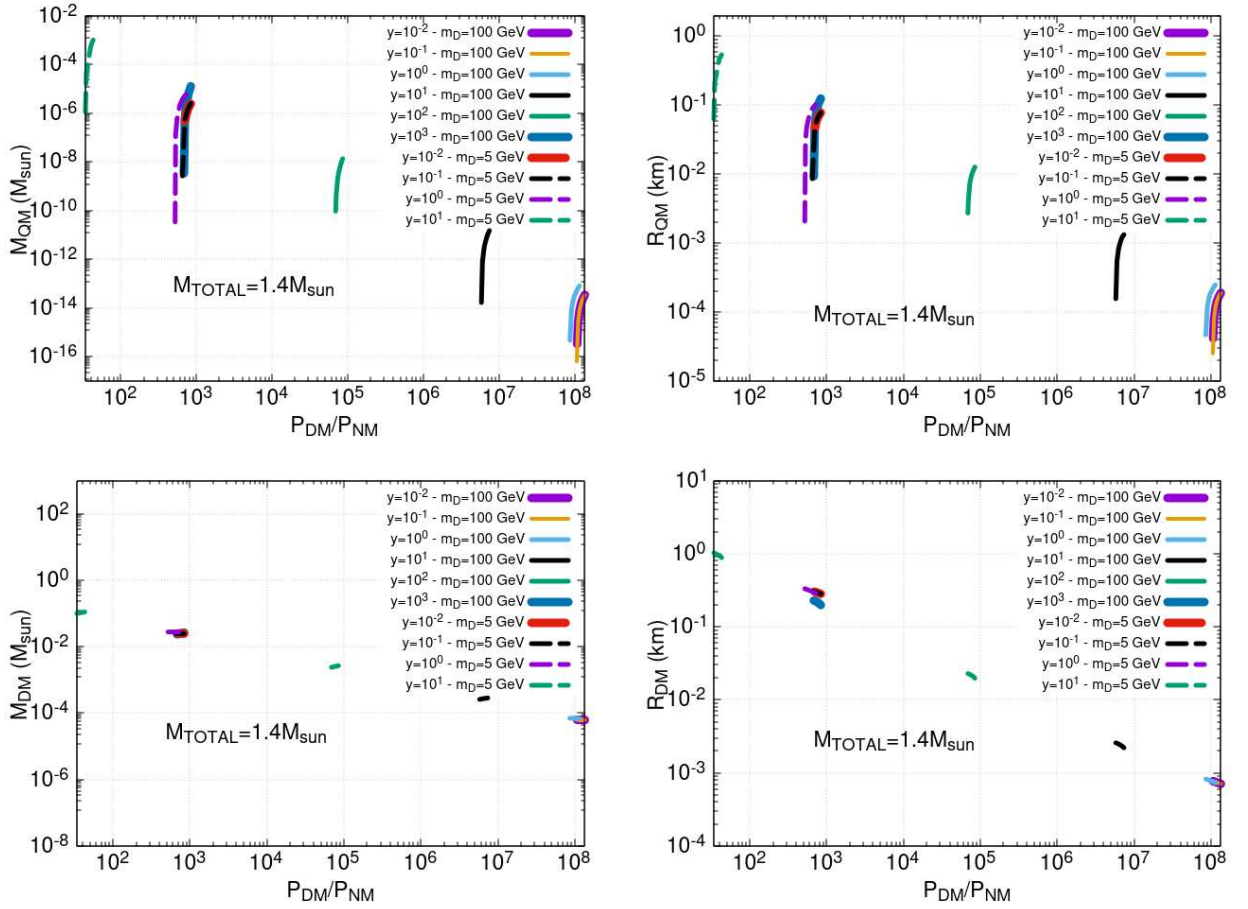


Figure 41 – Mass (top left) and radius (top right) of deconfined quark matter and mass (bottom left) and radius (bottom right) of dark matter in function of the pressure ratios P_{DM}/P_{NM} of $1.4M_{\odot}$ hybrid stars for different interacting strengths y and masses of the DM particle m_D .

We can see that only a small interval of pressure ratios P_{DM}/P_{NM} for each y and m_D value result in stable $1.4M_{\odot}$ hybrid stars. Here we include the results for more values of y than in the previous section, but the interval of ratios shown are the same when considering the same y and m_D shown in Fig. 39 after the phase transition line. So, the same conclusions drawn there remain valid here.

At first glance, it may seem that larger DM pressure ratios result in less deconfined QM and of DM mass, but we have to remember that each line shown in this plot corresponds to different types of DM, i.e., to different interacting strengths and particle masses, and, as discussed in the previous section, the values of y and m_D determine how much of DM needs to be added for the composition of the star to start to change. So, if we analyze each line separately, we see that what happens is exactly what is expected, the larger the DM pressure, the more deconfined QM we get. As

from the value of the central pressure up to zero pressure.

m_D (GeV)	y	ratios	$M_{DM}(M_\odot)$	$R_{DM}(km)$
100	10^{-2}	$1.07 \times 10^8 - 1.35 \times 10^8$	$6.25 \times 10^{-5} - 6.15 \times 10^{-5}$	$7.69 \times 10^{-4} - 7.05 \times 10^{-4}$
100	10^{-1}	$1.06 \times 10^8 - 1.35 \times 10^8$	$6.26 \times 10^{-5} - 6.16 \times 10^{-5}$	$7.72 \times 10^{-4} - 7.04 \times 10^{-4}$
100	10^0	$8.60 \times 10^7 - 1.15 \times 10^8$	$6.92 \times 10^{-5} - 6.86 \times 10^{-5}$	$8.22 \times 10^{-4} - 7.27 \times 10^{-4}$
100	10^1	$5.80 \times 10^6 - 7.40 \times 10^6$	$2.57 \times 10^{-4} - 2.84 \times 10^{-4}$	$2.57 \times 10^{-3} - 2.20 \times 10^{-3}$
100	10^2	$6.90 \times 10^4 - 8.50 \times 10^4$	$2.37 \times 10^{-3} - 2.66 \times 10^{-3}$	$2.28 \times 10^{-2} - 1.97 \times 10^{-4}$
100	10^3	$6.80 \times 10^2 - 8.40 \times 10^2$	$2.34 \times 10^{-2} - 2.64 \times 10^{-2}$	$2.28 \times 10^{-1} - 1.98 \times 10^{-1}$
5	10^{-2}	$7.00 \times 10^2 - 8.40 \times 10^2$	$2.49 \times 10^{-2} - 2.45 \times 10^{-2}$	$3.01 \times 10^{-1} - 2.81 \times 10^{-1}$
5	10^{-1}	$6.50 \times 10^2 - 8.40 \times 10^2$	$2.50 \times 10^{-2} - 2.46 \times 10^{-2}$	$3.09 \times 10^{-1} - 2.81 \times 10^{-1}$
5	10^0	$5.20 \times 10^2 - 7.20 \times 10^2$	$2.76 \times 10^{-2} - 2.73 \times 10^{-2}$	$3.31 \times 10^{-1} - 2.89 \times 10^{-1}$
5	10^1	$3.40 \times 10^1 - 4.30 \times 10^1$	$9.90 \times 10^{-2} - 1.10 \times 10^{-1}$	$1.02 - 8.80 \times 10^{-1}$
5	10^2	$1.205 - 1.207$	$4.74 \times 10^{-1} - 4.75 \times 10^{-1}$	$5.90 - 5.88$
5	10^3	never	-	-

Table 28 – Minimum and maximum DM pressure ratios P_{DM}/P_{NM} that result in stable $1.4M_\odot$ hybrid stars and its corresponding total DM mass for different values of the interacting strength y and DM particle mass m_D .

for the quantity of DM, its value is not as sensible as the QM to this small interval of pressure ratios, being that the mass remains almost constant and with the same order of magnitude. Nevertheless, from table 28 we can see that the mass slightly decreases with the increase of the pressure ratio for $10^{-2} < y < 1$ and increases for higher values of y for both m_D s. When we compare the deconfined QM mass with the DM mass we see that there is much more DM. For example, for $y = 10^{-2}$ and $m_D = 100$ GeV, the DM configuration to which we get the least QM and DM, we can get a mass up to $M_{QM} = 3.38 \times 10^{-14} M_\odot$ (or 6.72×10^{16} kg) whereas for the DM the maximum mass we get is $M_{DM} = 6.25 \times 10^{-5} M_\odot$ (or 1.24×10^{26} kg). And for the $y = 10^2$ and $m_D = 5$ GeV, not included in the plots, we get $M_{QM} = 1.95 \times 10^{-3} M_\odot$ (or 3.88×10^{27} kg) and $M_{DM} = 4.75 \times 10^{-1} M_\odot$ (or 9.45×10^{29} kg). So, it is clear that the major contribution to the total mass comes from the hadronic matter.

If we now look at the radii formed by each type of matter we see that the QM forms a very small radius at the core of the star and follows the behavior of the QM mass, smaller masses also mean smaller radii, and as the mass increases, the radius increases in the same proportion. Its values can be as small as 4.14×10^{-5} km (or 4.14 cm) and reach at most 0.539 km (for $y = 10^2$ and $m_D = 5$ GeV, not shown in the plot, we get a radius of 0.749 km). As for the DM radii, for all the cases shown here, its value is also small but slightly bigger than the QM radii, with values varying from 7.04×10^{-4} km (or 70.4 cm) up to 5.90 km. Its value is also not as sensible to the pressure ratios as the QM radius, but here, in all cases, for the same combination of y and m_D the radius decreases with the increase of the pressure ratio.

As we have shown here the results only for $1.4M_\odot$ hybrid stars, one may wonder what is the proportion of QM and DM for different masses. Stars with larger masses

already have a higher central pressure without DM, so, the DM pressure ratio needed to reach the hadron-quark phase transition pressure is smaller and, as a direct consequence, the quantity of DM in those stars will be smaller, but the amount of QM can be increased. Stars with less than $1.4M_{\odot}$ will require a lot more of DM pressure to reach the P_0 pressure and so those stars will have a bigger and more massive DM core, but a less massive QM core. At the same time the radius of the DM decreases, so that the DM core of those stars is denser. But, if we look back at Fig. 40 we see that, depending on the type of DM, there is a limit in the minimum mass of the star that can be hybrid. That is because, eventually, the DM renders the low mass stars unstable.

It is also worthy to call attention here to the fact that we get really very small quark cores as compared to what we obtained in the last chapter. But we have to remember that the goal of the present chapter is to show that the DM can trigger the appearance of QM and because of that, we chose a combination of NM parametrizations where there is no QM at all at a stable star. With the conclusion drawn here, we can now argue that for NM configurations that already allow the appearance of deconfined QM, with the addition of DM, the amount of QM can be even larger.

6.3.4 Dark Oysters

In this final section of results we want to take a closer look at the DM that is really bad at masquerading hybrid stars, i.e., we want to study the behavior of the strongly interacting DM $y = 10^3$ with a particle mass of $m_D = 5$ GeV.

From Fig. 36, top right, we can see that the mass-radius diagram is very sensible to this configuration of DM, already changing at very low P_{DM}/P_{NM} and, differently than what happens to the other DM configurations, the mass of the stars increases. If we also take a look at Fig. 37 we can have a clue why we call this section 'Dark Oysters', because the DM radius is larger than the NM radius so that the DM forms a halo around the NM core. In all the other DM configurations discussed in this chapter, the DM radius was smaller than the NM one, forming only a core inside the star. In this section, besides all the NM being mixed with DM, more DM forms a halo around the NM core.

In Fig. 38 we have shown the results only for observable radii and only for values smaller than 20 km. We next plot in Fig. 42 the equivalent, but showing instead the DM radius. At the left we show the total mass and at the right the DM mass in function of the DM radius. These plots leave even more evident the big halo that the DM forms, being that the R_{DM} is always bigger than the observable one. And, if we compare the total mass with the DM mass it is also evident that the DM dominates over the NM. Only for the $P_{DM}/P_{NM} = 10^{-1}$ the difference is noticeable.

In fact, for larger pressure ratios the first stable branch is of very low NM central pressures. In fact, the higher the DM pressure ratio the more the white dwarf branch

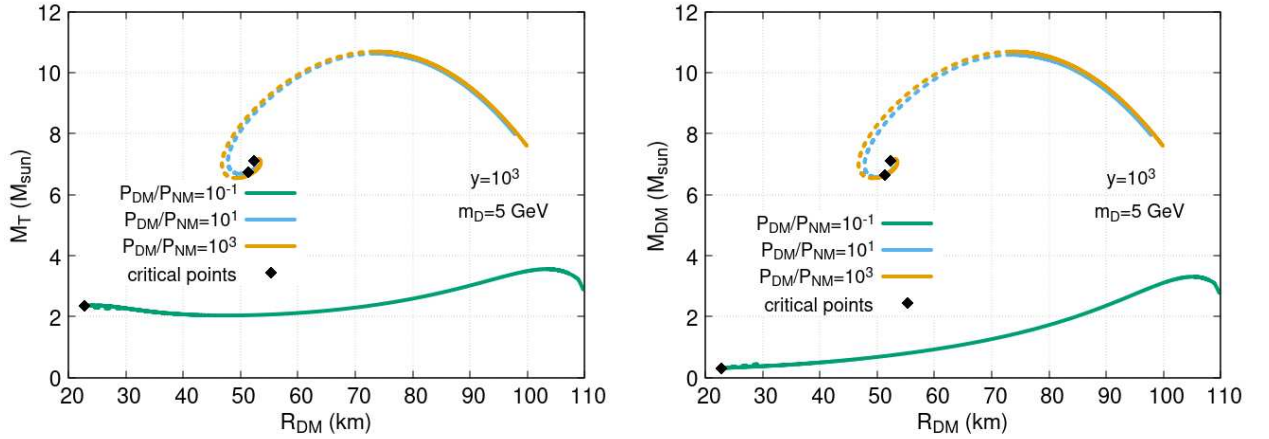


Figure 42 – Total mass (left) and DM mass (right) in function of the DM radius for strongly ($\gamma = 10^3$) interacting DM and $m_D = 5$ GeV for different pressure ratios for values of the observable radius $R_{NM} < 20$ km. The dots indicate the critical mass M_{crit} at which hybrid stars start to appear for each configuration. We also include the unstable results (dashed lines).

migrates to lower NM ratios, as can also be seen in Fig. 36.

6.4 CONCLUSIONS

In this chapter, we have investigated the effect of ADM on hybrid stars. To build the hybrid stars we used the same models as in the previous chapter, i.e., QHD-based model and MIT-based model, but with a different parametrization for the latter. For DM we used the EoS from (NARAIN; SCHAFFNER-BIELICH, Jürgen; MISHUSTIN, 2006) for a non-annihilating massive DM particle and explored different interacting strengths and two different masses for the DM particle, $m_D = 5$ GeV and $m_D = 100$ GeV.

In the first section of the results, we analysed the mass-radius relation for the NS and WD branches and showed that when one of the pressures is much larger than the other, the fluid with the largest central pressure dominates and the configuration behaves as a single fluid. The DM pressure ratio necessary for DM dominance depends on the interacting strength and the mass of the DM particle, confirming what was already discussed in (DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2022; TOLOS; SCHAFFNER-BIELICH, Jürgen, 2015; DENGLER; SCHAFFNER-BIELICH, Jürgen; TOLOS, 2021; DELIYERGIYEV et al., 2019). We also showed that for strong interacting DM and a mass of $m_D = 5$ GeV the mass of the COs increase.

Then, we concentrated only on the NS branch and showed that the DM increases the central pressure of an already compact star and that ultimately can allow the appearance or even the increase in the amount of deconfined QM in its core. We concluded that, except for the strong interacting DM and a mass of $m_D = 5$ GeV, we can get the same mass stars with closely the same radius, but with different central pressures,

depending on the amount of DM added, which, in turn, can allow the appearance of deconfined QM. We conclude that DM is masquerading hybrid stars. We also showed that the amount of DM necessary to enhance the appearance of QM depends both on the interacting strength and the mass of the DM particle, being less for $m_D = 5$ GeV than for $m_D = 100$ GeV and more for weakly interacting DM.

In the following section we investigated the amount of QM and DM such stars can accumulate and concluded that it is actually not a lot. For $1.4M_\odot$ stars we managed to get at most $1.95 \times 10^{-3} M_\odot$ with a radius of 0.749 km for the QM core and $4.75 \times 10^{-1} M_\odot$ with a radius of 5.90 km for the DM, remembering that this last one is always mixed with the NM. We also showed that only very small intervals of pressure ratios produce stable dark matter admixed hybrid star for each combination of interacting strength and m_D , but that, within this interval, the QM mass and radius increase as the ratio increases, whereas the DM mass and radius does not change much, but the R_{DM} always decreases.

At the last section of results we took a the DM configuration that behaves much different than the rest analyzed before, namely the strongly interacting DM with $m_D = 5$ GeV. This configuration produces stars where the R_{DM} is bigger than the NM one, so that the DM, besides being mixed with all the NM, also forms a halo around the NM. We called these objects dark oysters.

Most of the results presented in this chapter will appear in a future publication that is being prepared.

7 CONCLUSION

From the theoretical point of view, there are still many limitations for the QCD to be solved, which led physicists to attack the problem from two different perspectives: through lattice QCD (LQCD), and through effective models. Due to computational constraints and numerical difficulties, the LQCD method only achieves results for low chemical potentials. This makes effective models currently the only source to generate results that cover the QCD phase diagram at higher chemical potentials.

In this thesis we have proposed an improvement of an already well-known effective model to describe quark matter, the MIT bag model. In chapter 2 we proposed some improvements to this model, enabling it to satisfy recent astrophysical constraints, exploring different features and eventually, also combining it with another effective model, the QHD-based model.

In chapter 2 we revised the vector MIT Bag model and added a vector field mass term, absent on models already found in the literature. Considering the universal coupling and also using symmetry group to establish the relation between the meson-quark coupling constants for the different quarks, an innovation for this kind of model, we built the stability window and then were able to reproduce strange stars with masses up to $2.40M_{\odot}$ that satisfied all astrophysical constraints of mass, radius and also tidal deformability while also maintaining thermodynamic consistency. Later on, we also added a quartic term in the vector channel, which gave more malleability to the EoS at higher densities. So, with the proposed modifications to the MIT Bag model, we were able to reconcile the Bodmer-Witten conjecture with the existence of massive compact stars in agreement with recent astrophysical measurements.

In chapter 3 we explored new aspects of the proposed MIT-based model at finite temperatures and constructed QCD phase diagrams besides taking a look at the mass-radius relations for proto-quark stars. In order to obtain transition temperatures at vanishing chemical potentials that were in agreement with the results coming from LQCD, we had to further improve our model by transforming the Bag constant in a temperature-dependent Bag parameter. With this new implementation, we were able to construct QCD phase diagrams in agreement with the LQCD results, were able to fit the Cleymans line of experimental data related to the freeze-out (a completely hadronic process, totally inside the hadron phase) and also to obtain chemical potentials at $T = 0$ in the range $1050 < \mu_B < 1400$ MeV, satisfying all the restraints we aimed to satisfy in this chapter. However, the parametrization we used lies outside the stability window. As for the mass-radius relation, we did not observe significant changes for $T = 40$ MeV. In this chapter, we also showed that the conformal limit of the speed of sound ($v_s^2 < 1/3$) is always violated for the modified MIT Bag models.

In chapter 4 we built QCD phase diagrams combining the Modified MIT Bag

model with the QHD-based model using Gibbs' conditions as criteria for hadron-quark phase transition. Some of the same conclusions drawn in the previous chapter were also drawn in this one: the temperatures of phase transition at $\mu_B \rightarrow 0$ depend only on the Bag pressure value and on the matter hypotheses whereas the μ_B at $T = 0$ depends on all of the parametrizations and matter hypotheses and that the stellar matter favours the quark phase if compared with the symmetric matter. When combining two models, though, the effects of the Bag pressure value and the vector field at $T \rightarrow 0$ are increased. Only with a temperature-dependent Bag we were able to build QCD diagrams in agreement with all the restraints aimed. In this chapter, we also argued that calculations of the latent heat may be used to locate the CEP of the QCD phase diagram.

In chapter 5 we used the same models as in chapter 4 to build hybrid stars and concluded that, indeed, only with the modified versions of the MIT Bag model it is possible to reproduce hybrid stars within the restraints imposed by the most recent astrophysical measurements of compact stars. We showed that, within the Vector MIT Bag model it is possible to obtain hypermassive quark cores corresponding up to 84% of the total mass of the star.

And finally, in chapter 6, we investigated the effect of dark matter on hybrid stars considering a two-fluid system, with the dark matter only interacting gravitationally with the ordinary matter. We concluded that the presence of DM increases the central pressure of neutron stars which can trigger the appearance of deconfined quark matter in the core of those stars.

We showed throughout this thesis that the Modified MIT models introduced in chapter 2 are really good effective models in their simplicity, being that they perform very well within what they propose. More terms could be added to improve the models and satisfy even more restraints. More precise astrophysical measurements, as well as hot quark matter experimental data, would help in this regard. Other tests, involving dissipation effects in isolated rotating neutron-like stars or mergers could also be performed, as transport coefficients, such as bulk viscosity, are very EoS dependent (HERNÁNDEZ; MANUEL; TOLOS, 2024). The last chapter showed us that DM makes physics even more interesting and that more theoretical works are necessary to find ways of detecting directly or indirectly this still elusive matter.

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APPENDIX A – BODMER-WITTEN CONJECTURE AND THE STABILITY WINDOW

We usually assume that the ground state of matter is the state in which quarks are confined inside hadrons. And indeed, our everyday experience seems to confirm this idea. However, the confined state can simply be a very long-lasting state, but not an absolutely stable one. Bodmer (BODMER, 1971) and Witten (WITTEN, 1984), independently proposed the Strange Matter hypothesis, also called Bodmer-Witten Hypothesis, or even Bodmer-Witten Conjecture. This hypothesis proposes that the true and absolute ground state of strong interactions is the deconfined state of quark matter consisting of an approximately equal proportion of quarks u , d and s , such matter being called strange matter (SM) (GLENDENNING, 2000).

The Bodmer-Witten conjecture is conditional on the binding energy per baryon of the matter of stable deconfined quarks having a value lower than the energy per nucleon of the ^{56}Fe , the element that has the highest binding energy per nucleon. The binding energy per baryon is given by

$$\frac{E}{A} = \frac{\mathcal{E}}{n_B}, \quad (67)$$

where E is the binding energy, A is the baryonic number and $\mathcal{E}$ and n_B are the energy and baryon densities, respectively. According to this hypothesis, strange matter will be stable at zero pressure if the condition

$$\frac{\mathcal{E}_{(u,d,s)}}{n_{B(u,d,s)}} \leq 930 \text{ MeV} \quad (68)$$

is satisfied while, at the same time, the matter of two undefined flavors is not stable, that is,

$$\frac{\mathcal{E}_{(u,d)}}{n_{B(u,d)}} > 930 \text{ MeV}. \quad (69)$$

Otherwise atomic nuclei could be made only from deconfined matter, which contradicts experience.

If we are taking into account the Bodmer-Witten conjecture we should also discuss the stability window, identified as the model parameters that are consistent with the fact that two-flavour quark matter must be unstable (Eq. (69)) and SM must be stable (Eq. (68)). In this thesis the model parameters are the Bag constant B , the meson-quark coupling constant G_V , the constant of the self-interaction term b_4 and eventually also the mass of the quark s (m_s).

It is worth mentioning that the regions outside the stability window on the left (stable $u-d$ matter) and on the right (unstable $u-d-s$ matter), see, for example, Fig. 3, have very different meanings and consequences. If we use values of the constants

describing stable $u - d$ matter, we would be describing an universe in which matter composed of protons and neutrons would no longer exist. But if we use values that describe unstable $u - d - s$ matter, this would only imply that strange matter is not the ground state of matter. In short, the use of values that describe stable $u - d$ matter is never allowed, but the of unstable $u - d - s$ matter may be allowed, in fact this last condition enables the existence of deconfined quark matter in the cores of massive hybrid stars, as we discuss in chapter 5. On the other hand, the Bodmer-Witten conjecture is what could explain the existence of strange quark stars, because if strange matter is in fact the true ground state of matter, as soon as the core of the star converts to the quark phase, the entire star converts. So, in order to describe strange stars, we use the stability window to limit our choice of parameters. This is done in chapter 2 and 3.

In Ref. (TORRES; MENEZES, 2013) it is shown that slightly different results are obtained if we consider matter with identical quark chemical potentials, corresponding to equal quantities of u and d quarks in two-flavour matter, or charge neutral matter in beta-equilibrium, as expected in stars. For the models investigated in the afore mentioned reference, the stable parameter region is larger if the beta-equilibrium condition and charge neutrality are imposed. In chapter 2 of this thesis we consider only symmetric matter with with identical quark chemical potentials.

Understanding the stability of strange matter in protoquark stars is also important because stars cool down slowly after deleptonization, as we discuss in the text. When establishing the stability window at finite temperature the free energy per baryon comes into play. So we have $\mathcal{F}/A = f/n_B = (\varepsilon - Ts)/n_B$, where f is the free energy density, n_B the baryon density, ε the energy density, T the temperature and s the entropy density of the system. For zero temperature, the free energy density becomes the energy density and hence, the binding energy per baryon ($B/A = \varepsilon/n_B$) is analyzed at zero temperature in the search for stable matter. The lower limits of the stability window, given by the points where the two-flavor quark matter has an energy per baryon larger than 930 MeV, however, remain the same because the ^{56}Fe ground state is a zero temperature system. In (DEXHEIMER; TORRES; MENEZES, 2013) is showed that a wider range of constants becomes possible to ensure stable matter at finite temperatures. In this thesis, even though we also consider hot stellar matter, we only obtain the stability window for zero temperature.

APPENDIX B – STABILITY ANALYSIS

B.1 STABILITY ANALYSIS FOR A ONE FLUID SYSTEM

One of the most sophisticated and complete ways of checking the stability of a compact star is by taking a look at small radial perturbations of the equilibrium configuration. One adds a harmonic perturbation of the form $\xi(r)e^{-i\omega t}$ to the metric fields and then linearizes the Einstein field equations with respect to this perturbation. The result of this is a Sturm-Liouville eigenvalue equation. The eigenfunctions can be labeled by the number of nodes n with eigenfrequencies ω_n , which are ordered in energy as

$$\omega_0^2 < \omega_1^2 < \omega_2^2 < \dots \quad (70)$$

A negative ω_n leads to an exponential growth of the radial perturbation and collapse of the star. So, if $\omega_0^2 > 0$, all modes must be real and the star configuration in total is stable with respect to all possible radial oscillations (SCHAFFNER-BIELICH, Jürgen, 2020; SHAPIRO; TEUKOLSKY, 2008).

Another simpler way to analyze the stability of a compact star is by looking at the relation between the mass M and the radius R with the central energy density ε_c . The mass should always increase with increasing ε_c :

$$\frac{dM}{d\varepsilon_c} > 0 \quad (71)$$

Consequently, when $\frac{dM}{d\varepsilon_c} = 0$ there is a change in the stability of the star, which also means that a ω_n is changing its sign. Now, to know which mode is changing its sign one needs to look at the slope of the radius in function of the ε_c at the point of the extremum ($\frac{dM}{d\varepsilon_c} = 0$):

$$\left. \frac{dR}{d\varepsilon_c} \right|_{\text{extremum}} \begin{cases} < 0 & \rightarrow \omega_n^2 \text{ of an even mode changes sign} \\ > 0 & \rightarrow \omega_n^2 \text{ of an odd mode changes sign.} \end{cases} \quad (72)$$

This way of making the stability analysis becomes clearer if we take a look at Fig. B.1, where we display the mass and the radius versus the central energy density of white dwarfs¹ and neutron stars.

The points A to E mark the extrema of the mass as a function of the central energy density. Once we know that the results at low densities before point A are stable, we can analyse each point:

¹ White dwarfs are the final evolutionary state of stars whose mass are not high enough to become neutron stars or black holes. And, differently from NSs, they are composed mostly of electron-degenerate matter.

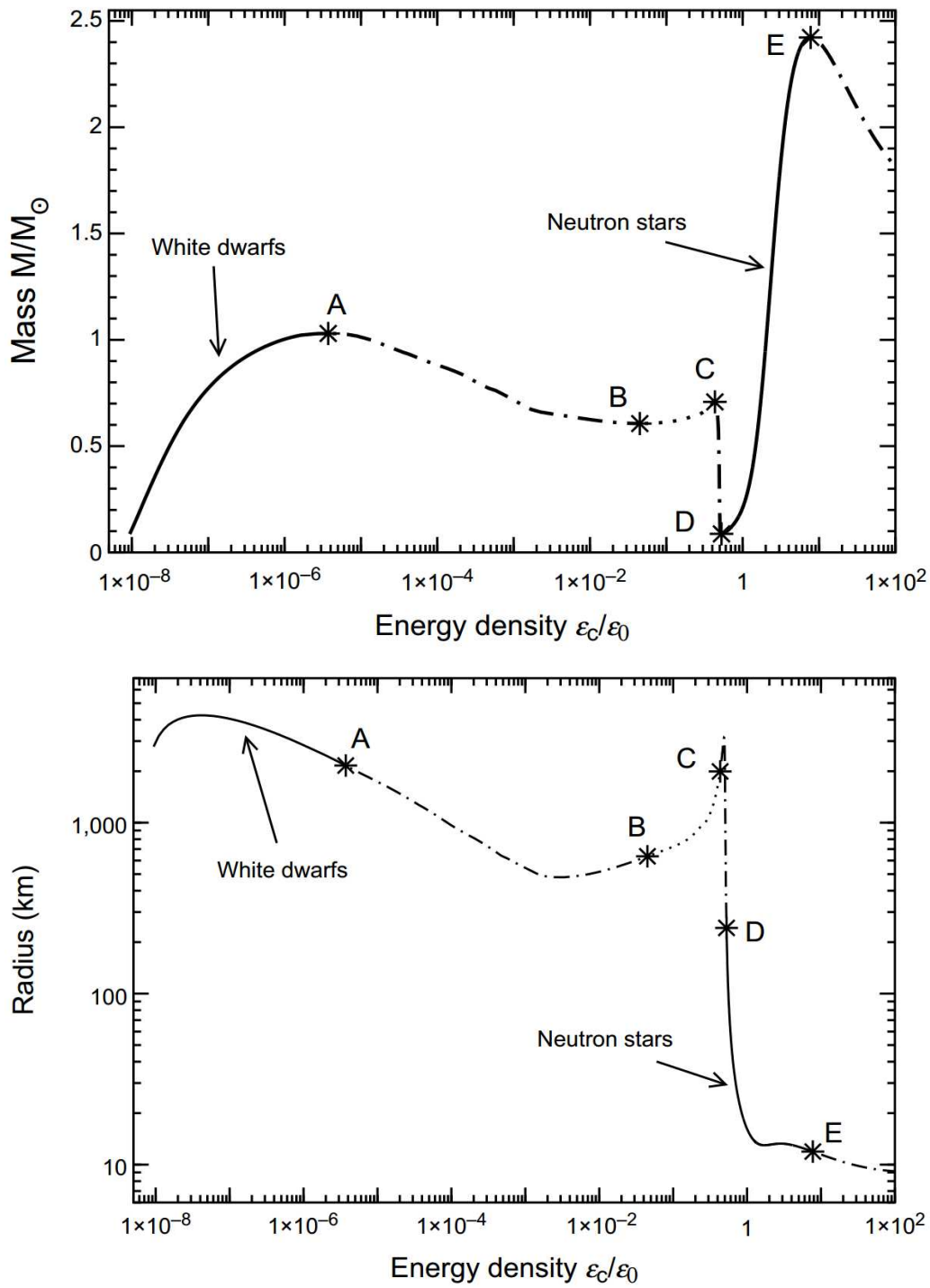


Figure B.1 – The mass (top) and radius (bottom) of white dwarfs and neutron stars versus the central energy density. Extracted from (SCHAFFNER-BIELICH, Jürgen, 2020).

- point A: as it is a maximum in the mass and the radius decreases with the ϵ_c , an even mode changes sign. It must be the lowest mode that changes, i.e., ω_0^2 , so the configurations after the point A are unstable;
- point B: as it is a minimum in the mass and now the radius is increasing with the ϵ_c , an odd mode changes sign, which can only be ω_1^2 . So, from this point

on two modes are unstable, ω_0^2 and ω_1^2 ;

- point C: this point is again a maximum in the mass, but this time the radius is increasing with the ε_C , so an odd mode changes sign, which means that ω_1^2 is back on being positive and stable again, but as ω_0^2 still remains unstable, we continue in an unstable configuration;
- point D: we are again at a minimum in the mass, but now the radius decreases with increasing ε_C , which means that an even mode is changing sign, i.e., ω_0^2 turns positive and stable again and so, from the point D to, at least, the point E we have stable configurations;
- point E: a maximum in the mass with decreasing radius, so, as on point A, an even mode changes sign and the configurations after are all unstable.

Now the only thing left to do is to correspond these points to a mass-radius diagram, as done in Fig. B.2. As can be seen, only the configurations with large radii up to point A (which corresponds to the white dwarfs branch) and with rather smaller radii between points D and E (corresponding to the neutron stars branch) are stable.

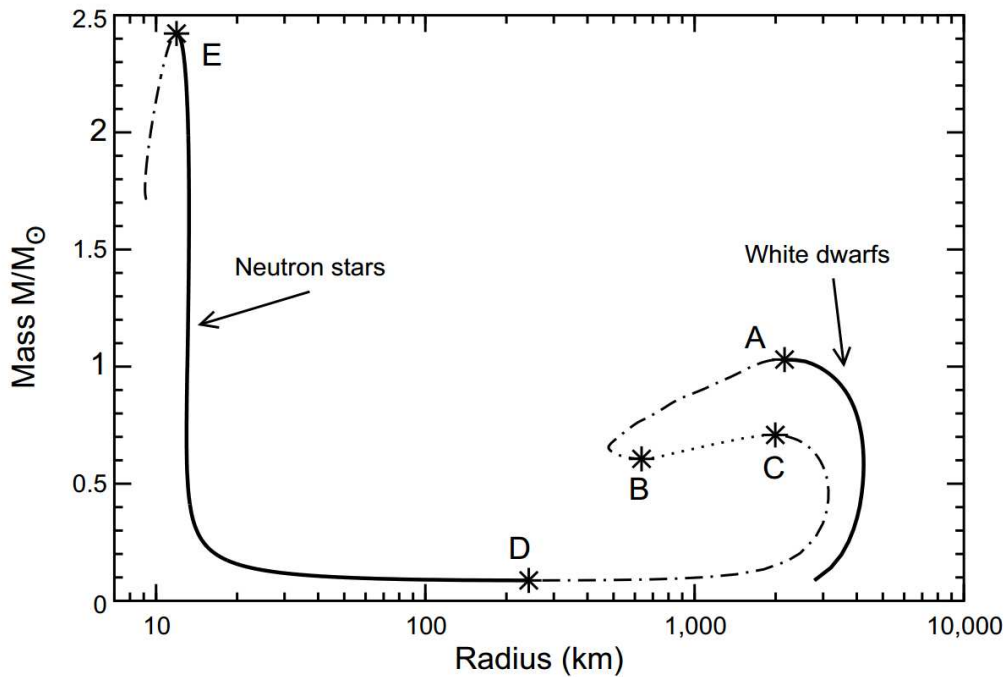


Figure B.2 – The mass-radius relation for white dwarfs and neutron stars. Extracted from (SCHAFFNER-BIELICH, Jürgen, 2020).

Another way to do this analysis is by only looking at the extremum of the mass-radius relation and examine if the curve is going clockwise or counterclockwise as the following rule:

Extremum	Clockwise	Counterclockwise
Maximum	stable	unstable
Minimum	stable	unstable

Table 29 – Conditions for modes getting stable or unstable for mass-radius relations of compact stars. Extracted from (SCHAFFNER-BIELICH, Jürgen, 2020).

To use this criterion one should start by looking at the results corresponding to very low densities, which normally corresponds to very large radii in the case of hybrid or hadron stars, and, assuming that these results are stable, search for the extremum and follow the rules given by table 29. This method is completely equivalent to the method of analysing the $M \times \varepsilon_c$ and $R \times \varepsilon_c$ curves explained above.

B.2 STABILITY ANALYSIS FOR A TWO FLUID SYSTEM

For a two-fluid system, as the one addressed in Chapter 6, one can also employ the simpler methods explained above for each fluid separately and consider stable only the results for which both fluids are stable.

However, with this method of analysis some stable results may be wrongly discarded as it is possible that the oscillation modes, regarded separately as unstable, when combined turn the star stable. For that reason, one may employ a method that takes this into account. Here we will make use of a different method based on the one developed by (HENRIQUES; LIDDLE; MOORHOUSE, 1990) and expanded in (KAIN, 2021; DI GIOVANNI et al., 2022; VALDEZ-ALVARADO et al., 2013; NYHAN; KAIN, 2022), more precisely, we will follow closely what was done in (HIPPERT et al., 2023). However, we have to point out that this criterion is less rigorous than solving the Sturm-Liouville problem for two fluids.

Going back to the harmonic perturbation of the form $\xi(r)e^{-i\omega t}$ added to the metric fields, at the critical point, where the star changes stability, i.e., when $\omega_0^2 = 0$, the field perturbations are time-independent and, therefore, this corresponds to an equilibrium configuration, as argued above. Once the equilibrium configurations are uniquely specified by central densities, we have that this small static perturbations correspond to the shifts $\varepsilon_c^n \rightarrow \varepsilon_c^n + \delta\varepsilon_c^n$. Besides that, if we reach $\omega_0^2 \rightarrow 0^+$ from the positive side, these configurations will be joined by arbitrarily slow oscillations, which must leave the total number of fluid elements N_i unchanged (HIPPERT et al., 2023).

In summary, this means that at the point where the stability changes, the total number of both fluid elements must be stationary under variations of ε_c :

$$\begin{pmatrix} \delta N_{NM} \\ \delta N_{DM} \end{pmatrix} = \begin{pmatrix} \partial N_{NM}/\partial \varepsilon_c^{NM} & \partial N_{NM}/\partial \varepsilon_c^{DM} \\ \partial N_{DM}/\partial \varepsilon_c^{NM} & \partial N_{DM}/\partial \varepsilon_c^{DM} \end{pmatrix} \begin{pmatrix} \delta \varepsilon_c^{NM} \\ \delta \varepsilon_c^{DM} \end{pmatrix} = 0, \quad (73)$$

where NM and DM stand for normal matter, or ordinary matter, and dark matter, respec-

tively. As $\delta\epsilon_C^i \neq 0$, we will have

$$\frac{\partial N_{NM}}{\partial \epsilon_C^{NM}} \frac{\partial N_{DM}}{\partial \epsilon_C^{DM}} - \frac{\partial N_{NM}}{\partial \epsilon_C^{DM}} \frac{\partial N_{DM}}{\partial \epsilon_C^{NM}} = 0. \quad (74)$$

This is the criteria for the onset of radial instability for two-fluid stars (HIPPERT et al., 2023).

The condition $\partial N/\partial \epsilon_C = 0$ is equivalent to the condition $\partial M/\partial \epsilon_C = 0$ discussed above for a one fluid system, but only if the TOV equations are satisfied, and, therefore, exclusively for equilibrium configurations. So we have that, at the onset of instability, $\partial N_A = 0$, $\partial N_B = 0$ and $\partial M = 0$ under variations of the central energy densities, $(\epsilon_C^A, \epsilon_C^B) \rightarrow (\epsilon_C^A + \delta\epsilon_C^A, \epsilon_C^B + \delta\epsilon_C^B)$.

For a two-fluid star the physical picture is less clear, so, we go one step further and, to determine the stable solutions of a two-fluid star, we diagonalize the matrix $\partial N_i/\partial \epsilon_j$, with $\{i, j\} = \{DM, OM\}$, in eq. (73):

$$\begin{pmatrix} \delta N_A \\ \delta N_B \end{pmatrix} = \begin{pmatrix} \kappa_A & 0 \\ 0 & \kappa_B \end{pmatrix} \begin{pmatrix} \delta\epsilon_C^A \\ \delta\epsilon_C^B \end{pmatrix} \quad (75)$$

where κ_A and κ_B are the eigenvalues. As N_A and N_B are linear combinations of N_{NM} and N_{DM} , they are also conserved and kept fixed as the star is perturbed. Changes in ϵ_C^A and ϵ_C^B should be done independently and the stable configurations will be the ones where both eigenvalues are positive

$$\kappa_A > 0 \quad \text{and} \quad \kappa_B > 0. \quad (76)$$

In chapter 6 we determine stability by using the condition in Eq. (76).

To obtain the total number of particles we solve the following equations together with the TOV equations

$$\frac{dN'_i}{dr} = 4\pi \frac{n'_i}{\sqrt{1 - \frac{2(m_{NM} + m_{DM})}{r}}} r^2 \quad (77)$$

where $n'_i = n_i/m_D^3$ and N_i are the dimensionless baryon number density and the total baryon number, respectively, of $i = \{NM, DM\}$.

ANNEX A – PAPERS

In the following we present the papers already published in the order they were presented in this thesis.

PAPER

Modified MIT bag Models—part I: Thermodynamic consistency, stability windows and symmetry group

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PAPER

Modified MIT bag Models—part I: Thermodynamic consistency, stability windows and symmetry group

Luiz L Lopes¹ , Carline Biesdorf² and Débora P Menezes² ¹ Centro Federal de Educação Tecnológica de Minas Gerais Campus VIII, CEP 37.022-560, Varginha, MG, Brazil² Departamento de Física, CFM—Universidade Federal de Santa Catarina; C.P. 476, CEP 88.040-900, Florianópolis, SC, BrazilE-mail: llopes@cefetmg.br**Keywords:** Modified MIT bag Models, stability window, thermodynamic consistency, massive stars, strange matter, canonical stars**Abstract**

In this work we study different variations of the MIT bag model. We start with the so called non-ideal bag model and discuss it in detail. Then we implement a vector interaction in the MIT bag model that simulates a meson exchange interaction and fix the quark-meson coupling constants via symmetry group theory. At the end we propose an original model, inspired by the Boguta-Bodmer models, which allows us to control the repulsion interaction at high densities. For each version of the model we obtain a stability window as predicted by the Bodmer-Witten conjecture and discuss its thermodynamic consistency.

1. Introduction

According to the quantum chromodynamics (QCD) phase diagram, deconfined quark matter existed in the early universe when the temperature was very high. The region, where it is foreseen is called quark-gluon plasma (QGP). At zero or low temperatures, however, deconfined quark matter may be present in the core of massive compact objects like neutron stars, as quark matter becomes energetically favorable over hadronic matter.

Moreover, Bodmer and Witten proposed that the ordinary matter we know, composed of protons and neutrons may be only meta-stable. The true ground state of strongly interacting matter would therefore consist of strange matter (SM), which in turn is composed of deconfined up, down and strange quarks [1, 2]. If this is true, as soon as the core of the star converts to the quark phase, the entire star will convert into what is called a strange star in a finite amount of time [3].

Also, it is worth pointing out that if the quark-quark interaction is attractive in any channel, then the true ground state of the system may not be the naked Fermi surface, but rather a complicated coherent state of particle and hole pairs - *Cooper pairs*, generally called *color superconductivity*. The quark pairs play the same role here as the Higgs particle does in the standard model; the color-superconducting phase can be thought of as the spontaneously broken (as opposed to confined) phase of QCD. A detailed study of the color-superconducting phase is beyond the scope of this work, but the reader can see more details in [4, 5].

There have been some models used to approach the SM hypothesis, the first of them [6] being the original MIT bag model [7]. A modified version of this original model, called a non-ideal bag model [8, 9] has also been used to test this hypothesis. Furthermore, the quark-mass density dependent (QMDD) [10] and the Nambu-Jona-Lasinio (NJL) [11–14] models have also been used to this same purpose. However, they present some issues: the NJL model does not satisfy the Bodmer-Witten conjecture [15] while the QMDD models present some thermodynamic inconsistencies. Although the original model [10] was revisited several times (see [16, 17] and references therein), the problem persisted. More recently, another QMDD model without this defect was introduced [18, 19] and recently also revisited in [20]. Regarding the original version of the MIT bag model, it was shown it is not able to reproduce massive stars as the ones recently detected [16].

In order to better describe quark stars, we would like to have a quark matter model that could reconcile thermodynamic consistency with the Bodmer-Witten conjecture, i.e, the SM being the true ground of the matter, with the existence of massive stars recently detected; this is the main motivation for this work. Towards

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this quest, we first revisit the original MIT bag model in section 2 and the non-ideal-bag model in section 3 and discuss their main caveats. To circumvent the problems we encounter, a improved version of the vector MIT bag model, introduced in [21–26], is proposed in section 4 with the help of symmetry group techniques. Finally, inspired by the Boguta-Bodmer [27, 28] non-linear terms in quantum hadrodynamics, a self-interacting vector field is introduced in section 5. These non-linear terms mimic the Dirac sea contribution, which is absent in mean field approximation [29]. Our work ends in section 6 with a quick discussion about the tidal deformation for quark stars and the comparison with the GW170817 event constraints. Finally, the overall conclusions are presented in section 7.

2. Original MIT bag model

The MIT bag model considers that each baryon is composed of three non-interacting quarks inside a bag. The bag, in its turn, corresponds to an infinity potential that confines the quarks. In this simple model the quarks are free inside the bag and are forbidden to reach its exterior. All the information about the strong force relies on the bag pressure value, which mimics the vacuum pressure. The MIT bag model Lagrangian reads [6]:

$$\mathcal{L} = \sum_{u,d,s} \{ \bar{\psi}_q [i\gamma^\mu \partial_\mu - m_q] \psi_q - B \} \Theta(\bar{\psi}_q \psi_q), \quad (1)$$

where m_q is the mass of the quark q of flavor u, d or s , ψ_q is the Dirac quark field, B is the constant vacuum pressure and $\Theta(\bar{\psi}_q \psi_q)$ is the Heaviside step function to assure that the quarks exist only confined to the bag.

Applying the Euler–Lagrange equations for the quarks, we have:

$$[i\gamma^\mu \partial_\mu - m_q] \psi_q = 0, \quad (2)$$

which gives us a energy eigenvalue for the quark q :

$$E_q = \sqrt{m_q^2 + k^2}, \quad (3)$$

where k is the momentum.

Quarks are fermions with spin 1/2. Hence, the number density and the energy density of the quark matter can be obtained via Fermi–Dirac distribution [30]. At zero temperature approximation the Fermi–Dirac distribution becomes the Heaviside step function, and the energy eigenvalue matches the chemical potential, $E_q = \mu_q$. Therefore the quark number density is:

$$n_q = 2N_c \int_0^{k_f} \frac{d^3k}{(2\pi)^3} \Theta(\mu_q - E_q), = N_c \frac{k_f^3}{3\pi^2} \quad (4)$$

while the energy density is:

$$\epsilon_q = \frac{N_c}{\pi^2} \int_0^{k_f} E_q k^2 dk, \quad (5)$$

where N_c is the number of colors and k_f is the Fermi momentum. On the other hand the bag contribution to the energy density is easily obtained through the Hamiltonian: $\mathcal{H} = -\langle \mathcal{L} \rangle = B$. Then, the total energy density is here the sum over the three lightest quarks plus the bag contribution. The pressure p can be obtained via thermodynamic relations:

$$\epsilon = \sum_q \epsilon_q + B \quad \text{and} \quad p = \sum_q n_q \mu_q - \epsilon = -\Omega \quad (6)$$

where n_q is the number density, μ_q is the chemical potential and Ω is the thermodynamic potential. It is also useful to write down the pressure (as well the thermodynamic potential) as the derivative of the energy with respect to the volume:

$$p_q = -\Omega_q = -\left(\frac{\partial E_q}{\partial V} \right)_T = \frac{N_c}{3\pi^2} \int_0^{k_f} \frac{k^4 dk}{E_q} \quad (7)$$

and the total pressure is the sum of the pressure of each quark flavor plus the contribution of the bag from the Lagrangian: $p = \langle \mathcal{L} \rangle$. Therefore we obtain:

$$p = \sum_q p_q - B. \quad (8)$$

Now, the bag constant is not totally arbitrary. In fact B needs to assure that a two-flavored quark matter is unstable and that it is not the ground state of the hadrons, i.e., its energy per baryon must be higher than 930 MeV at zero pressure, otherwise the protons and neutrons would decay into u and d quarks. On the other hand, if the Bodmer–Witten conjecture [1, 2], that states that the true ground state of the hadronic matter is not

Table 1. Stability window for the MIT bag model.

MIT	Min. $B^{1/4}$	Max. $B^{1/4}$
...	148 MeV	159 MeV

consisted of baryons, but strange matter consisting of $\mu_u = \mu_d = \mu_s$, is true, the three-flavored quark matter needs to be stable (energy per baryon lower than 930 MeV), while the two-flavored quark matter is unstable. Therefore, the bag pressure value B can only assume a range of values, known as the stability window [1, 2]. These values depend on the quark masses. In this work, we assume that the masses of the u and d quarks are 4 MeV. While in the past the mass of the s quark was very ambiguous, today it is known to be around 95 MeV, more precisely, 93_{-5}^{+11} MeV [31]. With these values, we display the range of B for the stability window in table 1. Slightly different values are obtained if different quark masses are chosen, as seen in [16]. Although the minimum value of B remains the same, no matter the mass of the quark s , the upper limit varies from $B^{1/4} = 162$ MeV for $m_s = 0$ to $B^{1/4} = 148$ MeV for $m_s = 240$ MeV, i.e., no stable strange matter is possible if $m_s > 240$ MeV.

3. Non-ideal bag model

The non-ideal bag model, presented in [8, 9], is an empirical correction to the thermodynamic potential - Ω - (or directly to the pressure as in [32]) to match QCD correction of $O(\alpha_s)$ where α_s is the strong coupling constant [8]. In this model, the deviation from the original MIT bag model comes from an adimensional parameter a_4 , where $a_4 = (1 - 2\alpha_s/\pi)$. Although in the non-ideal bag model this term is included in an *ad hoc* way to the thermodynamic potential to reproduce some results coming from lattice QCD, it can be derived both from perturbative QCD [33], and from the hard dense loop (HDL) approximation [34]. When $a_4 = 1$, the original MIT bag model is recovered. For massless quarks the thermodynamic potential reads [8, 9, 32]:

$$\Omega = -a_4 \frac{N_f}{4\pi^2} \mu^4 + B_{eff}, \quad (9)$$

where, μ is the chemical potential of massless quarks, N_f is the number of massless quarks, and B_{eff} is an effective bag pressure value. With this model, the energy and the pressure for each of the massless quarks are given by [9]:

$$\begin{aligned} p &= a_4 \frac{1}{4\pi^2} \mu^4 - B_{eff}, \\ \epsilon &= a_4 \frac{3}{4\pi^2} \mu^4 + B_{eff}, \\ \epsilon &= 3p + 4B_{eff}. \end{aligned} \quad (10)$$

Although this model does not modify the equation of state ($p(\epsilon)$), it was proposed so that the parameter a_4 could change the relation between the pressure and the chemical potential - $p(\mu)$. We discuss here some subtleties of this model.

As this model, in principle, is not constructed at the Lagrangian level, it is hard to understand what kind of interaction could produce such behavior. Also, this model can present thermodynamic inconsistencies if we do not proceed carefully. To maintain the consistency of the model, the number density needs to be scaled too, as in [15]:

$$n_q = a_4 \left(N_c \frac{k_f^3}{3\pi^2} \right) = a_4 n_q. \quad (11)$$

Therefore, here the model is thermodynamically consistent. In addition, assuming only equation (11), all equation (10) come as a consequence for massless quarks in the original MIT model. Therefore, the non-ideal bag model is actually a modification in the quark number, instead of a correction in the MIT model itself. On other hand, the price we have to pay is the use a modification of the Fermi–Dirac distribution [30]. This modification comes from a renormalization at $(4-2\epsilon)$ dimensions in perturbative QCD [33].

To summarize, the non-ideal bag model can be considered as an empirical correction from QCD. With this in mind we construct a stability window for the non-ideal bag model, but using massive quarks, with the same masses as used previously, in the original MIT model. Therefore, for each flavor, we have the following EoS:

$$\epsilon_q = a_4 \frac{N_c}{\pi^2} \int_0^{k_f} E_q k^2 dk, \quad (12)$$

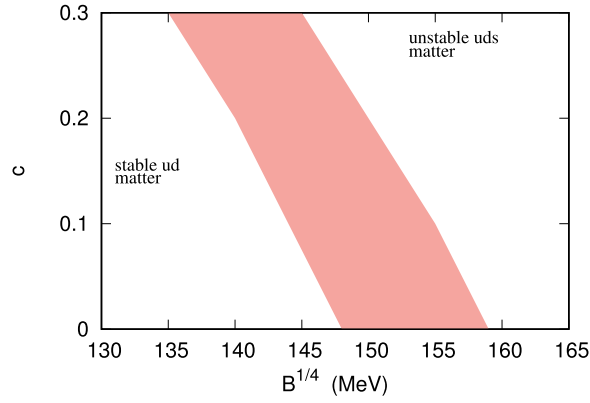


Figure 1. Stability window for the non-ideal bag model. If $c = 0$ we recover the original MIT bag model.

Table 2. Stability windows for the non-ideal bag model.

c	Min. $B^{1/4}$	Max. $B^{1/4}$
0.0	148 MeV	159 MeV
0.1	144 MeV	155 MeV
0.2	140 MeV	150 MeV
0.3	135 MeV	145 MeV

$$\begin{aligned}
 p_q &= a_4 \frac{N_c}{3\pi^2} \int_0^{k_f} \frac{k^4 dk}{E_q}, \\
 n_q &= a_4 N_c \frac{k_f^3}{3\pi^2}
 \end{aligned} \tag{13}$$

and

$$\epsilon = \sum_q \epsilon_q + B_{eff} \quad \text{and} \quad p = \sum_q p_q - B_{eff}. \tag{14}$$

Now let's redefine a_4 as $a_4 = 1 - c$ as done in [9]. In this case, if $c = 0$ we recover the original MIT bag model. As we increase c we deviate from the traditional model. We construct a stability window for values varying from 0 to 0.3, which coincides with the values used in [8, 9]. The stability window is displayed in figure 1 and the corresponding values are presented in table 2.

As we can see, as we increase the value of c , we displace the stability window within almost the same range. Increasing c from 0 to 0.3 displaces the minimum value of $B^{1/4}$ from 148 MeV to 135 MeV. On other hand, the range is almost the same, 11 MeV for $c = 0$ to 10 MeV for $c = 0.3$.

It is worth keeping in mind that the regions outside the stability window on the left and on the right have very different meaning and consequences. The $B^{1/4}$ cannot be lower than the minimum value presented in table 2 because this would imply that two-flavored quark matter is stable and our known universe composed of protons and neutron would no longer exist. But $B^{1/4}$ can be higher than the maximum value. This implies that the strange quark matter is not the ground state of the matter. In this case deconfined quark matter can only be present in the core of massive hybrid stars, instead of forming quark stars.

The free parameter $c = 2\alpha_s/\pi$ is here chosen below 0.3 to match the values used in [8, 9]. However it is worth pointing out that α_s allows values of $c \approx 2$, as can be seen in [35], which would produce negative a_4 values. Therefore, the non-ideal bag model also presents some limitations. Nevertheless, our aim is to compare this model to the original MIT bag model and its modifications suggested in the next sections. For this purpose, we choose the c values used in [8, 9].

As the u and d quark masses are the same, to avoid saturating the figure, (or display several ones) we plot in figure 2 the strangeness fraction instead of the individual particle population, for the values of c presented in table 2. The strangeness fraction is defined as:

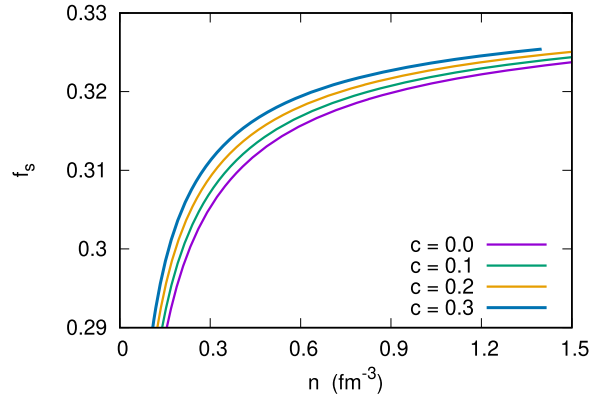


Figure 2. Strangeness fraction for different values of c . Increasing c causes an increase of f_s .

$$f_s = \frac{n_s}{\sum n_f} = \frac{1}{3} \frac{n_s}{n}, \quad (15)$$

where n is the total baryon number density, n_s is the strange quark number density and the sum runs over the three quark flavors. As we can see, when we increase the value of c we also increase the strangeness fraction at a fixed density. This was expected once this non-ideal bag model modifies the quark number, but not the chemical potential of the particles.

With the stability window we can now study what the maximum mass of a stable strange star is. To accomplish this, we need to construct a neutral, beta-stable quark matter. We add leptons as free Fermi gas and impose chemical equilibrium:

$$\mathcal{L}_{lep} = \sum_l \bar{\psi}_l [i\gamma^\mu \partial_\mu - m_l] \psi_l, \quad (16)$$

where the sum runs over the two lightest leptons (e and μ), and:

$$\begin{aligned} \mu_s = \mu_d = \mu_u + \mu_e \quad \text{and} \quad \mu_e = \mu_\mu, \\ n_s + n_\mu = \frac{1}{3}(2n_u - n_d - n_s). \end{aligned} \quad (17)$$

We display in figure 3 the EoS and the TOV [36] solution for the minimum and maximum allowed bag pressure values that produce stable strange quark matter. As we increase c we are able to use lower values of the bag pressure. Although we use massive quarks in the EoS, the masses are very low. This makes an almost linear EoS, as in the case of equation (10) for all values of c . The EoS differ from each other only by a displacement proportional to the bag pressure value. Also, lower values of the bag produce more massive quark stars, as pointed in [2, 9]. Using $c = 0.3$ we are able to produce a $2.21 M_\odot$ stable strange star within the non-ideal bag model, while the original MIT bag model produces a maximum mass of only $1.85 M_\odot$. Increasing the bag pressure value, the maximum strange quark star mass decreases, and lies outside the constraint imposed by the MSP J070 + 6620 [37], whose mass is $2.14^{+0.10}_{-0.09} M_\odot$ at 68% credibility interval (light blue in figure 3) and $2.14^{+0.20}_{-0.18} M_\odot$ at 95% credibility interval (light yellow in figure 3). Increasing the bag constant to values beyond the stability window produces unstable strange matter that can be still present in the core of massive hybrid stars. A curious feature is the fact that while the central density decreases with the increase of c , the strangeness fraction f_s remains almost the same.

Nowadays, an important discussion issue is the radii of canonical stars, $M = 1.4 M_\odot$. The main results of our model are shown in table 3. In the last decade, several studies pointed towards a small radius. Using x-ray telescopes in quiescent neutron stars, a radius between 10 and 13 km was obtained, as shown in [38, 39]. But, here we pay special attention to a recent study based on multi-messenger observations of the binary neutron-star merger GW170817. The authors [40] conclude that the canonical star radius cannot exceed 11.9 km. This result together with the existence of MSP J070 + 6620 puts strong constraints on the EoS of dense matter. As we can see, for $c = 0.2$ and $c = 0.3$ we fulfill both constraints.

However, it must be clear that although the results coming from [38–40] are observational constraints, they still use some nuclear model to fit the data. Therefore the limit value of 11.9 km must be faced with care.

To summarize this section, we state that we are able to produce massive stable strange stars using the modified MIT bag model. However we have to keep in mind that it is only possible to attain the thermodynamic consistency using a renormalized Fermi–Dirac distribution, which cannot be justified at the level of the simple MIT bag model.

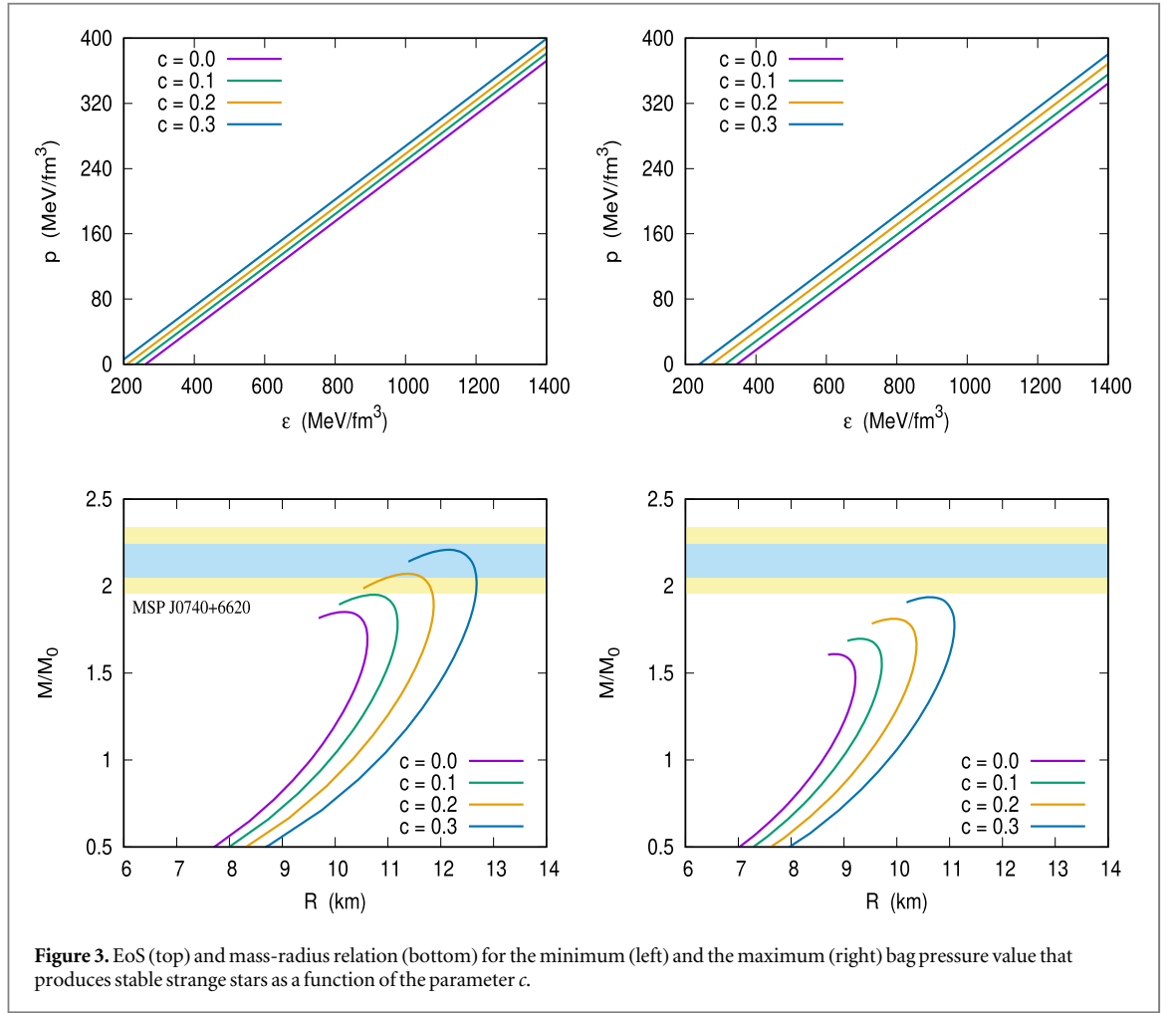


Table 3. Quark stars main properties for different values of c . $R_{1,4}$ is given in km.

c	$M/M_{\odot} - B_{(Min)}$	R (km)	ϵ_c (MeV/fm ³)	f_s	$R_{1,4}$	$M/M_{\odot} - B_{(Max)}$	R (km)	ϵ_c (MeV/fm ³)	f_s	$R_{1,4}$
0.0	1.85	10.17	1286	0.322	10.38	1.61	8.93	1726	0.323	9.20
0.1	1.95	10.72	1158	0.322	10.81	1.70	9.31	1540	0.323	9.64
0.2	2.07	11.37	1038	0.321	11.30	1.81	9.94	1359	0.323	10.17
0.3	2.21	12.18	888	0.321	11.89	1.94	10.61	1205	0.322	10.74

4. Vector MIT bag model

One way to introduce an interaction among the quarks in the MIT model is by coupling the quarks to a field. We next use a vector field that produces a repulsion between the quarks. The inclusion of vector channels in the MIT bag model is not new, as can be seen in [21–26]. Following [22, 23], we introduce the Lagrangian that becomes:

$$\mathcal{L} = \sum_{u,d,s} \{ \bar{\psi}_q [\gamma^\mu (i\partial_\mu - g_{qqV} V_\mu) - m_q] \psi_q - B \} \Theta(\bar{\psi}_q \psi_q), \quad (18)$$

where the quark interaction is mediated by the vector channel V_μ analogous to the ω meson in QHD [41]. Indeed, in this work we consider that the vector channel is the ω meson itself.

Unfortunately, in the previous papers, the authors missed the mass term of the vector channel. As we expose below, in mean field approximation, the vector channel becomes zero if the mass is zero. Therefore, we introduce the mass term of the V_μ (ω_μ) field as:

$$\mathcal{L}_V = \frac{1}{2} m_V^2 V_\mu V^\mu. \quad (19)$$

Now, assuming mean field approximation (MFA) ($V^\mu \rightarrow \langle V \rangle \rightarrow \delta_{0,\mu} V^0$), we obtain the eigenvalue for the energy of the quarks and the equation of motion for the V field respectively:

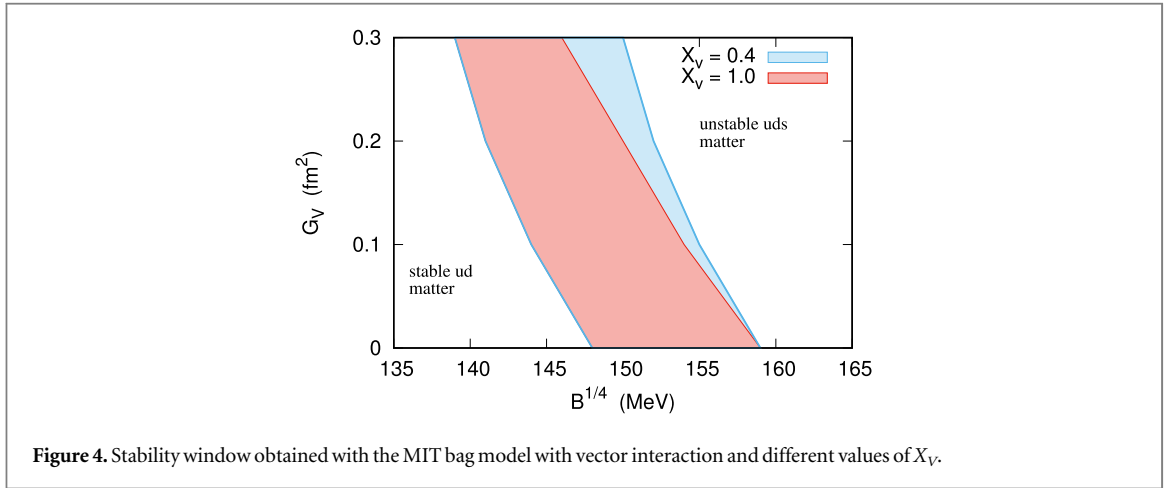


Figure 4. Stability window obtained with the MIT bag model with vector interaction and different values of X_V .

$$E_q = \mu = \sqrt{m_q^2 + k^2} + g_{qqV} V^0, \quad m_V^2 V_0 = \sum_{u,d,s} g_{qqV} \langle \bar{\psi}_q \gamma^0 \psi_q \rangle, \quad (20)$$

where the term $\langle \bar{\psi}_q \gamma^0 \psi_q \rangle$ can be recognized as the number density n_q for each quark q . It is clear from the expression above that if we do not take into account the mass of vector channel, the vector field itself needs to be zero. The energy density for the quarks is then:

$$\epsilon_q = \frac{N_c}{\pi^2} \int_0^{k_f} E_q k^2 dk. \quad (21)$$

We next need to compute the influence of the massive ω particle on the EoS. In MFA, we have: $\epsilon = -\langle \mathcal{L} \rangle$. So, the total energy density reads:

$$\epsilon = \sum \epsilon_q + B - \frac{1}{2} m_V^2 V_0^2, \quad (22)$$

the last term of equation (22) being absent in [21–23]. Moreover, the bag vacuum value is not independent of the vector field V_0 , which ultimately depends on the strength of the coupling constant. The pressure is obtained via the thermodynamic relation, $p = n\mu - \epsilon$, to guarantee thermodynamic consistency given in equation (6).

To construct a new stability window, we have to fix the coupling constant g_{qqV} , as well the mass of the vector field m_V . We consider that the vector channel is the physical ω meson, as in QHD models [41]. In relation to the coupling constant g_{qqV} , we have two concerns. The first one refers to its absolute value. There are very few studies trying to constrain its value, and the uncertainty is yet very high [42, 43]. Most of the models just consider it as a free parameter [44–46]. Our second concern is about the relative strength of the g_{qqV} constant for different quarks. In the literature, the g_{qqV} 's are universal, assuming the same value for all quark flavors [21–23, 42–48, 49]. Here we follow a new path. Instead of a universal coupling, we use symmetry group to fix the relative quark-vector field interaction. We obtain the relation:

$$g_{ssV} = \frac{2}{5} g_{uuV} = \frac{2}{5} g_{ddV}. \quad (23)$$

All the calculations are detailed in the appendix. The use of symmetry group to fix coupling constants is very common when we are dealing with baryons [48, 49, 50–56], but, as far as we know, it is an original approach in the quark sector.

In the following, we redefine $(g_{uuV}/m_V)^2 = G_V$ and define X_V as the ratio between g_{ssV} to g_{uuV} :

$$X_V = \frac{g_{ssV}}{g_{uuV}}. \quad (24)$$

We construct a stability window for G_V varying from 0 to 0.3 fm^2 for $X_V = 1$, which is usually found in the literature, and for $X_V = 0.4$, which is the value predicted by symmetry group. The results are presented in figure 4, as well as in table 4.

As we increase the value of G_V , the stability window is displaced to lower values. For $X_V = 1.0$, the stronger the vector field, the narrower the stability window. For $X_V = 0.4$, even though the window is also displaced to lower values, its interval remains the same, i.e., 11 MeV, regardless of the value of G_V . As $X_V = 1.0$ produces a stronger repulsion between the quarks, it yields a narrower stability window. As expected, the minimum value of the stability window is independent of X_V , and the differences increase as we increase G_V .

Table 4. Stability windows obtained with the vector MIT bag model.

$G_V (fm^2)$	X_V	Min. $B^{1/4}$	Max. $B^{1/4}$
0.0	...	148 MeV	159 MeV
0.1	1.0	144 MeV	154 MeV
0.1	0.4	144 MeV	155 MeV
0.2	1.0	141 MeV	150 MeV
0.2	0.4	141 MeV	152 MeV
0.3	1.0	139 MeV	146 MeV
0.3	0.4	139 MeV	150 MeV

Table 5. Quark star main properties for different values of G_V and X_V .

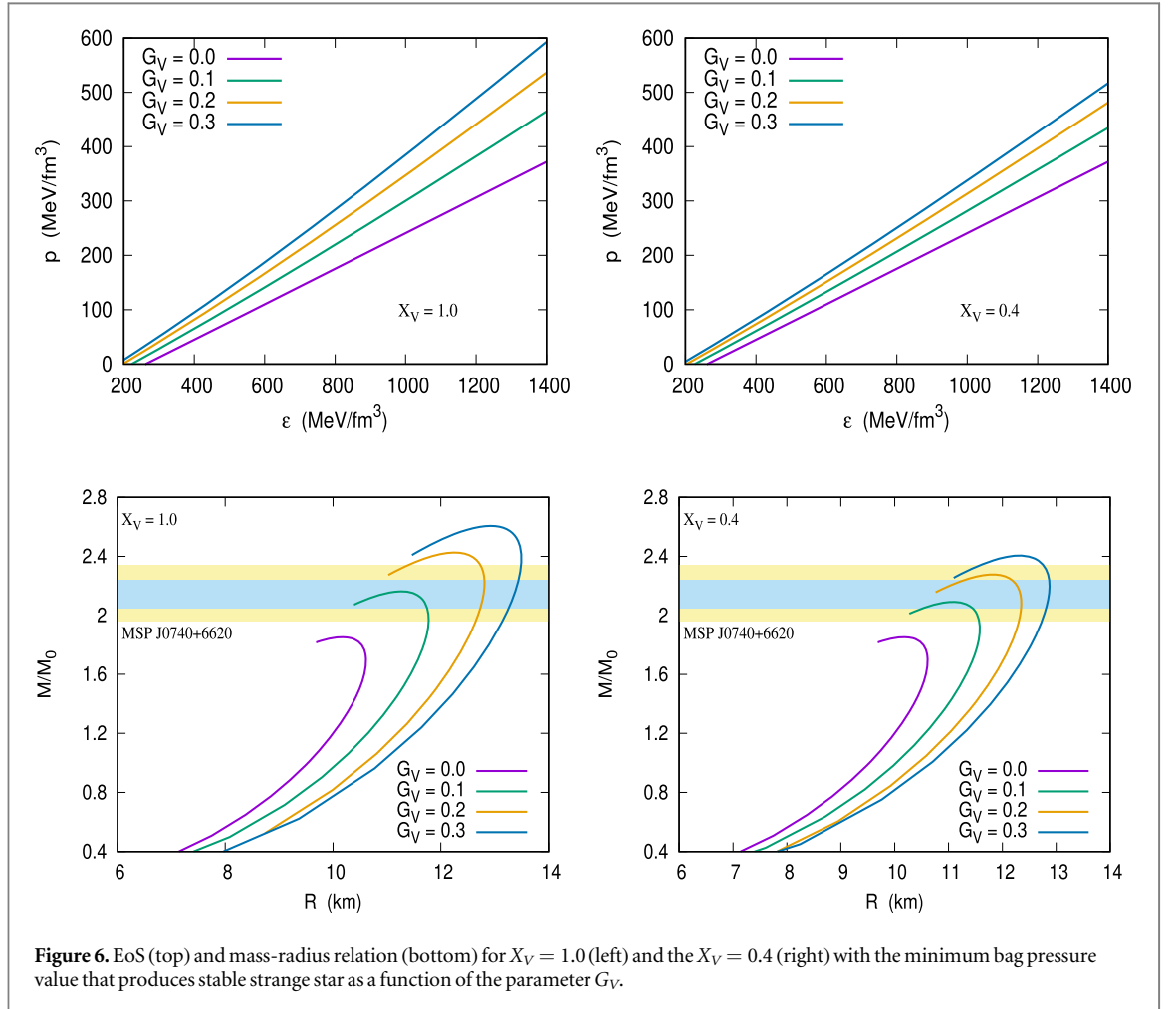
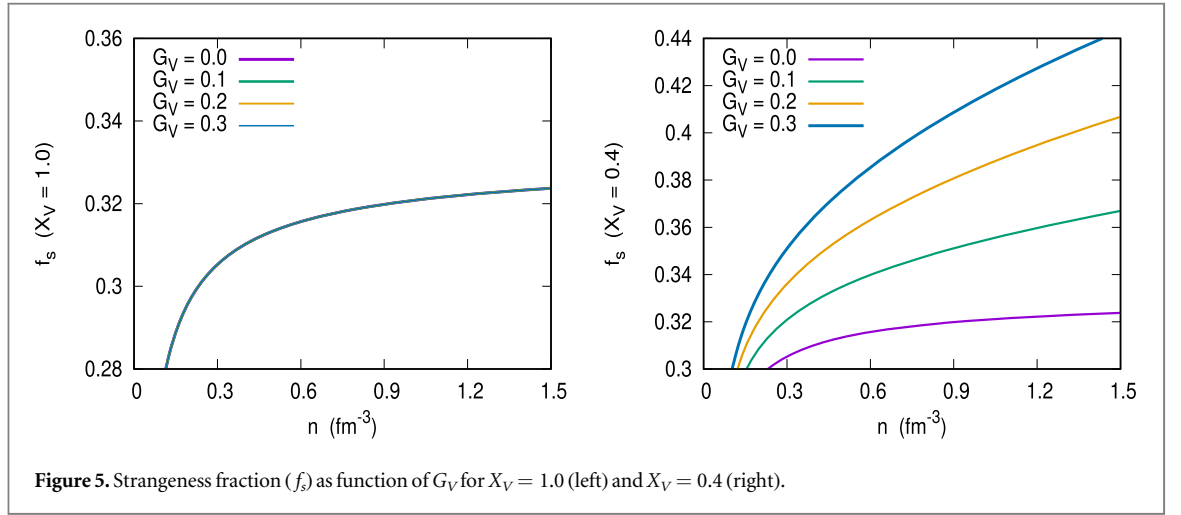
$G_V (fm^2)$	X_V	$M/M_\odot$ -		R (km)	ϵ_c (MeV/fm ³)	f_s	$R_{1.4}$	$M/M_\odot$ -		R (km)	ϵ_c (MeV/fm ³)	f_s	$R_{1.4}$
		$B_{(Min)}$	$B_{(Max)}$					$B_{(Min)}$	$B_{(Max)}$				
0.0	...	1.85	1.85	10.17	1286	0.322	10.38	1.61	1.61	8.93	1726	0.323	9.20
0.1	1.0	2.16	2.16	11.29	1051	0.320	11.11	1.92	1.92	9.98	1331	0.321	10.08
0.2	1.0	2.43	2.43	12.24	921	0.318	11.66	2.19	2.19	11.00	1112	0.320	10.79
0.3	1.0	2.61	2.61	12.97	795	0.317	12.08	2.40	2.40	11.85	978	0.319	11.34
0.1	0.4	2.09	2.09	11.11	1071	0.354	11.02	1.82	1.82	9.63	1450	0.379	9.85
0.2	0.4	2.28	2.28	11.83	962	0.379	11.50	2.01	2.01	10.36	1248	0.388	10.38
0.3	0.4	2.41	2.41	12.33	893	0.402	11.81	2.14	2.14	10.86	1154	0.413	10.72

We next study stable strange stars within the vector MIT bag model. As we are interested in massive strange stars, we only display in the figures the results for the minimum allowed bag vacuum value. Nevertheless, the star masses for the maximum values of the bag are presented in the text as well as in table 5.

We plot in figure 5 the strangeness fraction for different values of G_V and the two different values of X_V previously justified: a universal coupling ratio $X_V = 1.0$ and $X_V = 0.4$. The strangeness fraction is independent of the bag vacuum value. As expected, for the universal coupling, the strangeness fraction is independent of the strength of the coupling of the quarks with the vector field. This can be easily seen from equation (20). As all g_{qqV} have the same values, the chemical potential of the quarks are shifted by the same amount. This behaviour is similar to the Nambu Jona-Lasinio model for quarks, where the vector field does not affect the particle population either [44, 48, 49]. However, if the couplings are not the same, the s quark, which has the lower value of the coupling constant, has also the lower shift in the chemical potential. As we increase the G_V , the difference in the shift becomes larger. This causes, for instance, the strangeness fraction at $n = 1.0 \text{ fm}^{-3}$ to grow from 0.32 for $G_V = 0.0$ to 0.415 for $G_V = 0.3 \text{ fm}^2$.

In figure 6 we display the EoS and the macroscopic properties of stable strange stars for the minimum allowed value of the bag in the stability window. In this case, we have a *positive feedback* between the vector channel and the stability window. As we increase the parameter G_V we stiffen the EoS. Nevertheless, this also reduces the minimum value of the bag pressure in the stability window, which in turn, also stiffens the EoS. The higher the G_V value, the stiffer the EoS. Additionally, the higher the G_V , the lower is the minimum allowed value of the bag pressure. The lower the value of the bag pressure, the stiffer the EoS again [2]. As result of this combined effect we are able to produce very massive stable quark stars, fully compatible with the MSP J0740 + 6610 [37]. Our maximum mass can reach $2.61 M_\odot$ for $X_V = 1.0$. Also, assuming a universal coupling, $X_V = 1.0$, the strangeness fraction at the center of the quark stars decreases as we increase G_V but increases if we assume $X_V = 0.4$.

Unlike the minimum bag pressure value, the maximum allowed bag pressure value does depend on X_V as can be seen in figure 4. As $X_V = 0.4$ produces smaller repulsion, the *positive feedback* plays its role again. A small repulsion produces a softer EoS, which produces a higher value of the maximum allowed bag pressure value, which in turn also softens the EoS. As a consequence, the maximum quark star mass for $G_V = 0.3 \text{ fm}^2$ can vary from $2.14 M_\odot$ to $2.61 M_\odot$, a difference of $0.47 M_\odot$, i.e. 22%. We emphasize that our radii are in agreement with [40], except when we assume $B^{1/4} = 139 \text{ MeV}$, $X_V = 1.0$, and $G_V = 0.3 \text{ fm}^2$. This indicates that the bag pressure value is too low, or/and $X_V = 0.4$ is a better approach to dense quark matter. Anyway, it is clear from table 5 that, within the vector MIT bag model, we are able to produce stable strange stars with masses above $2.4 M_\odot$ that fulfill all astrophysical constraints.



We can also compare our results with different models. For instance, in [24], the authors used a chiral bag NJL-based model with vector interaction, and found that it is not possible to build stable strange matter, even after introducing a deconfinement bag constant. Nevertheless, non-stable strange matter can be important to investigate the possibility of hybrid stars. This is beyond the scope of this work but the reader can seek more information in [24–26].

In this section we have shown that we are able to obtain very massive quark stars, all models are thermodynamically consistent and no modifications on the Fermi–Dirac distribution are necessary. Another point worth mentioning: in our work the G_V values are around ten times smaller than the one used in [22, 23] where $G_V = 2.2$ fm². This is probably because the authors did not include the mass term of the vector field. Only

for a comparison, if we remove the mass term, equation (19), from our final Lagrangian, the maximum mass drops from $2.61 M_\odot$ to only $2.08 M_\odot$, a reduction of $0.53 M_\odot$!

5. Self-interacting vector field

The vector channel in mean field approximation takes into account only the valence quarks. This scenario is called ‘no sea approximation’, once the Dirac sea of quarks is completely ignored [29]. As the vector field is borrowed directly from quantum hadrodynamics (QHD), the vector MIT bag model also becomes renormalizable [41]. However instead of transforming the mean field approximation (MFA) into a more complex relativistic Hartree or Hartree–Fock approximation, we can take the Dirac sea into account throughout modifications on the effective Lagrangian as made in [29]. Here, we introduce a quartic contribution for the vector field: $(V_\mu V^\mu)^2$ as a correction for the EoS at high density which will mimic the Dirac sea contribution.

When we add the vector channel in the MIT bag model Lagrangian, it creates a repulsion term in the quark–quark interaction and, as result, the pressure (as well as the chemical potential and the energy density) increases. The stiffening of the EoS grows linear with the density. The quartic vector field makes the EoS more malleable. The introduction of self-interacting fields is not new in the relativistic models. Boguta and Bodmer [27] introduced self-interaction in the scalar sector to correct the compressibility of the symmetric nuclear matter. The same Bodmer also introduced quartic interaction in the vector sector [28] and others [57, 58] used quartic terms in order to correct the behaviour of nuclear matter at densities above $2n_0$. Now we introduce a self-interacting vector field in the vector MIT bag model as:

$$U(V^\mu) = b_4 \frac{(g^2 V_\mu V^\mu)^2}{4}, \quad (25)$$

where $g = g_{uuV}$ for short, and b_4 is a dimensionless parameter [29]. The self-interactions of the vector field allow us to construct either a softer or a stiffer EoS when compared with the linear case. It also plays a crucial role in the relation between pressure and chemical potential $-p(\mu)$. This relation is important in hadron–quark phase transitions [8, 9, 21, 48]. As in the hadronic case [57], we do not expect any significant modification in the EoS for densities below $2n_0$. Using a mean field approximation and solving the Euler–Lagrange equations of motion we obtain the following eigenvalues for the quarks and V field respectively:

$$\begin{aligned} E_q &= \mu = \sqrt{m_q^2 + k^2} + g_{qqV} V_0, \\ gV_0 + \left(\frac{g}{m_V}\right)^2 (b_4 (gV_0)^3) &= \left(\frac{g}{m_V}\right) \sum_{u,d,s} \left(\frac{g_{qqV}}{m_V}\right) n_q. \end{aligned} \quad (26)$$

Again, this result is very similar to the Boguta and Bodmer terms in the scalar and vector fields in hadronic models [27, 28]. In order to produce only small deviations we impose that $|b_4| \leq 1$. Due to the quartic nature of the non-linear term, the higher the density, the higher the deviation from the linear one. Also, as can be seen from equation (25), the quartic term has also a quartic dependence on the g constant. So, the higher the value of G_V , the higher the influence of the quartic term. The advantage of using non-linear terms is that we can modify the EoS at high density while keeping the stability window unaffected.

We are now in the position to construct an EoS in MFA, considering the Fermi–Dirac distribution of the quarks and the Hamiltonian of the vector field and the bag pressure value, $\mathcal{H} = -\langle \mathcal{L} \rangle$. We obtain:

$$\epsilon_q = \frac{N_c}{\pi^2} \int_0^{k_f} E_q k^2 dk. \quad (27)$$

$$\epsilon = \sum \epsilon_q + B - \frac{1}{2} m_V^2 V_0^2 - U(V_0). \quad (28)$$

The pressure is obtained via the thermodynamic relation, $p = n\mu - \epsilon$, to guarantee thermodynamic consistency.

In figure 7 we display a 3-D stability window. For each value of G_V we vary the dimensionless parameter b_4 from -0.4 to 1.0 . As pointed out, the advantage of the self-interaction is to preserve the original stability window. This can be seen in this 3-D graphic. Each color wall is a stability window for a fixed G_V . The results are the same as the ones shown in table 4, being independent of b_4 .

As can be seen from equation (25), the influence of the self-interaction does not only dependent on b_4 but also on g , as presented in [59] for the scalar meson. Hence, the higher the value of G_V , the stronger the influence of the quartic term. So we display here only the results for $G_V = 0.3 \text{ fm}^2$. As for lower values, the influence is significantly lower. We can also increase the strength of G_V or b_4 , but they would not modify the qualitative aspects and therefore, are beyond the scope of this work. We start by studying the influence of the non-linear term in the strangeness fraction f_s . If b_4 is negative, then the vector field V^0 increases with density when compared

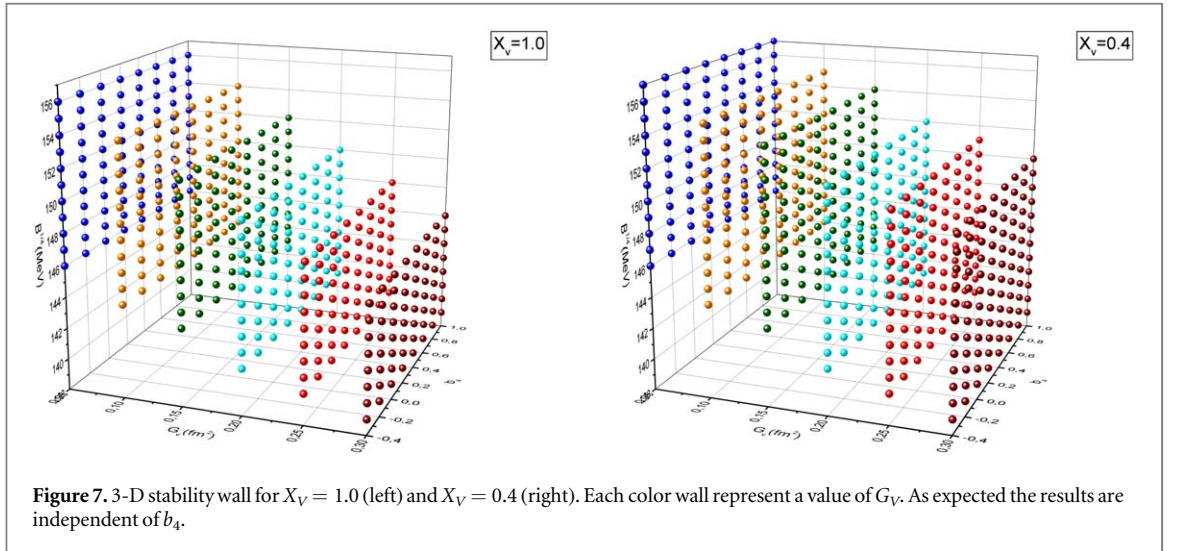


Figure 7. 3-D stability wall for $X_V = 1.0$ (left) and $X_V = 0.4$ (right). Each color wall represent a value of G_V . As expected the results are independent of b_4 .

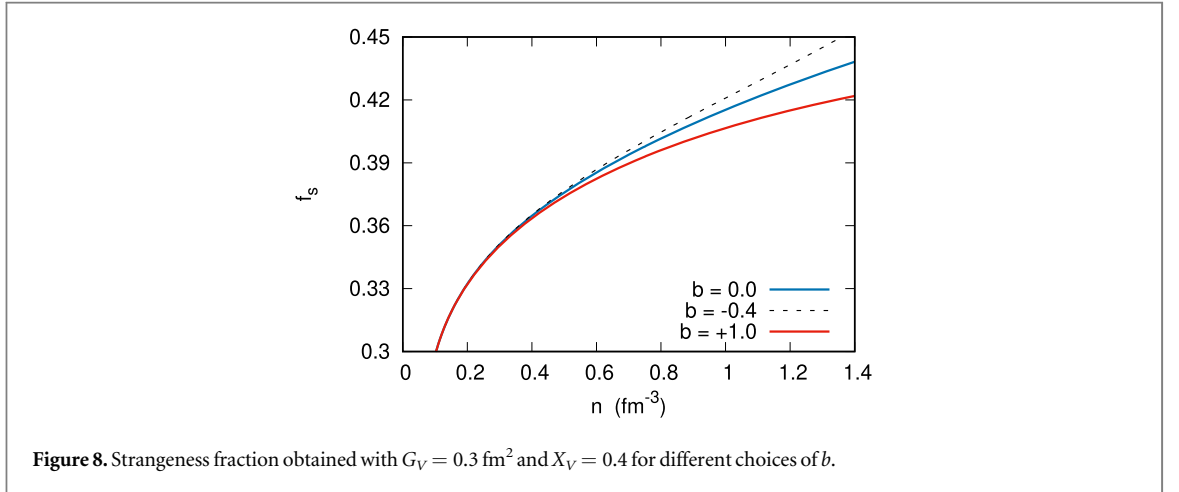


Figure 8. Strangeness fraction obtained with $G_V = 0.3 \text{ fm}^2$ and $X_V = 0.4$ for different choices of b .

with the pure linear case. On the other hand, if b_4 is positive, then V^0 is lower when compared with the linear case. This effect is also reflected in the strangeness fraction if $X_V \neq 1.0$. As already pointed out in the last section, if $X_V = 1.0$ then the strangeness fractions is independent of the vector field. Due to these considerations, here we only discuss the strangeness fraction for $G_V = 0.3 \text{ fm}^2$ and $X_V = 0.4$, as predicted by symmetry group. The results are displayed in figure 8.

As can be seen, the strangeness fraction is the same up to densities about 0.4 fm^{-3} . From this point on, the results begin to differ from the linear case. For negative b_4 (-0.4), we have an increase of the vector field, which makes the differences of the coupling constant more evident, increasing the fraction of s quarks. On the other hand, a positive b_4 (1.0) reduces the vector field as well as the strangeness fraction when compared with the linear case. For instance, fixing $n = 1.0 \text{ fm}^{-3}$ we have f_s equal to 0.415 , 0.421 , and 0.406 for b_4 equal to 0.0 , -0.4 and $+1.0$ respectively.

We now discuss the effect of the self-interacting vector field on the EoS and on macroscopic properties of strange stars. The results for the minimal allowed bag pressure value for $G_V = 0.3 \text{ fm}^2$ are displayed in figure 9.

As expected, the EoS is almost the same until the energy density reaches values around $400\text{--}500 \text{ MeV fm}^{-3}$. Then, as the strangeness fraction behavior, it starts to deviate from the linear case. We also see that the influence of the quartic term is higher for $X_V = 1.0$ as it produces a higher V^0 value. The softening of the EoS for a positive value of b_4 is analogue to the hadronic case, as shown in [57, 58]. The results of the EoS are reflected in the mass-radius relation. The maximum quark star mass is higher for $b_4 = -0.4$ and lower for $b_4 = 1.0$. Also, from figure 9 and table 6, we can see that the influence of the quartic term is larger for the maximum bag pressure value when compared with its minimum value. The influence is also stronger for $X_V = 1.0$ when compared with $X_V = 0.4$. For instance, the maximum star mass varies by $0.08 M_\odot$ for $X_V = 1.0$ for the minimum bag pressure value and by $0.10 M_\odot$ for the maximum bag pressure value. For $X_V = 0.4$, the mass variation is $0.04 M_\odot$ for the minimum bag pressure value and $0.06 M_\odot$ for its maximum. The variation in the radius are not significant. Also, for $X_V = 0.4$

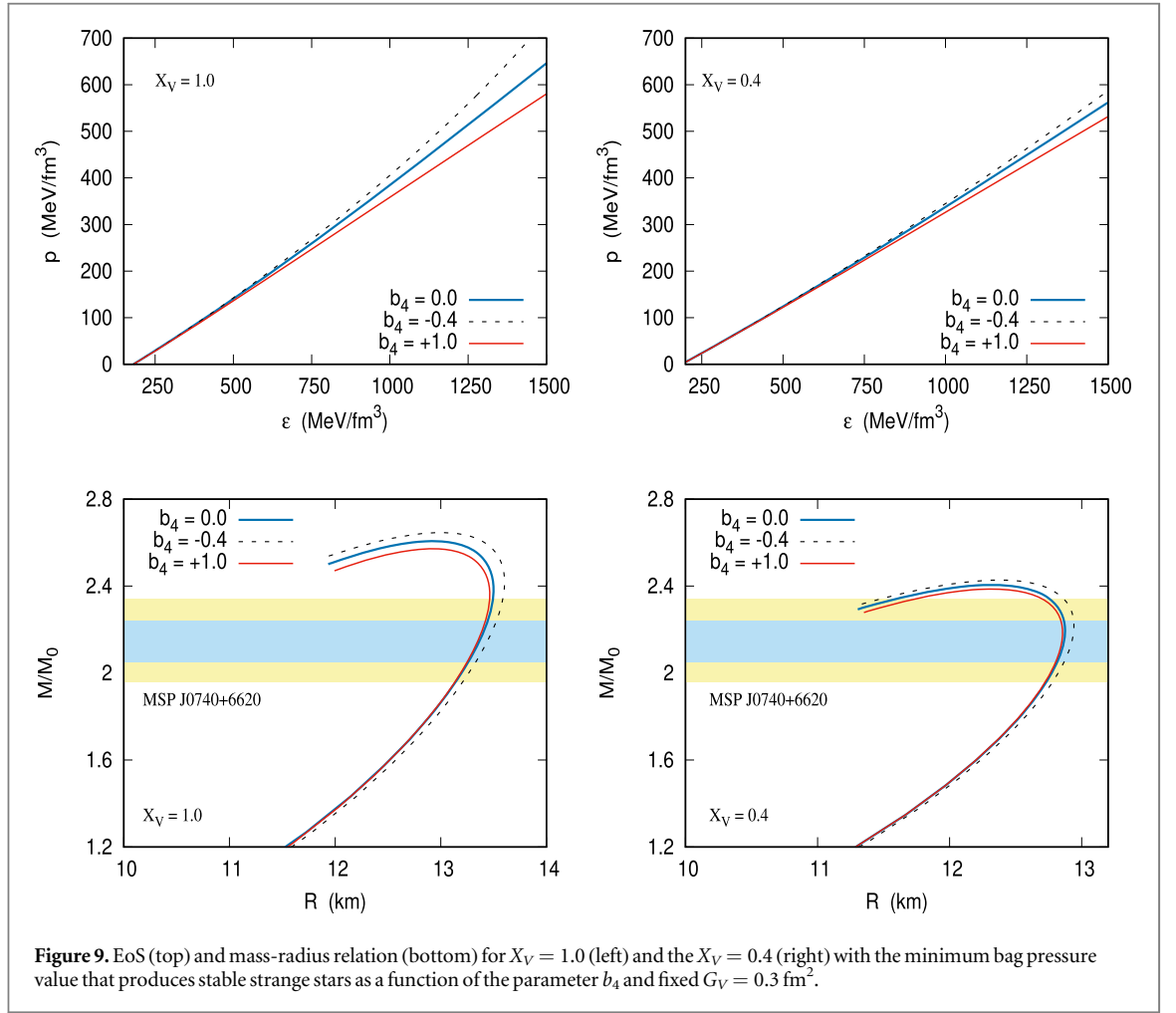


Table 6. Quark star main properties for different values of b_4 and X_V .

b_4	X_V	$M/M_\odot$ -					$M/M_\odot$ -				
		$B_{(Min)}$	R (km)	ϵ_c (MeV/fm ³)	f_s	$R_{1.4}$	$B_{(Max)}$	R (km)	ϵ_c (MeV/fm ³)	f_s	$R_{1.4}$
-0.4	1.0	2.65	13.02	797	0.317	12.13	2.44	11.85	986	0.319	11.36
0.0	1.0	2.61	12.97	795	0.317	12.08	2.40	11.85	978	0.319	11.34
+1.0	1.0	2.57	12.96	791	0.317	12.08	2.34	11.73	994	0.319	11.27
-0.4	0.4	2.43	12.45	903	0.407	11.84	2.16	10.88	1188	0.417	10.88
0.0	0.4	2.41	12.33	893	0.402	11.81	2.14	10.86	1154	0.413	10.86
+1.0	0.4	2.39	12.35	869	0.396	11.80	2.10	10.77	1185	0.406	10.69

the strangeness fraction at the center of the maximum mass star increases if b_4 is negative and decreases if b_4 is positive.

We finish this section indicating that although for the specific case that $B^{1/4} = 139 \text{ MeV}$, $G_V = 0.3 \text{ fm}^2$ and $X_V = 1.0$ the radii of canonical stars are in disagreement with [40], in all other cases we are able to produce stable strange stars in agreement with this study. Indeed, within the self-interacting vector field, even a $2.44M_\odot$ star is obtained.

6. Tidal deformability

Before we finish we make a brief discussion about the tidal deformability (Λ) and the recent constraints presented in the literature coming from VIGO and LIGO gravitational wave observatories and the GW170817 event [60].

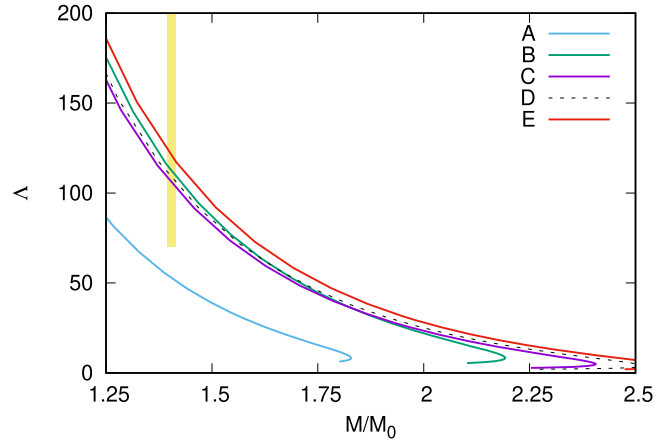


Figure 10. Dimensionless tidal parameter as a function of the gravitational mass for different modified MIT models. Model A is the original MIT, model B is the non-ideal bag with $c = 0.3$, model C(D) is the linear vector MIT with $X_V = 0.4$ (1.0) and model E is the vector MIT with Dirac's sea contribution with $X_V = 1.0$ and $b_4 = -0.4$. The yellow line is the LIGO/VIRGO constraint $70 < \Lambda_{1.4} < 580$ for the $1.4M_\odot$ star.

Table 7. Dimensionless tidal parameter for the canonical mass ($\Lambda_{1.4}$) for different models discussed in the text. Only the original MIT (model A) does not fulfill the LIGO/VIRGO constraint.

...	A	B	C	D	E
$\Lambda_{1.4}$	54	113	107	111	122

If we put an extended body in an inhomogeneous external field it will experience different forces throughout its extent. The result is a tidal interaction. The tidal deformability of a compact object is a single parameter λ (sometimes called Love parameter) that quantifies how easily the object is deformed when subjected to an external tidal field. A larger tidal deformability indicates that the object is easily deformable. On the opposite side, a compact object with a smaller tidal deformability parameter is smaller, more compact, and it is more difficult to deform it.

The tidal deformability is defined as the ratio between the induced quadrupole Q_{ij} and the perturbing tidal field $\mathcal{E}_{ij}$ that causes the perturbation:

$$\lambda = -\frac{Q_{ij}}{\mathcal{E}_{ij}}. \quad (29)$$

However, in the literature is more commonly found the dimensionless tidal deformability parameter (Λ) defined as:

$$\Lambda \doteq \frac{\lambda}{M^5} \equiv \frac{2k_2}{3C^5}, \quad (30)$$

where M is the compact object's mass and $C = M/R$ is its compactness. The parameter k_2 is called Love number and is related to the metric perturbation. A complete discussion about the Love number and its calculation is both, very extensive and well documented in the literature. Therefore, it is out of the scope of the present work. We refer the interested reader to see [49, 61, 62–64] and the references therein.

Now we can discuss how the different modifications of the MIT affect its tidal deformability. In all chosen models, we use the minimum bag value to fulfill Bodmer-Witten conjecture. Also, we use $c = 0.3$ in the non-ideal bag model, and $G_V = 0.3 \text{ fm}^2$ in all vector couplings. The tidal deformabilities for the canonical star $1.4M_\odot$ ($\Lambda_{1.4}$) for different models are presented in table 7 and the tidal deformabilities for stars up to $2.5M_\odot$, as well the VIGO/LIGO constraint ($70 < \Lambda_{1.4} < 580$) provided in by [60] are showed in figure 10.

Before we proceed, its good to remark that, although [60] uses data of the LIGO/VIRGO observatories, the authors still use some of nuclear model to fix the parameters. Therefore, as in the radius constraint, the tidal deformability constraint also must be faced with care.

We can see that, with the exception of the original MIT bag model, all discussed models fulfill the LIGO/VIRGO constraint. Indeed, the values for $\Lambda_{1.4}$ are all lower than 290, the most probable value discussed in [60].

This is the opposite to what normally happens with common used models of neutron stars, which in general present a higher value for the $\Lambda_{1.4}$ as showed in [49]. Even the model E, which has a radius beyond 11.9 presents a relatively small tidal deformability. This reflects the fact that the Love number k_2 has a higher dependence on the microscopic EoS than the macroscopic mass and radius as pointed out in [64].

7. Conclusions

In this work we revisited the MIT bag model, as well as some of its modified versions found in the literature [8, 9, 21–23]. We started from the Lagrangian density and obtained the stability window for the original model. Then we revisited the so called non-ideal bag model, obtained the stability windows for some values of the parameter c . To preserve the thermodynamic consistency, the Fermi–Dirac statistics needs to be modified. In fact, the only modification required is in the quark number distribution. Once it is modified, the correction in the energy density and pressure comes as consequence. Therefore this model modifies the statistics rather than the MIT bag model itself. A stable strange star of mass as high as $2.21M_\odot$ and a radius of 11.89km, which agrees with both mass and radius constraints [37, 40], were produced. However, we must remember of using a renormalized Fermi–Dirac distribution.

We can overcome the thermodynamic issues by introducing a vector field in the Lagrangian density. We first corrected this introduction, as originally proposed in [21–24], by taking into account the mass term of the vector field. With the corrected Lagrangian density we reduced the value of G_V from 2.2 fm^2 to values ten times smaller. Moreover, besides the traditional universal coupling for all three quarks, with the help of symmetry group arguments, we calculated a new coupling constant for the s -quark mass, which is 40% of the u and d quark couplings. We constructed a stability window for this model and we were able to reproduce a star with a mass of $2.61M_\odot$, but in disagreement with the maximum radius obtained in [40]. Yet, we were able to produce a $2.41M_\odot$ that fulfils all the astrophysical constraints while keeping the thermodynamic consistency.

At the end we proposed a modification on the linear vector field, inspired in the models of QHD [27, 28, 57, 58], by including a quartic term in the vector channel. We emphasize here that this is meant to be only a correction in order to mimic the contribution of the Dirac quark sea, which is absent in MFA. This quartic term allowed us to modify the EoS while keeping the stability window unaltered. We saw that a positive quartic term contribution causes a softening of the EoS at high densities, while a negative one causes its stiffening. Ultimately a stable strange star with mass of $2.44M_\odot$ that fulfils the above mentioned astrophysical constraints was obtained.

To finish our analyses, we calculated the tidal deformability of quark stars. We saw that the canonical star (with $1.4M_\odot$), if faced as a quark star, presents a very small tidal deformability ($\Lambda_{1.4}$). The constraint obtained from GW170817 is not very restrictive and allows values between 70 and 580. In all of our models, this tidal deformability is lower than 130, and lies outside the observational constraint for the original MIT model, with the chosen bag parameter.

In the second part of this study we investigate the influence of the temperature on the vector MIT bag model, and also check if the model is able to reproduce results coming from both LQCD and from the experimental determination of the chemical freeze-out.

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Appendix A. SU(3) and SU(6) symmetry group

To fix the quark coupling constant to the vector channel we use the hybrid group SU(6), which is invariant under both SU(3) flavor symmetry and SU(2) spin symmetry. We start from the Yukawa Lagrangian [52]:

$$\mathcal{L} = -g(\bar{\psi}_q \psi_q)M, \quad (\text{A1})$$

where ψ_q is the quark Dirac field, and M is the field of an arbitrary meson. This Lagrangian belongs to the irreducible representation $\text{IR}\{1\}$, a unitary singlet. The $u - d - s$ quarks belong to the $\text{IR}\{3\} = \text{D}(0,1)$, while the anti-quarks belong to the $\text{IR}\{3^* = \text{D}(0,1)\}$ [52]. The meson field can belong either to $\text{IR}\{8\}$ or to $\text{IR}\{1\}$. The direct product of $\{3\} \otimes \{3^*\} = \{8\} \oplus \{1\}$ [65]. So, to preserve the $\text{IR}\{1\}$ of the Lagrangian, the $(\bar{\psi}_q \psi_q)$ needs to belong to the $\text{IR}\{8\}$ if the meson M belongs to $\text{IR}\{8\}$, or belong to $\text{IR}\{1\}$ if the meson belongs to $\text{IR}\{1\}$. The

coupling constant for each quark can be written as [50]:

$$\mathcal{L} = -g_2^8 \mathcal{C}(\bar{\psi}_q \psi_q) M, \quad (\text{A2})$$

for the mesons belonging to $\text{IR}\{8\}$. If the mesons belong to $\text{IR}\{1\}$ we have:

$$\mathcal{L} = -g_1(\bar{\psi}_q \psi_q) M, \quad (\text{A3})$$

where the $\mathcal{C}$ is the $\text{SU}(3)$ Clebsch–Gordan (CG) coefficient [52]. We calculate the CG with the help of the algorithm presented in [66] and the tables presented in [67]

As the quarks are in $\text{IR}\{3\}$ and the anti-quarks in $\text{IR}\{3^*\}$; in $\{3\} \otimes \{3^*\}$ we have only one $\text{IR}\{8\}$ as result, in opposition to the baryon case, where the baryons and anti-baryons are in $\text{IR}\{8\}$. In $\{8\} \otimes \{8\}$ there are two possible $\{8\}$, typically called symmetric and anti-symmetric ones. This also implies that the α_v defined in [52] is always equal to 1 in our case. The coupling constants read:

$$\begin{aligned} g_{uu\omega_8} &= \left(\frac{1}{\sqrt{6}}\right) \times \left(\frac{1}{\sqrt{8}}\right) = g_8 \frac{1}{\sqrt{48}}, \\ g_{dd\omega_8} &= \left(-\frac{1}{\sqrt{6}}\right) \times \left(-\frac{1}{\sqrt{8}}\right) = g_8 \frac{1}{\sqrt{48}}, \\ g_{ss\omega_8} &= \left(-\frac{2}{\sqrt{6}}\right) \times \left(\frac{1}{\sqrt{8}}\right) = -g_8 \frac{2}{\sqrt{48}}, \\ g_{uu\phi_1} &= g_{dd\phi_1} = g_{ss\phi_1} = g_1. \end{aligned} \quad (\text{A4})$$

The coupling of the strange quark is twice the value of the non-strange ones, and has the opposite sign of the $u - u$ and $d - d$ coupling. This result is exactly the same as the $\Sigma - \Sigma$ when compared with the N-N one (the reader can consult the table in [67]). Nevertheless, as happens in the baryon octet case, this weird value will be washed out when we impose the mix of the singlet and octet states.

In nature, the observed ω and ϕ meson are not the theoretical ω_8 and ϕ_1 but a mixture of them [50, 51]. So, the coupling constant of the real vector mesons with the quarks now reads:

$$\begin{aligned} g_{uu\omega} &= g_{dd\omega} = g_1 \cos \theta + g_8 \frac{1}{\sqrt{48}} \sin \theta, \\ g_{ss\omega} &= g_1 \cos \theta - g_8 \frac{2}{\sqrt{48}} \sin \theta. \end{aligned} \quad (\text{A5})$$

Now, to eliminate the last of the free parameters, we impose $\text{SU}(6)$ symmetry group, which give us an ideal mixing angle, ($\theta = 35.264^\circ$), and $g_8 = \sqrt{6} g_1$ [50, 51, 53]. We finally obtain:

$$g_{ss\omega} = \frac{2}{5} g_{uu\omega} = \frac{2}{5} g_{dd\omega}. \quad (\text{A6})$$

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PAPER

Modified MIT Bag Models—part II: QCD phase diagram and hot quark stars

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Modified MIT Bag Models—part II: QCD phase diagram and hot quark stars

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In the present work we use the modified versions of the MIT bag model, on which both a vector field and a self-interacting term are introduced, to obtain hot quark matter and to investigate the QCD phase diagram. We first analyze two-flavored quark matter constrained to both the freeze-out and the liquid-gas phase transition at the hadronic phase. Later, three-flavored quark matter subject to β equilibrium and charge neutrality is used to compute quark star macroscopic properties, which are confronted with recent observational massive and canonical star radius results. Finally, a comparison with QCD phase diagrams obtained from the Nambu-Jona-Lasinio model is performed.

1. Introduction

One of the most important features of the quantum chromodynamics (QCD) is the asymptotic freedom, which predicts that strongly interacting matter undergoes a phase transition from hadrons (constituted of confined quarks) to deconfined quarks and gluons—the quark gluon plasma (QGP) - at some high temperature as well as high density [1, 2]. Therefore, the correlation between two external parameters arises: for a fixed temperature, what is the density (or equivalently, the baryon chemical potential), at which the phase transition occurs in equilibrium QCD?

To study the QGP as well as the hadron-quark phase transition several experiments have been proposed and performed in recent years at LHC, RHIC and others [3–6]. Studying Au+Au and Pb+Pb collisions, Cleymans [7] was able to trace the chemical freeze-out line, obtained when inelastic collisions between particles cease such that the abundance ratios do not change anymore [8]. Although the chemical freeze-out is not directly related to the hadron-quark phase transition, the hadron multiplicities in central high energy nucleus-nucleus collisions are established very close to the phase boundary between hadronic and quark matter [9], especially at very low baryon chemical potential, when the chemical freeze-out temperature and the critical temperature are expected to lie at the same error bar [9, 10]. Moreover, the chemical freeze-out is expected to be a pure hadronic process, therefore, its trace needs to be in the hadron phase. This feature acts as a constraint for hadron-quark phase transition modeling.

In the standard model, the tool to describe strong interacting matter is the QCD. For low chemical potential and high temperature, the lattice QCD (LQCD) can be employed yielding satisfactory results. For instance, LQCD predicts the existence of a smooth crossover around a temperature of 160 MeV at low chemical potentials, while at higher densities a first order phase transition [11–13] is generally obtained from effective models. Furthermore, the first order phase transition must end at an unique point where a second order phase transition takes place, the critical end point (CEP), although its existence and exact location are not well-established [14, 15]. Another important region in the QCD phase diagram is the liquid-gas instability region related to nuclear fragmentation [16–18]. For low temperatures and chemical potential, the nucleons are confined into the nuclei [19], which can be regarded as a liquid phase. As the temperature increases, the nuclei start to dissolve into a diluted interacting gas of nucleons. A critical temperature, above which only the gas phase

survives, is expected [20]. Like the chemical freeze-out, the liquid-gas phase transition is a pure hadronic process, where the contribution from the quark degrees of freedom can be neglected. Nevertheless, the region expected to undergo this phase transition can also be used as a constraint.

In this work we use the modified versions of the MIT bag model, as originally introduced in [21] - on which a vector field is added in a minimal coupling scheme, as well as a self-interacting term that mimics the contribution from the quark Dirac sea [22] - to study the QCD phase diagram and hot quark matter. We start considering symmetric two-flavored quark matter, $\mu_d = \mu_u$, and check if we can fit both the freeze-out and the liquid-gas phase transition at the hadronic phase. Then, we verify if it is possible to fulfill these constraints alongside the existence of stable strange quark matter (SQM) as proposed by the Bodmer-Witten conjecture [23, 24]. If this is true, therefore, the nuclear matter as we know, made of protons and neutrons is only meta-stable, and the true ground state of all matter are not the baryons but three-flavored deconfined quark matter ($\mu_d = \mu_u = \mu_s$).

The next step is to construct a QCD phase diagram for three-flavored quark matter in β equilibrium and zero electrical charge using the modified versions of the MIT bag model. At $T = 0$, this study is important if one wants to describe quark (or strange) stars, as well as quark matter in the core of massive hybrid stars [25]. In the case where the strange quark is present, there are two possibilities for the strength of its interaction with the vector field: an universal coupling, when the strength of the s quark is equal to the u and d quarks, and one that we calculated from the group theory approach that fixes the s quark coupling constant to $2/5$ of the u and d quarks [21]. In all cases, we impose that β stable three-flavored quark matter needs to reproduce massive pulsars like MSP J070+6620 [26]. Another important constraint is the radius of the canonical $1.4 M_\odot$ neutron star. A recent study using multi-messenger observations indicates that this radius is in the range of 10.4 km to 11.9 km [27], although less restrictive constraints using x-ray telescopes have not been discarded [28, 29].

However, it must be clear that although the results coming from [27–29] are observational constraints, they still use some nuclear model to fit the data. Therefore the limit value of 11.9 km must be faced with care.

As will become clear in the manuscript, for reasonable values of the bag pressure parameter, the MIT bag model and its extensions are not suitable to describe quark matter at low chemical potential and high temperature, once its critical temperature is far below those (pseudo) temperatures predicted by LQCD [11, 14, 15], as well as the experimental line of the chemical freeze-out [7]. Due to these facts, inspired by an old recipe [30, 31], we propose a simple parametrization for the Bag, where it increases with temperature. With this simple modification we are able to produce massive quark stars and ensure that the freeze-out and liquid-gas phase transition happen in the confined phase. At the end we study the effects of finite temperature on hot quark stars (with a fixed temperature of 40 MeV) and on the speed of the sound of the quark matter. Finite temperature is important at the early stages of a quark star and in the quark core of a massive proto-hybrid star. A recent study suggests that the presence of quark matter is linked to the behaviour of the speed of sound in massive hybrid stars [25].

It is also worth pointing out that in the limit of vanishing quark masses the QCD Lagrangian presents chiral symmetry [32]. Although at low energy scale this symmetry is dynamically broken, it is restored at high energy scales. Besides, for low temperature and high chemical potentials, the existence of a color superconducting phase is possible. Some studies indicate that the related gaps in the fermion spectrum could be of the order of 100 MeV [33]. Both features are beyond the QCD characteristics that one can study using the MIT-like models presented in this work, but for the sake of comparison, we add a section where results obtained with the Nambu–Jona-Lasinio model [34] are shown and commented.

2. Formalism

The MIT bag model considers that each baryon is composed of three non-interacting quarks inside a bag. The bag, in turn, corresponds to an infinity potential, which confines the quarks. In this simple model the quarks are free inside the bag and are forbidden to reach out. All the information about the strong force lies in the bag constant, also called the vacuum pressure. The MIT Lagrangian density reads [35]:

$$\mathcal{L} = \sum_{u,d,s} \{ \bar{\psi}_q [i\gamma^\mu \partial_\mu - m_q] \psi_q - B \} \Theta(\bar{\psi}_q \psi_q), \quad (1)$$

where m_q is the q quark mass running from u , d and s , whose values are 4 MeV, 4 MeV and 95 MeV respectively [36]; ψ_q is the Dirac quark field, B is the constant vacuum pressure and $\Theta(\bar{\psi}_q \psi_q)$ is the Heaviside step function that is included to assure that the quarks exist only confined inside the bag.

As in [21], we introduce a quark interaction via minimal coupling described by a vector channel V_μ analogous to the ω meson in quantum hadrodynamics:

$$\mathcal{L}_V = \sum_{u,d,s} g_{qqV} \{ \bar{\psi}_q [\gamma^\mu V_\mu] \psi_q \} \Theta(\bar{\psi}_q \psi_q), \quad (2)$$

as well as the mass term and a self-interaction on the vector field:

$$\mathcal{L}_V = \frac{1}{2} m_V^2 V_\mu V^\mu + b_4 \frac{(g^2 V_\mu V^\mu)^2}{4} \quad (3)$$

where g_{qqV} is the coupling constant of the quark q with the vector field V^μ . As pointed out earlier, there are two possibilities: an universal coupling with $g_{ssV} = g_{uuV} = g_{ddV}$, as well as a ratio that comes from symmetry group calculations [21]: $g_{ssV} = 2/5 \cdot g_{uuV} = 2/5 \cdot g_{ddV}$; m_V is the mass of the vector field, taken to be 780 MeV, b_4 is a dimensionless parameter to modulate the self-interaction of the vector field, and $g = g_{uuV}$ for short.

Now, assuming mean field approximation (MFA) ($V^\mu \rightarrow \langle V \rangle \rightarrow \delta_{0,\mu} V^0$), we obtain the eigenvalue for the energy of the quarks and the equation of motion for the V field, respectively:

$$\begin{aligned} E_q &= \sqrt{m_q^2 + k^2} + g_{qqV} V^0, \\ gV_0 + \left(\frac{g}{m_V} \right)^2 (b_4 (gV_0)^3) &= \left(\frac{g}{m_V} \right) \sum_{u,d,s} \left(\frac{g_{qqV}}{m_V} \right) n_q \end{aligned} \quad (4)$$

where the term $\langle \bar{\psi}_q \gamma^0 \psi_q \rangle$ was replaced by the number density n_q for the q quark.

Now, quarks are Fermions with spin 1/2, then the number density, the pressure and the energy density of the quark matter can be obtained via Fermi–Dirac distribution [37]:

$$n_q = \langle \bar{\psi}_q \gamma^0 \psi_q \rangle = 2 \times N_c \int \frac{d^3k}{(2\pi)^3} (f_{q+} - f_{q-}), \quad (5)$$

$$\epsilon_q = 2 \times N_c \int \frac{d^3k}{(2\pi)^3} E_q (f_{q+} + f_{q-}), \quad (6)$$

$$p_q = \frac{2 \times N_c}{3} \int \frac{d^3k}{(2\pi)^3} k \frac{\partial E_q}{\partial k} (f_{q+} + f_{q-}), \quad (7)$$

where $N_c = 3$ is the number of colors. The Bag contribution, as well as the contribution of the vector field to the energy density and the pressure are easily obtained through the Hamiltonian:

$$\mathcal{H} = -\langle \mathcal{L} \rangle = B - \frac{1}{2} m_V^2 V_0^2 - b_4 \frac{(gV_0)^4}{4}, \quad (8)$$

and

$$\epsilon = \sum_q \epsilon_q + \mathcal{H}. \quad \text{and} \quad p = \sum_q p_q - \mathcal{H} \quad (9)$$

with f_{q+} (f_{q-}) being the Fermi–Dirac distribution of the quarks (anti-quarks), given by:

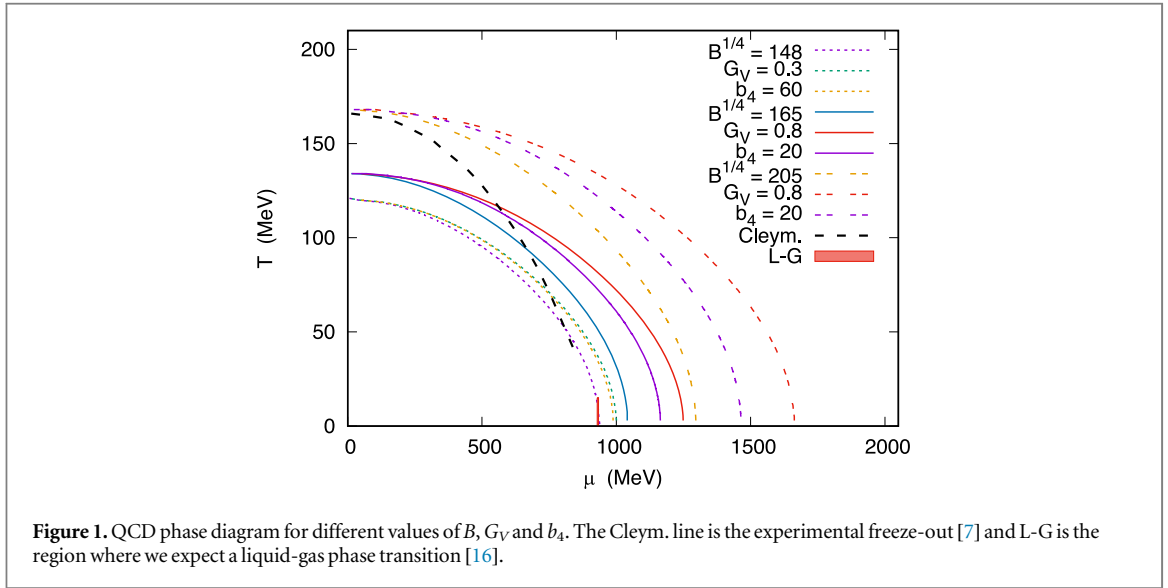
$$f_q \pm = \frac{1}{1 + \exp[(E_q^* \mp \mu_q^*)/T]}. \quad (10)$$

Here, $E_q^* = \sqrt{k^2 + m_q^2}$ and $\mu_q^* = \mu_q - g_{qqV} V_0$ is the effective chemical potential.

3. Phase diagram and quark stars

We start by studying how the vector field as well as the self-interacting term influence the QCD phase diagram obtained with the modified MIT model for different bag pressure values. In MIT-based models the interpretation of the QCD phase diagram is simple: for low values of temperature and chemical potential we have confined quarks, indicated by a negative pressure due to the bag B . This is the hadronic phase. When matter is heated, light hadrons, preferentially pions, are created thermally, which increasingly fill the space between the nucleons. Because of their finite spatial extent, the pions and other thermally produced hadrons begin to overlap with each other and with the bags of the original nucleons such that a network of zones with quarks, anti-quarks, and gluons is formed. At a certain critical temperature T_c these zones fill the entire volume in a ‘percolation’ transition. This new state of matter is the quark-gluon plasma (QGP) [8]. A similar picture emerges when matter is strongly compressed. In this case the nucleons overlap at a critical number density n_c and form a cold degenerate QGP consisting mostly of quarks. Therefore, in our work the transition line between confined quark matter and QGP is indicated by $p = 0$. It is important to point out that, although in our MIT-based model we always treat it as a first order phase transition, at low chemical potential the LQCD predicts a smooth crossover.

Before we proceed, we introduce some definitions. As in our work g_{uuV} and g_{ddV} are always equal (and called only g for short), we define X_V as the ratio between g_{ssV} and g_{uuV} . Also, the strength of the vector channel is



directly related to $(g/m_V)^2$, so we define G_V as this quantity:

$$X_V \doteq \frac{g_{ssV}}{g_{uuV}} \quad \text{and} \quad G_V \doteq \left(\frac{g}{m_V} \right)^2. \quad (11)$$

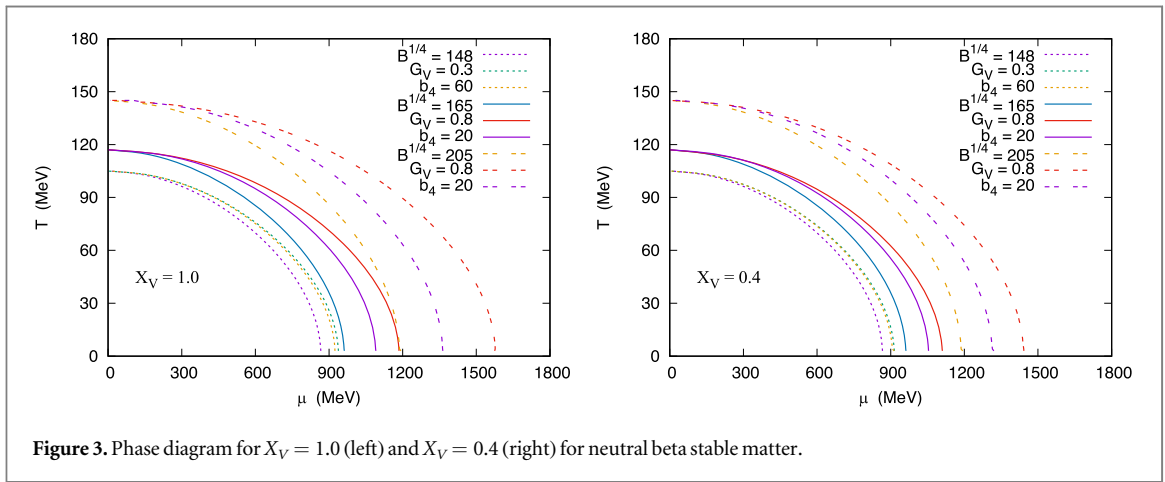
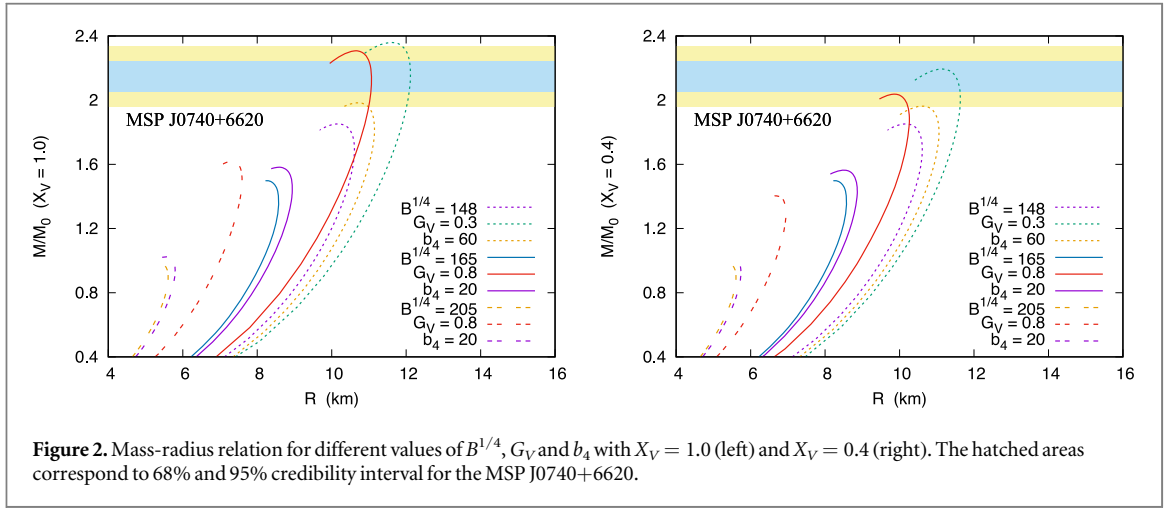
We start with two-flavored symmetric matter with $\mu_u = \mu_d$ to make a direct comparison with the chemical freeze-out as presented in [7], as well as with the liquid-gas phase transition. We use three different values for the bag parameter, $B^{1/4} = 148$ MeV, 165 MeV and 205 MeV. $B^{1/4} = 148$ MeV is the lowest allowed value for non-interacting bag model because even lower values would predict stable $u - d$ matter. A good model to simulate the hadron-quark phase transition needs to ensure that both the chemical freeze-out and the liquid-gas phase transition lie in the confined (hadronic) phase. At $T = 0$ we also expect that the phase transition takes place at values of the chemical potential at least higher than $\mu = 1050$ MeV, as shown in [1] using the Polyakov loop formalism. Also, using NJL-based models, the works of [32, 38] indicate that the critical chemical potential is around 1080 MeV to 1100 MeV. There is no experimental evidence of the maximum chemical potential which preserves the hadronic phase, however, a recent study points out that quark matter inside massive neutron stars is not only possible but probable [25]. Using chiral bag model with $B^{1/4} = 170$ MeV, [39] found a critical chemical potential around 1250 MeV. For β -stable matter, using relativistic density functional approach, [40] indicates that it should occur around $\mu = 1200$ MeV; for two-flavored symmetric matter we assume a maximum value of $\mu = 1400$ MeV as a more conservative estimate. Also, there are strong evidences that the (pseudo) critical temperature obtained from LQCD is very close to the critical temperature of chemical freeze-out [41]. Once these constraints are investigated, we proceed by checking whether the model is able to reproduce massive quark stars, and if the parameters are in favor or against the Bodmer-Witten conjecture of stable strange matter, i.e., the energy per baryon of strange matter $\mu_u = \mu_d = \mu_s$ must be below 930 MeV [21, 23, 24].

Moreover, for each bag pressure value there are also three possibilities: the original MIT bag model without any interaction, a vector channel via minimum coupling and a vector channel with minimal coupling alongside a self-interacting term to simulate the quark Dirac sea contribution. In order to reproduce massive quark stars, we use here $G_V = 0.3 \text{ fm}^2$ for a bag pressure value of $B^{1/4} = 148$ MeV, and $b_4 = 60$. For $B^{1/4} = 165$ MeV and 205 MeV, we use $G_V = 0.8 \text{ fm}^2$ and $b_4 = 20$. Notice that the self-interacting term depends on both, b_4 and g as shown in equation (3). The results are presented in figure 1. We also use these values and construct the equation of state (EoS) of a β -stable matter with zero electrical charge, namely [42]:

$$\mu_d = \mu_s = \mu_u + \mu_e \quad \text{and} \quad \mu_e = \mu_\mu$$

$$n_e + n_\mu = \frac{1}{3}(2n_u - n_d - n_s). \quad (12)$$

We then use the EoS at zero temperature as an input to the TOV equations [43] to produce the mass-radius diagram. The results for different values of X_V are presented in figure 2, as well as the MSP J070+6620 pulsar, whose mass range is $2.14_{-0.09}^{+0.10} M_\odot$ at 68 % credibility interval (light blue) and $2.14_{-0.18}^{+0.20} M_\odot$ at 95 % credibility interval (light yellow) [26]. We also display in figure 3 the phase diagrams for three-flavored β -stable matter. Although zero temperature is generally assumed for calculations of stable neutron and quark stars, higher temperatures can be important in the early stages of the proto-star. For instance, the neutron matter deconfinement occurs on a strong interaction time scale of 10^{-23} s [44], while the chemical equilibrium happens



at a weak time scale of 10^{-8} s. On the other hand, the cooling of the newborn neutron star by neutrino diffusion takes a few seconds [45]. Therefore, the phase diagram for values up to $T = 50$ MeV in neutral β -stable matter is important [46]. For higher temperatures the discussion is more pedagogical, once we do not expect β -stable matter in this region [47, 48].

From figure 1 we can see that when there is no interaction term, increasing bag pressure values favor the hadron phase. The higher the B value, the higher both the critical temperature and the critical chemical potential are. The vector field causes an additional repulsion, increasing the critical chemical potential. However, as shown in equation (4), the vector channel couples to the density and causes very little effects at low chemical potential. Indeed, the vector field does not change the critical temperature for $\mu = 0$. The self-interacting term reduces the repulsion at large densities. In our previous work [21], we have used low values of b_4 to not change the stability window, but here we relax this condition in order to study the strong coupling effects. We see that for $B^{1/4} = 148$ MeV (the only value in this work with which it is possible to reproduce SQM) the critical temperature for zero chemical potential is 121 MeV. This value is way lower than the experimental results from chemical freeze-out [7] and the LQCD [14, 15]. At the low temperature limit, we see that without the vector channel, even the region related to the liquid-gas phase transition is not fully contained in the hadron phase. We overcome this issue by adding the vector field, but it is not enough to push the critical chemical potential for values larger than 1000 MeV, which is still below the minimum of 1050 MeV. For $B^{1/4} = 165$ MeV, the critical temperature at zero chemical potential is 134 MeV, which is still not enough to match the LQCD and freeze-out results. Without the vector channel, the critical potential at zero temperature is 1040 MeV, but with G_V as well as with the b_4 term, the critical chemical potential is always in the range of $1050 \text{ MeV} < \mu_c < 1400 \text{ MeV}$. Finally, for $B^{1/4} = 205$ MeV we have both the freeze-out line and the LQCD results in agreement with our model, i.e. the lines are contained in the confined (hadronic) phase. But the vector channel pushes the critical chemical potential for values above 1400 MeV.

Although $B^{1/4} = 205$ MeV seems to describe reasonably well the low chemical potential region of the QCD phase diagram, it can be seen from figure 2 that it results in quark stars of very low masses. Indeed, in most cases the maximum mass barely reaches $1.0M_\odot$. With $B^{1/4} = 148$ MeV we are able to reproduce a $2.19 M_\odot$ quark star

within the SQM conjecture and a $2.36 M_{\odot}$ in general. For $B^{1/4} = 165$ MeV a maximum mass of $2.31 M_{\odot}$ arises, a result very close to $2.36 M_{\odot}$ with $B^{1/4} = 148$ MeV. However, although the maximum masses are very close, there is a huge difference on the radii of the canonical stars. For $B^{1/4} = 148$ MeV a radius of 11.15 km is obtained, while for $B^{1/4} = 165$ MeV the radius is only 10.20 km, about 1 km smaller. The vector channel can help to differentiate the values of the bag pressure, even with similar maximum masses. The radii are very sensitive to the bag pressure value. We also see that the maximum mass is sensitive to X_V in absence of the self interacting term. Within a strong coupling constant b_4 , the effect of X_V is secondary. Indeed, the self-interacting term makes the results closer to the ones obtained with the non-interacting quark gas than the ones with the vector field only. Another important feature is the radius of the canonical star. We see that the only models whose radii are $10.4 \text{ km} < R_{1.4} < 11.9 \text{ km}$ [27] are those with bag equal to 148 MeV, which is within (or very close to) the stability window of SQM [21]. Bag pressure values that are far from the stability window produce too low radii, even if their maximum masses are compatible with the MSP J070+6620 [26].

Finally, from figure 3 we see that the β -stable matter disfavors the hadronic phase when compared with symmetric two-flavored matter at figure 1, as it always produces a lower value for the critical temperature (at zero chemical potential), as well as a lower value for the critical chemical potential (at zero temperature). Also, when we compare $X_V = 1.0$ to $X_V = 0.4$ we see that higher values of X_V push the critical chemical potential to higher values.

The results of this section are summarized in table 1, where the critical temperature and chemical potential in two and three-flavored matter are presented, the constraints discussed in the text from both freeze-out and neutron stars observations and what parametrizations give us a SQM for $\mu_u = \mu_d = \mu_s$. We also need to notice that not all the EoS satisfy the Bodmer-Witten conjecture.

4. Temperature-dependent bag model

It is clear from the last section that we are not able to reproduce both the freeze-out critical temperature and massive quark stars. Even if we increase the value of G_V for $B^{1/4} = 205$ MeV, it pushes the critical chemical potential even further, in disagreement with the probable existence of a quark core as presented in [25], and reproduces quark stars with too low radii, in disagreement with the measurements from [27]. Because of these behaviors, different bag pressure values are generally used for different energy scales. When the interest lies in the low chemical potential, a bag pressure value higher than 200 MeV is employed, while for constructing quark stars, values around 150–170 MeV are generally used [49, 50]. Hence, it makes it impossible to use a single bag pressure value to describe the whole energy spectrum with MIT-based models. We then propose an alternative, a temperature dependent bag model, a prescription that is not new. For instance, [51] uses a temperature dependent bag model derived from QCD sum rules and [30, 31] introduce a phenomenological temperature dependence in the bag constant, which reads:

$$B = B_0 \left[1 - \left(\frac{T}{T_0} \right)^4 \right], \quad (13)$$

where T_0 is a free parameter. However, as noted in [52], this bag model is the thermodynamic potential itself: $B = \Omega$. Another possibility is a density-dependent bag model, as proposed in [53]. Notice that in [54], the authors introduce a bag parametrization that depends on both, temperature and chemical potential.

Here we propose a bag model similar to [30], but where the bag increases with the temperature instead of decreasing:

$$B(T) = B_0 \left[1 + \left(\frac{T}{T_0} \right)^4 \right]. \quad (14)$$

Now T_0 is adjusted to reproduce the LQCD and freeze-out (pseudo) critical temperature at zero chemical potential. Therefore $T_0 = 131$ MeV for $B_0^{1/4} = 148$ MeV, and $T_0 = 155$ MeV for $B_0^{1/4} = 165$ MeV. The increase of the bag pressure value at high temperature can be justified not only to reproduce the chemical freeze-out results [7], but also to increase the quark surface at high temperatures. As shown in [55], the surface tension above 100 MeV increases monotonically with T . This model decouples the bag parameter from the thermodynamic potential, and two distinct couplings appear: the vector field only couples to the density while the bag parameter only couples to the temperature.

The QCD phase diagram with two-flavored $\mu_u = \mu_d$ matter is presented in figure 4. As can be seen, we obtain the critical temperature T_c around 168 MeV for both B_0 values. However, for $B_0^{1/4} = 148$ MeV, the freeze-out line is not entirely inside the confined (hadron) phase. Even for the upper limit of the stability window we can't satisfy this restrain. This indicates that, for this model, we are not able to reproduce both, SQM and the QCD

Table 1. Critical temperature (T_c), chemical potential (μ_c), quark star main properties, and some observational constraints for two and three-flavored quark matter within different values of $B^{1/4}$, G_V , b_4 and X_V .

...				Two-flavored matter					Three-flavored β -matter					same μ	
$B^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	T_c (MeV)	μ_c (MeV)	Cleymans?	L-G?	μ_c^2	M ($M_\odot$)	$R_{1.4}$ (km)	$T_{c(\beta)}$	$\mu_{c(\beta)}$	$1.96M_\odot$?	$R_{1.4}$?	SQM?
148	—	0.0	0.0	121	938	No	No	No	1.85	10.36	105	866	No	No	Yes
165	—	0.0	0.0	134	1040	No	Yes	No	1.50	8.58	117	962	No	No	No
205	—	0.0	0.0	168	1296	Yes	Yes	Yes	0.98	—	145	1187	No	No	No
148	1.0	0.3	0.0	121	1000	No	Yes	No	2.36	11.15	105	938	Yes	Yes	No
148	1.0	0.3	60	121	987	No	Yes	No	1.99	10.73	105	924	Yes	Yes	Yes
165	1.0	0.8	0.0	134	1249	No	Yes	Yes	2.31	10.20	117	1186	Yes	No	No
165	1.0	0.8	20	134	1163	No	Yes	Yes	1.58	8.94	117	1091	No	No	No
205	1.0	0.8	0.0	168	1662	Yes	Yes	No	1.62	7.60	145	1575	No	No	No
205	1.0	0.8	20	168	1464	Yes	Yes	No	1.02	—	145	1364	No	—	No
148	0.4	0.3	0.0	121	1000	No	Yes	No	2.19	10.93	105	915	Yes	Yes	Yes
148	0.4	0.3	60	121	987	No	Yes	No	1.96	10.67	105	914	Yes	Yes	Yes
165	0.4	0.8	0.0	134	1249	No	Yes	Yes	2.04	9.78	117	1111	Yes	No	No
165	0.4	0.8	20	134	1163	No	Yes	Yes	1.56	8.86	117	1055	No	No	No
205	0.4	0.8	0.0	168	1662	Yes	Yes	No	1.41	6.72	145	1442	No	No	No
205	0.4	0.8	20	168	1464	Yes	Yes	No	1.00	—	145	1320	No	—	No

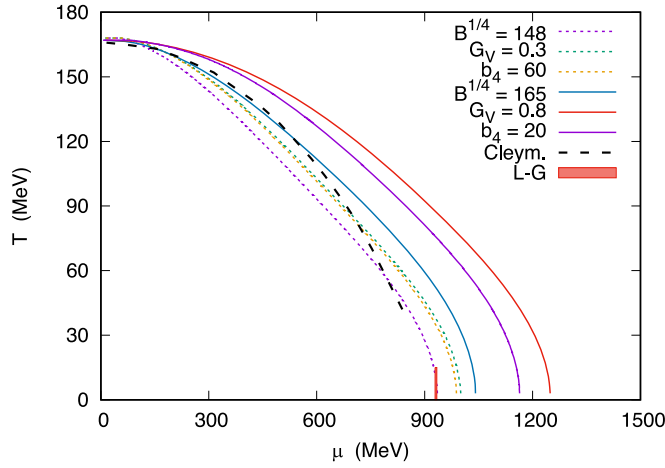


Figure 4. QCD phase diagram for temperature dependent bag model.

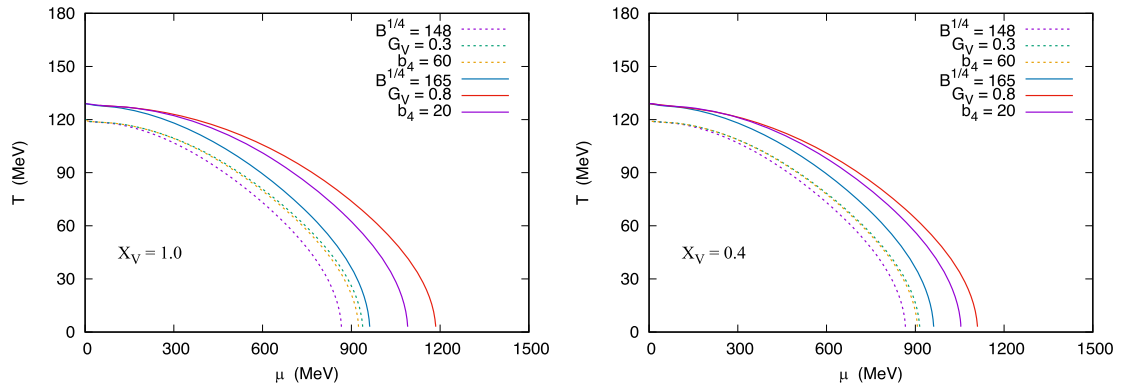


Figure 5. Phase diagram for $X_V = 1.0$ (left) and $X_V = 0.4$ (right) for neutral beta stable matter within $B(T)$ formalism.

phase diagram. Also, as for temperatures $T \simeq 0$, so that $B(T) \simeq B_0$, this bag pressure value produces a low critical chemical potential. On the other hand, when we use $B_0^{1/4} = 165$ MeV, all the freeze-out region lies within the hadronic phase, except for the non-interacting case. Also, the critical chemical potential at zero temperature lies between $1050 \text{ MeV} < \mu_c < 1400 \text{ MeV}$, resulting on a good description of the QCD diagram. In the same sense, as shown in figure 2, we are able to reproduce massive stars at $T = 0$ approximation, in agreement of [26]. Nevertheless, this bag pressure value is far above the stability window, even for non-interacting bag model.

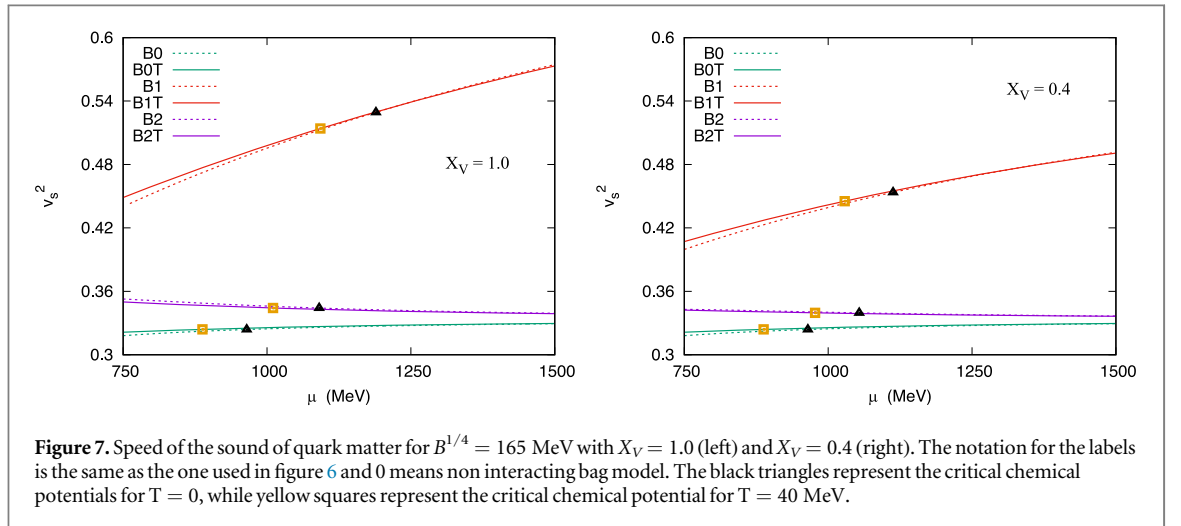
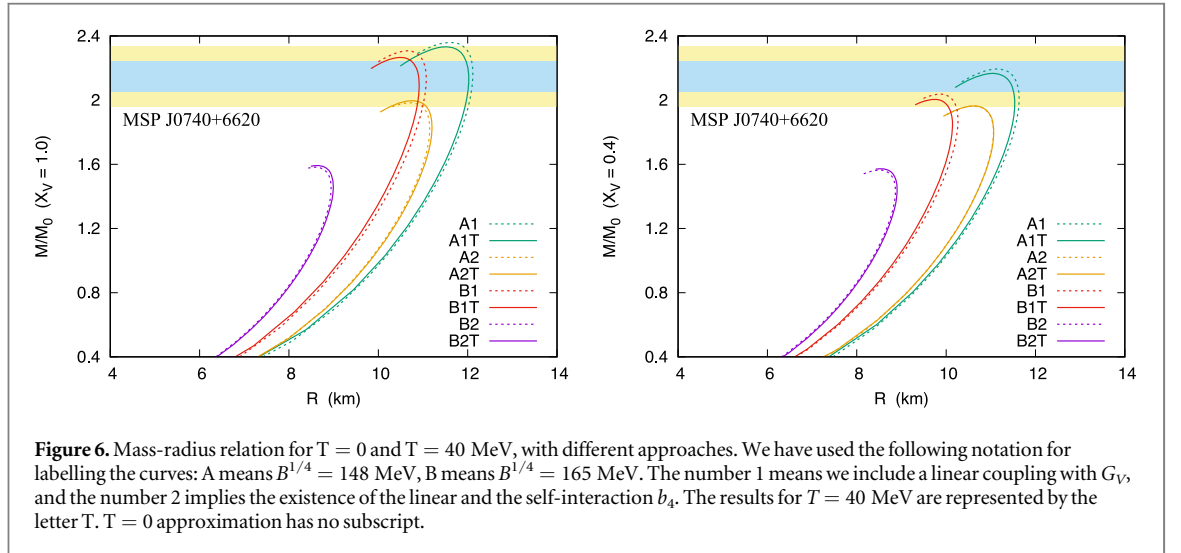
In figure 5 we display the phase diagram for tree-flavored β -stable matter for $X_V = 1.0$ and $X_V = 0.4$. For low temperatures figure 5 and figure 2 are similar. But at low chemical potential we have a increase of the critical temperature due to the dependence of the bag on the temperature. When compared with two-flavored symmetric matter, there is a decrease from the critical temperature at low chemical potential, as expected. All the numerical values are presented in table 2. As can be seen, for $B_0^{1/4} = 165$ MeV, with the linear vector channel, both values of X_V are able to simultaneously give a good description of the QCD phase diagram and to reproduce massive quark stars. However these results point against the existence of SQM, as they are far from the stability window. This is reinforced by the low radii of the canonical stars, which are in disagreement with [27], indicating that canonical stars are probably hadronic neutron stars.

5. Hot quark stars and the speed of sound

As pointed out in [45, 56], a newborn neutron star is formed in the aftermath of a successful supernova explosion as the stellar remnant becomes gravitationally decoupled from the expanding ejecta, reaching a temperature as high as 500 billion Kelvin in the core (around 50 MeV). And while it takes a dozen of seconds to cool down, deconfinement and chemical equilibrium are reached in a timescale millions and millions of times lower [44].

Table 2. Critical temperature (T_c), chemical potential (μ_c), quark star main properties, and some observational constraints for two and three-flavored quark matter within different values of $B^{1/4}$, G_V , b_4 and X_V within B(T) formalism.

...				Two-flavored matter					Three-flavored β -matter						same μ SQM?
$B^{1/4}$ (MeV)	X_V	$G_V(\text{fm}^2)$	b_4	T_c (MeV)	μ_c (MeV)	Cleymans?	L-G?	$\mu_c?$	$M (M_\odot)$	$R_{1.4}$ (km)	$T_{d(\beta)}$	$\mu_{d(\beta)}$	$1.96M_\odot?$	$R_{1.4}?$	
148	—	0.0	0.0	168	938	No	No	No	1.85	10.36	118	866	No	No	Yes
165	—	0.0	0.0	167	1040	No	Yes	No	1.50	8.58	128	962	No	No	No
148	1.0	0.3	0.0	167	1000	No	Yes	No	2.36	11.15	118	938	Yes	Yes	No
148	1.0	0.3	60	167	987	No	Yes	No	1.99	10.73	128	924	Yes	Yes	Yes
165	1.0	0.8	0.0	168	1249	Yes	Yes	Yes	2.31	10.20	118	1186	Yes	No	No
165	1.0	0.8	20	168	1163	Yes	Yes	Yes	1.58	8.94	128	1091	No	No	No
148	0.4	0.3	0.0	167	1000	No	Yes	No	2.19	10.93	118	915	Yes	Yes	Yes
148	0.4	0.3	60	167	987	No	Yes	No	1.96	10.67	118	914	Yes	Yes	Yes
165	0.4	0.8	0.0	168	1249	Yes	Yes	Yes	2.04	9.78	128	1111	Yes	No	No
165	0.4	0.8	20	168	1163	Yes	Yes	Yes	1.56	8.86	128	1055	No	No	No



Therefore the study of hot quark matter is important in both macroscopic properties of quark stars as well as to how the temperature affects the critical chemical potential.

Here we study how a temperature of 40 MeV affects the mass-radius relation of quark stars. Although some studies point out that an isentropic formulation is more realistic [46, 56, 57], we choose an isothermic one, as made in previous works [47, 48, 58–60], which allows a direct comparison with the phase diagram presented in figure 2. We also use B_0 instead of $B(T)$, as for 40 MeV the increase of the bag pressure value is always below 1% (even below 0.5% for $B^{1/4} = 165$ MeV.) In figure 6 the mass-radius relation is presented for $B^{1/4} = 148$ MeV and $B^{1/4} = 165$ MeV with $X_V = 1.0$ and $X_V = 0.4$.

From figure 6 we see that $B^{1/4} = 148$ MeV and $B^{1/4} = 165$ MeV, have the same qualitative behavior for all G_V , b_4 and X_V . In a non-interacting gas (not shown in order not to saturate the figure) we have just a small increase of the maximum mass for $T = 40$ MeV when compared to $T = 0$. For a linear coupling, there is a more significant decrease of the mass instead of an increase. But when we include the vector with self-interaction we recover an increase of the mass with the temperature. The decrease of the maximum mass within a linear vector field is due to the reduction of the central density with the temperature. The main contribution of the temperature is the reduction of the critical chemical potential. In all cases studied, the critical chemical potential has a reduction around 7%–10%.

In figure 7 we show the square of the speed of sound, v_s^2 , of quark matter as a function of the chemical potential for $B^{1/4} = 165$ MeV (as the speed of sound does not depend on the bag pressure value, the qualitative results are maintained),

$$v_s^2 = \frac{\partial p}{\partial \epsilon}. \quad (15)$$

Table 3. Critical chemical potential (μ_c), maximum mass and speed of the sound in function of several parameters, as discussed in the text

$B^{1/4}$ (MeV)	X_V	G_V (fm ²)	b_4	T (MeV)	μ_c (MeV)	M ($M_\odot$)	v_s^2 at μ_c
148	—	0.0	0.0	0	866	1.85	0.32
148	—	0.0	0.0	40	781	1.86	0.32
165	—	0.0	0.0	0	962	1.50	0.32
165	—	0.0	0.0	40	885	1.52	0.32
148	1.0	0.3	0.0	0	938	2.36	0.42
148	1.0	0.3	0.0	40	847	2.33	0.40
148	1.0	0.3	60	0	924	1.99	0.35
148	1.0	0.3	60	40	836	2.00	0.35
165	1.0	0.8	0.0	0	1186	2.31	0.53
165	1.0	0.8	0.0	40	1096	2.26	0.51
165	1.0	0.8	20	0	1091	1.58	0.34
165	1.0	0.8	20	40	1009	1.59	0.34
148	0.4	0.3	0.0	0	915	2.19	0.39
148	0.4	0.3	0.0	40	826	2.16	0.37
148	0.4	0.3	60	0	914	1.96	0.34
148	0.4	0.3	60	40	819	1.98	0.34
165	0.4	0.8	0.0	0	1111	2.04	0.46
165	0.4	0.8	0.0	40	1249	2.00	0.45
165	0.4	0.8	20	0	1055	1.56	0.34
165	0.4	0.8	20	40	974	1.56	0.34

As recently pointed out in [25] the speed of sound of the quark matter is closely related to the mass and radius of the quark core in hybrid stars. The authors found that if the conformal bound ($v_s^2 < 1/3$) is not strongly violated, massive neutron stars are predicted to have sizable quark-matter cores. We plot the black triangles (for $T = 0$) and yellow squares (for $T = 40$ MeV) that represent the critical chemical potential of each model. For the linear vector field, we obtain a monotonically increase of the speed of sound with the chemical potential. Also, the model with linear vector field presents both the higher speed of sound and higher critical chemical potential. For the non-interacting bag model, as well as for the bag model with self-interacting vector field we have an almost constant speed of the sound. However the vector field displaces the speed of the sound to slightly higher values. The non-interacting bag model produces the lower speed of the sound and the lower critical potential. When we compare $X_V = 1.0$ to $X_V = 0.4$ we see that only with the linear vector field there is a significant change in the speed of the sound. The self-interacting field washes out the role of X_V . The main results of this section are presented in table 3.

6. Comparison with the Nambu–Jona-Lasinio model

In this section we also reproduce some results obtained with the Nambu–Jona-Lasinio (NJL) model to compare with the ones we have obtained in the present work. Although the simplest versions of the NJL model does not reproduce the asymptotic freedom behavior of QCD, and thus cannot describe the quark confinement/deconfinement transition, it can be interpreted as a schematic quark model for many situations where chiral symmetry breaking/restoration is one of the most relevant features of QCD [32].

The original version of the model [34] can be extended by the inclusion of a vector-isoscalar interaction term, along the same lines discussed in [21] and written in equation (2) for the MIT bag model, such that, considering two quark flavor fields $\psi_q = [\psi_u \ \psi_d]^T$, it is given by the following Lagrangian density

$$\mathcal{L} = \bar{\psi}_q (i\gamma^\mu \partial_\mu - \hat{m}) \psi_q + G_s [(\bar{\psi}_q \psi_q)^2 + (\bar{\psi}_q i\gamma_5 \vec{\tau} \psi_q)^2] - G_v (\bar{\psi}_q \gamma^\mu \psi_q)^2, \quad (16)$$

where $\hat{m} = \text{diag}(m_u, m_d)$ are the quark bare masses, $\vec{\tau}$ is the Pauli isospin matrix, and G_s and G_v are the coupling constants. In the mean-field level, Lagrangian (16) can be rewritten via the bosonization of the model through auxiliary fields given by the non-vanishing scalar and vector condensates, from where it is straightforward to write the grand-canonical thermodynamic potential as [32]:

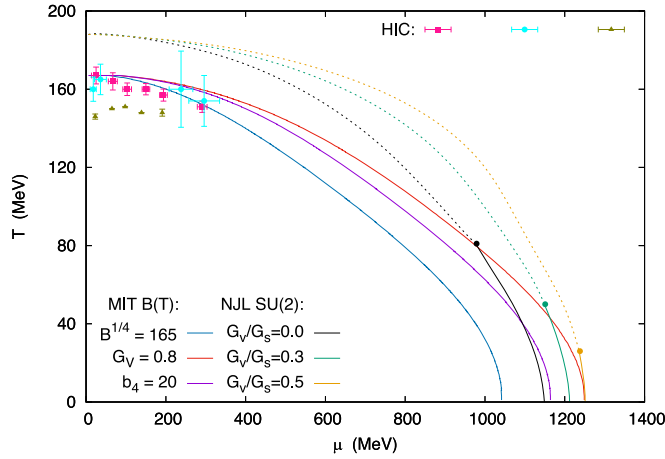


Figure 8. QCD phase diagram for different values of G_v/G_s from the SU(2) NJL model [63]. A solid line represents a first order transition and a dashed line represents a crossover, the intersection is the CEP. Selected curves from the temperature dependent bag model are included for comparison, as well as some estimations of the chemical freeze-out parameters in heavy ion collisions [65–67].

$$\Omega = \Omega_M + \frac{(M - m)^2}{4G_s} - \frac{(\mu_q - \tilde{\mu}_q)^2}{4G_v}, \quad (17)$$

with

$$\begin{aligned} \Omega_M = & -2N_c \int \frac{d^3k}{(2\pi)^3} \{E_k + T \ln [1 + e^{-(E_k - \tilde{\mu}_q)/T}] \\ & + T \ln [1 + e^{-(E_k + \tilde{\mu}_q)/T}]\}, \end{aligned} \quad (18)$$

being the displaced Fermi gas contribution, where $E_k = \sqrt{k^2 + M^2}$. In this process, the constituent mass and effective chemical potential were introduced, respectively, through the self-consistent gap equations [32]:

$$M = m - 2G_s \langle \bar{\psi}_q \psi_q \rangle, \quad (19)$$

$$\tilde{\mu}_q = \mu_q - 2G_v \langle \bar{\psi}_q \gamma^0 \psi_q \rangle. \quad (20)$$

The vector condensate is identified as the number density $\langle \bar{\psi}_q \gamma^0 \psi_q \rangle = n_q$, as in equation (5), and the scalar condensate can be evaluated applying standard techniques of thermal field theory from the dressed fermion propagator $S = (\gamma^\mu k_\mu - M + i\varepsilon)^{-1}$ as

$$\langle \bar{\psi}_q \psi_q \rangle = -2 \int \frac{d^3k}{(2\pi)^3} \frac{M}{\sqrt{k^2 + M^2}} (1 - f_{q+} - f_{q-}). \quad (21)$$

Here the isospin symmetry is assumed in the Lagrangian level, i.e., $m_u = m_d = m$, and the situation of chemical equilibrium is considered, i.e., $\mu_u = \mu_d = \mu_q$, with $\mu_q = \mu/3$. The parameters G_s , m and Λ are fitted to reproduce the quark condensate value, the pion mass and its decay constant. In the following, we set $\Lambda = 590$ MeV, $G_s \Lambda^2 = 2.435$ and $m = 6.0$ MeV [32]. The parameter G_v is usually taken as a free parameter, which some proposed constraints suggesting values between $0.25G_s$ and $0.5G_s$ [61, 62]. We consider $0 \leq G_v/G_s \leq 0.5$.

The effective quark mass M is obtained solving the gap equations (19)–(20). This constituent mass is larger than the bare quark mass m at lower temperatures and/or densities, generating dynamically the larger particle mass expected in this region and breaking the chiral symmetry of the model. As the temperature or the density increase, M approaches the value of the current mass m , thus restoring the chiral symmetry. A chiral phase transition $\mu - T$ diagram can be drawn determining the behavior of the thermodynamic potential minima with respect to M , for given chemical potentials [63]. In the low temperature regime, several effective models predict a first order chiral phase transition to occur. Results from LQCD for the low chemical potential region, however, point to a crossover transition. These two seemingly contradictory pictures suggest that the first order transition line starting at $T = 0$ ends at a critical end point (CEP), from which it turns into a crossover [64]. Figures 8 and 9 show that this behavior is reproduced by the NJL model. Notice that, as the contribution associated to the vectorial coupling vanishes at zero chemical potential, there is no vacuum correction consequences due to the value of the coupling constant G_v . With the chosen parametrization, this model renders the crossover temperature at $\mu = 0$ being equal to 188 MeV, which is higher than the values obtained from previously discussed MIT-type model calculations, but also from the estimations of the chemical freeze-out parameters in heavy ion collisions and expected from LQCD results, as can be seen in figures 8 and 9. As stated above, the NJL model produces a first order phase transition for temperatures below the CEP, where it acquires a second order

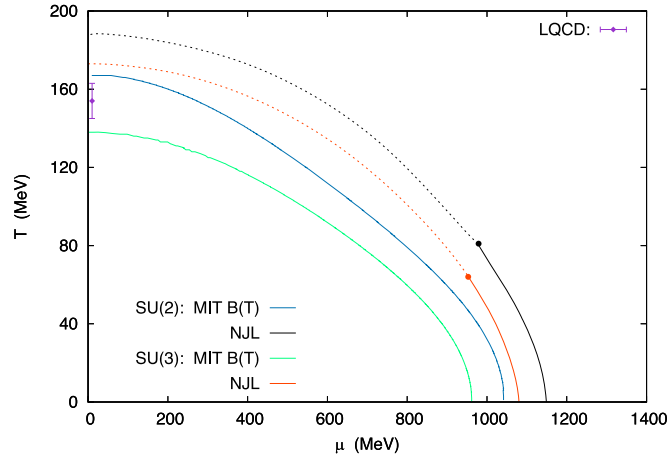


Figure 9. QCD phase diagram for SU(2) and SU(3) NJL model ($G_v/G_s = 0.0$) [63, 68] and temperature dependent bag model ($B^{1/4} = 165$ MeV and $G_v = 0$). A solid line represents a first order transition and a dashed line represents a crossover, the intersection is the CEP. The LQCD predicted critical temperature for 2+1 quark flavors [69] is included for comparison

phase transition point, before the crossover region. Also, increasing the vector term weakens and delays the first order phase transition of the chiral restoration, favoring the crossover transition on the majority of the QCD phase diagram high temperature-low baryonic density part.

The purpose of the present section is to compare different model frameworks, and hence it is useful to display both two and three-flavored matter results obtained from the NJL-type models too. The extension of the NJL SU(2) to NJL SU(3) is not as straightforward as the inclusion of the s quark in the MIT-like models. Thus, we refer the interested reader to [32]. The expressions for the grand-canonical potentials, the related gap equations and the chosen parametrizations are the same as in [68], taking $\Lambda = 631.4$ MeV, $G_s \Lambda^2 = 1.835$, $K \Lambda^5 = 9.29$, $m_u = m_d = 5.5$ MeV and $m_s 135.7$ MeV. As extensively discussed in [31], the strange quark is largely responsible for shaping the phase diagram of QCD since its mass controls the nature of the chiral and deconfinement transitions. In [1], the schematic figure of the Columbia phase diagram in 3-flavour QCD points to the importance of the strange quark mass, which also plays a decisive role on the possible existence of a quark core in collapsed stars. Hence, to consider strangeness conservation without relying on the inclusion of β -stable matter, which, as discussed below, is justified only at the lower temperature range of the QCD phase transition diagram, we show the curves obtained with the same chemical potential for the three quarks, i.e., $\mu_u = \mu_d = \mu_s$ in figure 9, for the NJL and the MIT models. One can clearly see that within the SU(3) version of both models, the transition temperature at low chemical potential is lowered considerably in relation to the SU(2) curve and at low temperatures, the transition chemical potentials are also shifted towards lower values. Also, the effects of the vector interaction are the same as the ones just discussed for the SU(2) case, e.g. the transition point at $\mu = 0$ would be also kept fixed had different values of the vector interaction been plotted, as explained above.

Analyzing this framework and the results presented in previous sections together, it is possible to argue that the phase transition in QCD can take place either in one or two different steps, depending on the parameter choice adopted for the MIT-type model. From figures 1 and 4, we can see that both MIT models allow the deconfinement phase transition to take place around $\mu = 1200$ MeV in the low temperature region, at least for some sets. If this is the case considered, it suggests that both deconfinement and chiral transition occur simultaneously in the QCD phase diagram. It does happen at $T = 0$, e.g., when the parameters of the MIT model are taken to be $B^{1/4} = 165$ MeV and $G_v = 0.8$ fm², while the vector coupling of the NJL is set as $G_v/G_s = 0.5$, at $\mu = 1250$ MeV. However, even in such case, the MIT and NJL transition curves diverge rapidly for finite temperatures, as the dependence of the transition temperature on the chemical potential is noticeably more intense in the MIT curves, specially for chemical potential greater than the μ at the CEP predicted for the NJL-type models.

The description of stellar matter cannot be done using the two-flavor formalism presented in this section, since strangeness is necessary to fulfill the Bodmer-Witten conjecture (but it is still a possibility in the framework of quark-meson models, as pointed out in [70]). Both hybrid and quark stars with vector NJL models have been described in several studies [71–77]. It is worth noting, however, that these models do not produce stable quark matter at zero temperature and/or magnetic field [78], but they can certainly describe the inner matter of a hybrid star [25], which is enough to justify the application of this type of model in theoretical studies, mainly the ones involving phase transitions. Increasing the vector term stiffens the equation of state, thus sustaining larger maximum stellar masses, but the macroscopic properties of the compact star depend strongly on the remaining parameter choice. As example, models used in [79] do not reach the $1.96 M_\odot$ even for higher vector couplings ($G_v/G_s > 0.6$).

7. Final remarks

In this work we have extended the modified versions of the MIT bag model, recently introduced in [21], on which a vector field and a self-interacting term are added, to consider finite temperature effects and then build the QCD phase diagram. We first consider symmetric two-flavored quark matter constrained to both the freeze-out and the liquid-gas phase transition at the hadronic phase. Then, we have verified that it is difficult to reconcile these constraints with the existence of stable strange quark matter as proposed by the Bodmer-Witten conjecture [23, 24].

We have next constructed the QCD phase diagram for three-flavored quark matter in β equilibrium and zero electrical charge with two different possibilities for the strength of the strange quark interaction with the vector field: an universal coupling, when the strength of the s quark is equal to the u and d quarks, and one coming from the group theory approach that fixes the s quark coupling constant to $2/5$ of the u and d quarks [21]. These conditions were then utilized to obtain EoS at zero and finite temperature and used as input for the TOV equations and to compute the sound velocity. The related mass-radius diagrams were displayed and discussed. Within different choices of parameters, massive stars can be described, but the radii of the canonical stars are generally too small, indicating that they are more likely to be hadronic stars.

Finally, we have revisited QCD phase diagrams obtained with the Nambu-Jona-Lasinio model with and without a vector interaction. Despite the fact that the nature of the transition described by the MIT-like and NJL models framework is different, we have seen that both formalisms can produce transition curves with the same overall behavior as the inclusion of the vector interaction is concerned, specially in the low temperature region. Moreover, depending on the parameters choice for both models, the NJL chiral symmetry transition can take place at roughly the same temperatures and chemical potentials as the MIT-like model deconfinement transition, meaning that both first order phase transitions can take place at the same time.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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QCD Phase Diagrams via QHD and MIT-Based Models

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Abstract

In this paper the QCD phase diagram is obtained by the crossing of two types of effective models: the MIT bag-based models are used to describe quark matter and QHD-type models to describe hadronic matter, with the former being something new to this kind of approach and the latter having improved parameterizations as compared with previous calculations. We use Gibbs' conditions to establish the crossing points of the pressure in function of the chemical potential obtained in both phases. We first analyse two-flavour symmetric matter constrained to both the freeze-out and the liquid–gas phase transition at the hadronic phase. Later, we analyse the results for β -stable and charge neutral stellar matter and compare two different prescriptions: one that assumes flavour conservation, so that the quark phase is completely determined from the hadronic phase, a prescription never applied to finite temperatures before, and the other based on the Maxwell construction, where the quark phase is also β -stable. At the end, we compute the latent heat to find a signature of the critical end point.

Keywords QCD phase diagram · Effective models · Gibbs' conditions · Flavour conservation · Maxwell construction · Latent heat

1 Introduction

It is currently understood that, at an elementary level, all matter is made up of leptons and quarks. Quarks are subject to all four fundamental forces, as they have mass and electrical charge, which subject them to gravitational and electromagnetic force; they have a color charge, which subject them to the strong force, and can change flavour in decays, which is due to the weak force. Quantum chromodynamics (QCD) is the theory that explains the interaction between quarks through the strong force.

The ordinary matter, however, is formed by hadrons. That is, under normal conditions, quarks are always confined inside hadrons. The reason behind confinement remains unknown, and a 1 million dollar prize will be awarded for the correct explanation [1]. QCD has also a property called asymptotic freedom, proposed in 1973 by Frank Wilczek and David Gross [2], and independently by David Politzer in

the same year [3], which says that the binding to which the quarks are subject decreases in intensity at small distances or high energies. This means that in extreme situations of high energy, such as the ones found in particle colliders or in the early universe, or at very high densities, such as the ones found inside compact stars, quarks can be deconfined. In fact, since the discovery of asymptotic freedom, the idea of the existence of a quark-gluon plasma (QGP) has been increasingly reinforced. The first experimental corroboration of the existence of the QGP occurred at RHIC in 2005 [4].

From the theoretical point of view, there are still many limitations for the QCD to be solved, which led physicists to attack the problem from two different perspectives: through QCD calculations on the lattice, the so-called lattice QCD (LQCD), and through effective models. Due to computational constraints and numerical difficulties, such as the sign problem, the LQCD method only achieves results for low chemical potentials [5–9]. This makes effective models currently the only source to generate results that cover the QCD phase diagram at higher chemical potentials.

The LQCD predicts the existence of a smooth crossover around a temperature of 160–170 MeV at low chemical potentials, while the effective models predict a first-order phase transition at higher densities [10–12]. Moreover, the first-order phase transition must end at a unique point where a second-order phase transition takes place, the

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critical end point (CEP), even though its existence and exact location are not yet well established [13, 14].

In a previous work [15], we have obtained an estimation of the QCD phase diagram based only on the extensions of the MIT bag model. Towards a more realistic description, in this work, we use two relativistic effective models to describe the hadronic and quark phases of matter. For the quark phase, we use again an extended version of the MIT bag model, as introduced in Lopes et al. [16], while for the hadronic phase, the quantum hadrodynamics (QHD) model with non-linear terms and $\omega - \rho$ interaction is used, since it reconciles the theory with experimental results [17, 18]. In both models, the relative strength of the vector channel is fixed by symmetry group arguments, which allows us to build a unified scheme for the strong interaction. Gibbs' conditions are used to establish the crossing points of the pressure as a function of the chemical potentials obtained in both phases [19, 20].

To reproduce reliable results, some restrictions must be imposed when choosing the models mentioned above. The MIT-based models are used only with constant values that do not allow stable u-d matter, and QHD-based models, in turn, are restricted to parameterizations that satisfy well-known nuclear and astrophysical properties [21, 22]. Other model choices are certainly possible, and depending (mainly) on the slope of the symmetry energy and other model properties, the phase transition points can be different.

Similar works combining two effective models to construct the QCD phase diagram can be found in the literature, as, for example, [23], where the authors combined relativistic mean field and the MIT bag models to analyse symmetry energy effects on the mixed hadron-quark phase, and [24], where the Nambu-Jona-Lasinio (NJL) model was used to describe the quark matter. NJL is a model that takes into account chiral symmetry aspects, which is not the case of the vector MIT bag model. In Lopes et al. [15], we can see that they foresee different values for the phase transition line if it is computed with the quark matter model only. In the present study, the same can happen.

The modified vector MIT bag model used next to describe the quark matter and the QHD parametrizations consistent with recent astrophysical observations are new to this kind of approach. We also compare different prescriptions to obtain the crossing points, generally used in separate studies. Furthermore, the use of these two models allows us to apply the same technique to both phases, once both models are relativistic and are dealt with at the mean field level. Although our results are model-dependent, we do believe that the general trends would be the same had we chosen different models, despite the fact that the temperature and chemical potential values in all scenarios are not expected to be the same.

We first obtain phase diagrams for two-flavour symmetric matter, i.e., we take the hadronic matter to be constituted only by nucleons with equal chemical potentials $\mu_p = \mu_n$ and the quark matter constituted only by quarks u and d with $\mu_u = \mu_d$. We then confront these results with the chemical freeze-out line obtained by Cleymans [25], which is known to be a pure hadronic process, and therefore, the freeze-out line needs to be in the hadronic phase. The liquid-gas phase transition [26], which is also a pure hadronic process that takes place at low temperatures, must be confined to the hadronic part of the QCD phase diagram.

At $T = 0$, we also expect the phase transition to occur at chemical potential values at least higher than $\mu = 1050$ MeV, as shown in [27] using Polyakov loop formalism. In [28, 29], using NJL-based models, it is shown that the chemical potential of the phase transition at $T = 0$ occurs in the range $\mu = 1080 - 1100$ MeV. In [30], using a chiral bag model, the obtained value is $\mu = 1250$ MeV. Although there is no experimental evidence of the maximum chemical potential which preserves the hadron phase, in [31], it is pointed out that quark matter inside neutron stars is not only possible, but probable. For β -stable matter, using a relativistic density functional approach, [32] indicates that the transitions should occur around $\mu = 1200$ MeV. As done in our previous work [15], for two-flavour symmetric matter, we assume a maximum value of $\mu = 1400$ MeV as a more conservative estimate.

Next, we obtain the phase diagram for stellar matter, i.e., β -stable and charge neutral matter, and compare two different prescriptions that affect the quark phase, one where we impose flavour conservation during the phase transition, so that the quark matter is totally determined from the hadronic matter [33] and the other where the quark matter is also β -stable and charge neutral, the so-called Maxwell prescription, usually taken to build hybrid stars [34, 35]. The prescription of flavour conservation has already been used for $T=0$ [33, 36], but for finite temperatures relevant to protoneutron star evolution, it is something completely new.

It is important to stress that the prescriptions we have just mentioned can only provide first-order phase transitions since they depend on relativistic models within mean field approximations (MFA) and quantum fluctuations are completely disregarded. Another aspect is that one should be aware of the limitations imposed by extrapolations of stellar matter conditions to high temperatures. For completeness, we analyse the dependence of the results on the models used at both ends of the curve, i.e., high temperatures and low chemical potentials and low temperatures and high chemical potentials, but our results at low chemical potentials and high temperatures serve as a guide only and should be regarded with a grain of salt.

Finally, we calculate the latent heat based on two different simple expressions to try to estimate the position

(temperature and corresponding chemical potential) of the critical end point.

2 Effective Models

2.1 Hadronic Matter—Relativistic Mean Field QHD Models

To describe hadronic matter, we use the Walecka model [37] with non-linear terms [17]. In this model, four types of mesons are included to describe the interactions between baryons, so we have the scalar σ , isoscalar-vector ω^μ , isovector-vector ρ^μ and strange isoscalar-vector ϕ^μ meson fields. As in [18, 38–40], we also consider the ω - ρ meson coupling terms as this term influences the symmetry energy and its slope, resulting in equations of state (EoSs) that can satisfy all important nuclear matter saturation properties and observational constraints. The inclusion of the ϕ meson does not affect the properties of the nuclear matter as it does not couple to the nucleons, but it stiffens the EoSs that include hyperons. The Lagrangian density is as follows:

$$\begin{aligned} \mathcal{L}_{NLWM} = & \sum_B \bar{\psi}_B [\gamma_\mu (i\partial^\mu - g_{B\omega}\omega^\mu - g_{B\rho}\frac{\vec{\tau}_B}{2}\vec{\rho}^\mu - g_{B\phi}\phi^\mu) - m_B^*] \psi_B \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3!}\kappa\sigma^3 - \frac{1}{4!}\lambda\sigma^4 + \\ & + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\Phi^{\mu\nu}\Phi_{\mu\nu} \\ & - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu + \Lambda_v g_{N\omega}^2 g_{N\rho}^2 \omega_\mu\omega^\mu \vec{\rho}_\mu\vec{\rho}^\mu, \end{aligned} \quad (1)$$

where the Dirac spinor ψ_B represents the baryons with the effective mass $m_B^* = m_B - g_{B\sigma}\sigma$, $\vec{\tau}_B$ are the corresponding Pauli matrices, g_{Bi} are the coupling constants of the mesons $i = \sigma, \omega, \rho, \phi$ with the baryon B , m_i is the mass of the meson i , $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{R}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g_\rho(\vec{\rho}_\mu \times \vec{\rho}_\nu)$ and $\Phi_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$. κ and λ are scalar self-interaction constants introduced by [17] and Λ_v is the coupling constant of the mixed quartic isovector-vector interaction. The B sum extends over the octet of the lightest baryons $\{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ or only over the nucleons, depending on the scenario considered (see Section 4).

After applying the mean field approximation (MFA), the EoS can be easily obtained from Eq. (1), and detailed calculations can be seen in [21], for instance.

As it is well known, it is easy to find in the literature several parametrizations for this model, but not all of them

Table 2 Properties at saturation of the models eL3 $\omega\rho$ and NL3* $\omega\rho$. We present the saturation density (n_0), energy per particle (E/A), compressibility (K), and effective nucleon mass (M^*/M) in symmetric nuclear matter and also the symmetry energy (E_{sym}) and slope of the symmetry energy (L) at n_0

Model	eL3 $\omega\rho$	NL3* $\omega\rho$
n_0 (fm $^{-3}$)	0.156	0.150
E/A (MeV)	16.2	16.3
K (MeV)	256	258
M^*/M	0.69	0.59
E_{sym} (MeV)	32.1	30.7
L (MeV)	66	42

satisfy the constraints imposed by experimental results for nuclear matter, as shown in [21]. Moreover, it is desirable that the EoSs reproduce neutron star masses that are heavier than $2 M_\odot$, as imposed by the NICER results for PSR J0740 + 6620 ($2.08 \pm 0.07 M_\odot$) [41] and the recently detected PSR J0952-0607, with a mass of $2.35 \pm 0.17 M_\odot$ [42], yet to be confirmed.

In the present work, we choose two parametrizations: eL3 $\omega\rho$ and NL3* $\omega\rho$. The eL3 $\omega\rho$ is the parametrization proposed in [43] with a slightly modification to adjust the symmetry energy and its slope accordingly to [44] and the NL3* $\omega\rho$ is the NL3* parametrization proposed in [45] with the addition of the $\omega\rho$ -channel as done in [46]. All of them satisfy the symmetric nuclear matter properties at the saturation density and also reproduce maximum star masses above $2 M_\odot$ even when hyperons are included. The main parameters of these parametrizations are presented in Table 1, and the main nuclear properties are presented in Table 2.

We consider the hyperon masses to be $m_\Lambda = 1116$ MeV, $m_\Sigma = 1193$ MeV, and $m_\Xi = 1318$ MeV. The couplings of the hyperons to the vector mesons are related to the nucleon couplings, $g_{N\omega}$ and $g_{N\rho}$, by assuming SU(6)-flavour symmetry, according to the ratios [47–51]:

$$\begin{aligned} g_{\Lambda\omega} : g_{\Sigma\omega} : g_{\Xi\omega} : g_{N\omega} &= \frac{2}{3} : \frac{2}{3} : \frac{1}{3} : 1, \\ g_{\Lambda\rho} : g_{\Sigma\rho} : g_{\Xi\rho} : g_{N\rho} &= 0 : 2 : 1 : 1, \\ g_{\Lambda\phi} : g_{\Sigma\phi} : g_{\Xi\phi} : g_{N\omega} &= -\frac{\sqrt{2}}{3} : -\frac{\sqrt{2}}{3} : -\frac{2\sqrt{2}}{3} : 1, \end{aligned} \quad (2)$$

noting that $g_{N\phi} = 0$. The coupling of each hyperon to the σ field is adjusted to reproduce the hyperon potential in strange nuclear matter (SNM) derived from hypernuclear observables. We fix this potentials as $U_\Lambda(n_0) = -28$ MeV,

Table 1 Parameter sets for the two models discussed in the text. The meson masses m_σ , m_ω , and m_ρ are all given in MeV. The nucleon and the ϕ meson masses were fixed at $M = 939$ MeV and $m_\phi = 1020$ MeV, respectively, in both models

Model	m_σ	m_ω	m_ρ	$g_{N\sigma}$	$g_{N\omega}$	$g_{N\rho}$	κ	λ	Λ_v
eL3 $\omega\rho$	512.000	783.000	770.000	9.0286	10.5970	9.4381	$0.008280 \cdot g_\sigma^3$	$-0.023400 \cdot g_\sigma^4$	0.0283
NL3* $\omega\rho$	502.574	782.600	763.000	10.0944	12.8065	14.4410	$0.004417 \cdot g_\sigma^3$	$-0.017422 \cdot g_\sigma^4$	0.045

$U_{\Sigma}(n_0) = +30$ MeV and $U_{\Xi}(n_0) = -4$ MeV and obtain the coupling constants presented in Table 3.

2.2 Quark Matter—MIT bag-based models

To describe the quark matter, we choose the simple MIT bag model [52] and a modification of that model that includes a vector field as presented in [16]. In the original MIT bag model, the quarks are free inside the bag and confined inside it. All the information about the strong force relies on the bag constant B , which mimics the vacuum pressure. With the inclusion of a vector field V_μ , the quark interaction inside the bag is mediated by the ω meson, in a similar way as in the QHD model with the baryons. In [16], a self-interacting vector field which allows more malleability on the stiffness of the EoSs has also been added. However, in the present study, we opted not to include this term as its influence is barely noticeable for relatively low densities. The Lagrangian density of the model is as follows:

$$\mathcal{L}_{MIT} = \sum_q \left\{ \bar{\psi}_q \left[\gamma^\mu (i\partial_\mu - g_{qqV} V_\mu) - m_q \right] \psi_q + \frac{1}{2} m_V^2 V_\mu V^\mu - B \right\} \Theta(\bar{\psi}_q \psi_q) - \frac{1}{2} \bar{\psi}_q \psi_q \delta_S, \quad (3)$$

where the Dirac spinor ψ_q represents the quark with mass m_q , g_{qqV} the coupling constant, and m_V the mass of the meson. $\Theta(\bar{\psi}_q \psi_q)$ is a Heaviside function that ensures that the quarks are confined inside the bag, and δ_S is a Dirac function that guarantees continuity of the fields of the quarks on the surface of the bag. If we take $g_{qqV} = m_V = 0$, we obtain the original MIT bag model. Using mean field approximation, we easily obtain the EoS. For more details, the interested reader can see reference [16].

In all cases, we choose the quark masses as being $m_u = m_d = 4$ MeV and $m_s = 95$ MeV [53]. The meson mass m_V is the same as the ω meson mass m_ω considered in the QHD models.

As we have done in the hadronic phase, for the relation between the coupling constants, we opt to use the ones obtained via symmetry group, which allows us to build an unified scheme for the strong interaction. In such approach, we have

$$g_{ssV} = \frac{2}{5} g_{uuV} = \frac{2}{5} g_{ddV},$$

Table 3 Hyperon- σ coupling constants adjusted to reproduce the hyperon potential in SNM derived from hypernuclear observables

Model	$g_{\Lambda\sigma}/g_{N\sigma}$	$g_{\Sigma\sigma}/g_{N\sigma}$	$g_{\Xi\sigma}/g_{N\sigma}$
eL3 $\omega\rho$	0.610	0.406	0.269
NL3* $\omega\rho$	0.613	0.461	0.279

as done in [16]. We also define $G_V = (g_{uuV}/m_V)^2$ and choose here $G_V = 0.05$ fm², $G_V = 0.1$ fm² and $G_V = 0.3$ fm².

As for the bag pressure value, we choose a temperature-dependent bag model in order to be able to obtain higher transition temperatures at low chemical potentials and at the same time maintain the transition chemical potentials at low temperatures within a certain range. More details on this discussion can be seen in [15] and [54]. So, the B in Eq. (3) is substituted by the following:

$$B(T) = B_0 \left[1 + \left(\frac{T}{T_0} \right)^4 \right], \quad (4)$$

where T_0 is adjusted to reproduce the LQCD and freeze-out (pseudo) critical temperature at zero chemical potential. Thus, we use $T_0 = 131$ MeV for $B_0^{1/4} = 148$ MeV and $T_0 = 155$ MeV for $B_0^{1/4} = 165$ MeV. As for the values of B_0 , we choose $B_0^{1/4} = 148$ MeV, which is the lowest value within the stability window [16, 55] that satisfies the Bodmer-Witten conjecture [56, 57], and $B_0^{1/4} = 165$ MeV, which is outside the stability window, but allows us to satisfy another constraint that will become clear latter on.

3 Conditions for Phase Coexistence—Gibbs' Conditions

Two states of matter which can exist simultaneously in equilibrium with each other and in contact are described as different phases. The equilibrium conditions can be used to determine when the transition from one phase to another occurs. For the two phases to be in equilibrium, it is necessary that, first of all, their temperatures are the same. Also, as the forces exerted by the two phases on each other at their surface of contact must be equal and opposite, the pressures have also to be equal. And, finally, the chemical potentials have to be identical [19].

As in our case, the two phases considered are the hadronic phase (H) and the quark phase (Q), and we can write the following:

$$\begin{aligned} T^H &= T^Q = T, \\ P^H &= P^Q = P_0 \\ \mu^H(P_0, T) &= \mu^Q(P_0, T) = \mu_0, \end{aligned} \quad (5)$$

These are the necessary conditions for thermodynamic equilibrium of the hadronic and quark phases, also called Gibbs' conditions. The detailed calculation of the relevant physical quantities can be easily found in the literature for both the hadronic (refs. [21, 24, 37]) and the quark (Lopes et al. [15, 16] phases. The total baryonic chemical potential for the hadron (μ^H) and the quark (μ^Q) phases is given in terms of its individual constituents [58]:

$$\begin{aligned}\mu_B^H &= \frac{(\sum_H n_H \mu_H + \sum_l n_l \mu_l)}{\sum_H n_H}, \\ \mu_B^Q &= \frac{3(\sum_Q n_Q \mu_Q + \sum_l n_l \mu_l)}{\sum_Q n_Q},\end{aligned}\quad (6)$$

where H , l , and Q stand for hadrons, leptons, and quarks, respectively. It is also worth emphasising that due to the model limitations and the prescriptions we use, Eq. (5) gives rise to a first-order phase transition, while a more realistic approach should reproduce a smooth crossover at low chemical potentials [10, 11]. In the last section of this paper, we analyse possible ways to limit the temperature and chemical potentials at which the critical end point is reached and, hence, define the limits of our calculation.

In Fig. 1, we show examples where Gibbs' conditions are met for various temperatures. In this case, we use symmetric matter for both phases, with $\mu_n = \mu_p$ for the hadron phase and $\mu_u = \mu_d$ for the quark phase, but in the present work, we will also explore constructions using stellar matter, i.e., a charge neutral matter in β -equilibrium, as already explained in the introduction of the paper. In all cases, the phase transition points are found as shown in Fig. 1.

4 Different Matter Hypotheses

Here, we discuss the conditions that we impose to the hadronic matter before it goes through a first-order phase transition and also the sort of conservation that we impose when the transition occurs.

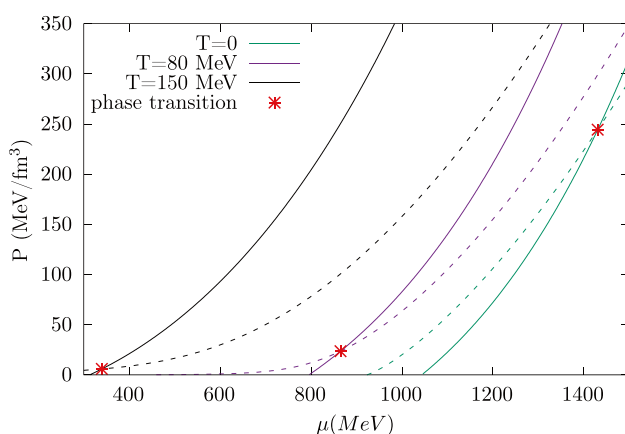


Fig. 1 Relation between pressure and chemical potential for the hadron (dashed lines) and quark (solid lines) phases, respectively, described by the eL3 $\omega\rho$ parametrization and $B_0^{1/4} = 165$ MeV considering three different temperatures. Both phases are of symmetric matter. The red dots are the points where Gibbs' conditions are satisfied

4.1 Based on Symmetric Matter

The first situation we consider is a two-flavour symmetric matter where the nucleons have equal chemical potentials ($\mu_n = \mu_p$) as well as the quarks ($\mu_u = \mu_d$).

The main reason to consider this scenario is to adjust our parametrizations in order to satisfy the constraints imposed by the chemical freeze-out line [25], which, as being a pure hadronic process, needs to be in the hadronic phase.

4.2 Based on Stellar Matter

Stable neutron and quark stars are normally described by zero temperature matter, but temperatures up to $T = 50$ MeV can be important in the early moments of the star as the cooling of a newborn neutron star by neutrino diffusion takes a few seconds [59]. For higher temperatures, we do not expect β -stable matter [60, 61]; nevertheless, here, we extend our study beyond the temperature of $T = 50$ MeV for educational purposes.

For a description of neutral and stable stellar matter, we include a non-interacting lepton gas to both the hadronic matter and the quark matter. For that, we add the following Lagrangian density to Eqs. (1) and (3):

$$\mathcal{L}_l = \sum_l \bar{\psi}_l (i\gamma^\mu \partial_\mu - m_l) \psi_l, \quad (7)$$

where the sum extends over the leptons e^- and μ^- with mass $m_{e^-} = 0.511$ MeV and $m_{\mu^-} = 105.66$ MeV, respectively [53].

Furthermore, as neutron stars present internal densities that can be up to 10 times higher than the nuclear saturation density, the onset of hyperons is expected because their appearance is energetically favourable as compared with the inclusion of more nucleons in the system [62], and so, here, we add the six lightest hyperons to hadron matter so that the sum in Eq. (1) extends over the baryon octet. Nevertheless, at the end of this work, we also obtain diagrams for stellar matter without strangeness, so then, in this case, the sum in Eq. (1) extends only over the nucleons.

We also have to impose β -equilibrium and electric charge neutrality to the hadronic matter:

$$\mu_B = \mu_n - q_B \mu_e \quad \text{and} \quad \mu_{e^-} = \mu_{\mu^-}, \quad (8)$$

$$n_p + n_{\Sigma^+} = n_{e^-} + n_{\mu^-} + n_{\Sigma^-} + n_{\Xi^-}. \quad (9)$$

Next, we consider two scenarios. In the first one, we impose flavour conservation at the point of the phase transition, so that the quark phase is completely determined from the initial hadronic matter through the bond:

$$y_q = \frac{1}{3} \sum_i n_{qi} y_i, \quad (10)$$

where $i = n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Xi^+$, $y_i = n_i/n_B$, being n_i the baryon density of baryon i and n_B the total baryon density, and n_{qi} the number of quarks with flavour q that constitute baryon i [63]. Thus, it is generally assumed that under certain circumstances, the electrically neutral and in chemical equilibrium hadronic matter is metastable and can be converted into an energetically favoured deconfined quark phase. Due to the imposition shown in Eq. (10), this matter will not be in β -equilibrium [33, 36].

Within this scenario, we also briefly analyse two sub-scenarios: one where the lepton fraction, defined as the fraction between lepton density and the baryonic density of each phase, is preserved at the point of the phase transition, and the other where we have the same lepton density, and consequently the same energy density and pressure, for both phases at the point of the hadron-quark phase transition.

We also consider a second scenario, where we do not impose flavour conservation, but charge neutrality and chemical equilibrium to both phases, so that we have, for the quark matter, the following:

$$\mu_s = \mu_d = \mu_u + \mu_e \quad \text{and} \quad \mu_{e^-} = \mu_{\mu^-}, \quad (11)$$

$$n_e + n_\mu = \frac{1}{3}(2n_u - n_d - n_s). \quad (12)$$

This second scenario is the so-called Maxwell construction (or prescription), most commonly used to construct the EoSs to describe hybrid stars, as done, for instance, in [58].

5 Phase Diagrams

In this section, we present the results first for two-flavour symmetric matter and then for stellar matter.

5.1 Symmetric Matter

In Fig. 2, we present the phase diagrams obtained considering two-flavour symmetric matter for both phases, with $\mu_n = \mu_p$ and $\mu_d = \mu_u$. In each graphic, we present three curves, one where we use the NL3 $\omega\rho$ parametrization (blue line) and the other where we use the eL3 $\omega\rho$ parametrization (purple line) in combination with the MIT bag model, and we also present the curve obtained using only the MIT bag model (red line), i.e., at this last one, instead of constructing the phase diagram with two models, quark matter pressure is simply set to be zero (see Lopes et al. [15], for details). The graphics at the top are obtained using a bag pressure value $B_0^{1/4} = 148$ MeV, and for the ones at the bottom, we use $B_0^{1/4} = 165$ MeV. From left to right, we vary the value of the vector channel of the MIT bag model where we use $G_V = 0.0$, $G_V = 0.05$ fm² and $G_V = 0.1$ fm².

We first take a look at the region of low chemical potentials. As can be seen, all curves converge more or less to the same point, so that the maximum temperatures (by maximum temperature T_{max} , we mean merely the highest temperature where we are able to satisfy the phase transition conditions at $\mu \rightarrow 0$) obtained are above the freeze-out line. This happens because we choose the values of T_0 exactly to satisfy the restraint imposed by the LQCD and freeze-out results, which states that the temperatures at very low chemical potentials should be around $T = 168$ MeV. In [54], we have already verified that the maximum temperature obtained depends only on the bag pressure value: the higher the $B^{1/4}$, the higher the maximum temperature. The same conclusion can be drawn here. Following, it is interesting to note that the low chemical potential results obtained using the combination of the two models are the same as the results using only the quark model. That is because Gibbs' conditions at this region are met at very low pressures ($P_0 \leq 5$ MeV/fm³), as can be seen in Fig. 1, and so, as the condition for phase transition when using only the MIT bag model is $P = 0$, it comes with no surprise that these results are very much alike. The same is true when we use a bag pressure value with no dependence on the temperature, as done in [54].

Now, we look at the region of the diagrams of high chemical potentials, i.e., $T \rightarrow 0$. We see that the results are different for each parametrization. Let us first analyse the effect of the different MIT bag-based model parametrizations. The most evident effect is that a higher bag pressure value results in a higher chemical potential. This follows what happens when using only the MIT model, a case easy to understand: in the equation for the pressure, the value of the bag subtracts from the value of the pressure of the quarks (likewise, the meson ω when $G_V \neq 0$), and so, the higher the bag value, the higher must the contribution for the pressure coming from the quarks be in order to meet the condition for phase transition ($P = 0$), and consequently, the chemical potential is also higher. When we use two models, the increase in the chemical potential is even greater because Gibbs' conditions are met at pressures higher than zero ($P_0 > 0$) so that the pressure realised by the quarks and ω meson is even higher, which also increases the chemical potential at the transition point μ_0 .

The inclusion and increase of the G_V value also follows what happens when we use only one model; the higher its value, the higher the chemical potential. As already stated in [15], the vector field causes an additional repulsion between the quarks, increasing the chemical potential of the phase transition. And again, as $P_0 > 0$, the effect when using two models is increased, so much so that here, we use smaller values for G_V than used in our previous work [15].

As for the influence of the different parametrizations of the QHD-based model, we can see that the eL3 $\omega\rho$

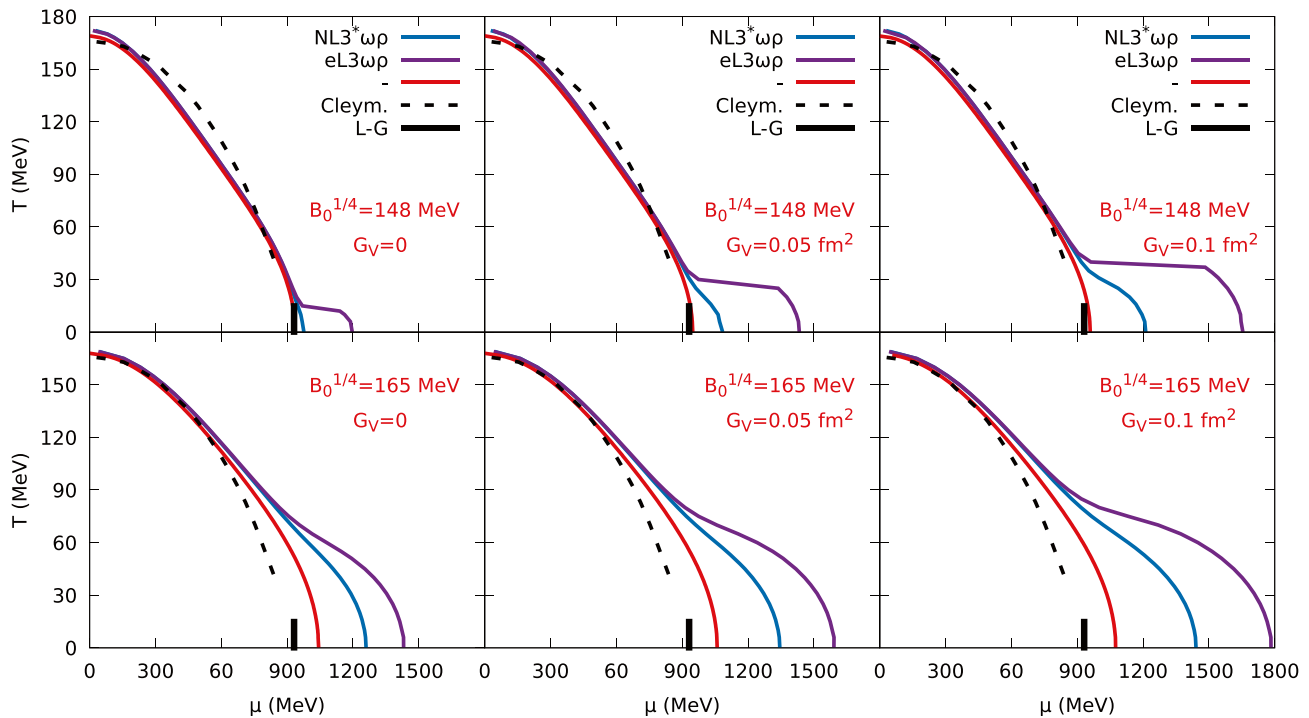


Fig. 2 Phase diagrams for two-flavour symmetric matter considering the NL3* $\omega\rho$ and eL3 $\omega\rho$ parametrizations for the hadronic matter and different MIT bag-based models for two temperature dependent bag $B(T)$ values for the quark matter. The Cleym. line is the experimental freeze-out [25] and L-G is the region where we expect a liquid–gas

phase transition [26]. The red lines correspond to the results obtained with the MIT-based bag models only, where the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in [15]

parametrization always results in higher chemical potentials, and, as the NL3* $\omega\rho$ is a stiffer EoS, we conclude the same as already noticed in [54], a stiffer hadron EoS results in a lower chemical potential.

The behaviour of the diagrams at low temperatures is not so smooth, as we can observe an abrupt increase of the chemical potential, especially when using a bag pressure value of 148 MeV. This happens because the lower the temperature, the smaller the change in the graphs of $\mu \times P$ for different temperatures. And more so, at lower temperatures, the curves for the hadron and the quark phases are very close to each other. As a result, a small change in the temperature makes the crossing point of the lines take a big leap resulting in an abrupt change in the chemical potential on the phase diagram.

In Table 4, we present the values of the maximum chemical potential obtained for each combination of parametrizations used in Fig. 2. By maximum chemical potential μ_{max} , we mean the chemical potential where a phase transition at $T = 0$ takes place. Only a few combinations of parametrizations result in a chemical potential within the range $1050 \leq \mu \leq 1400$ MeV: NL3* $\omega\rho$ in combination with $B_0^{1/4} = 148$ MeV + $G_V = 0.05$ fm² and $G_V = 0.1$ fm² and $B_0^{1/4} = 165$ MeV + $G_V = 0.0$ and $G_V = 0.05$ fm² and eL3 $\omega\rho$ in combination with $B_0^{1/4} = 148$ MeV. When using only the MIT-based model, the

results for $B_0^{1/4} = 165$ MeV + $G_V = 0.05$ fm² and $G_V = 0.1$ fm² are also within this range.

As we chose T_0 in order to obtain higher temperatures at low chemical potentials, we are able to obtain a maximum temperature around $T = 168$ MeV in all combinations of parametrizations, but we are only able to fit the freeze-out line entirely inside the confined (hadron) phase for the combinations that include a $B_0^{1/4} = 165$ MeV and a QHD-based model, which means that we were able to obtain only two combinations of parametrizations that satisfy both constraints for low and high temperatures: NL3* $\omega\rho$ + $B_0^{1/4} = 165$ MeV and NL3* $\omega\rho$ + $B_0^{1/4} = 165$ MeV + $G_V = 0.05$ fm².

5.2 Stellar Matter

Here, we present the results for charge neutral matter in β -equilibrium for two scenarios: one considering flavour conservation, which results in quark m

atter that is not in β -equilibrium, and another with no flavour conservation where both phases are in β -equilibrium, a typical Maxwell prescription used to construct hybrid stars, as done in [43], for example. But first, we analyse the results for a sub-scenario within the scenario of flavour conservation, where we analyse different

Table 4 Chemical potentials ($\mu_{\max}$) at $T = 0$ for all combinations of parametrizations considering **symmetric** matter. The first line corresponds to the results obtained with the MIT-based bag models only,

Models	$B_0^{1/4} = 148 \text{ MeV}$	$B_0^{1/4} = 148 \text{ MeV} +$ $G_V = 0.05 \text{ fm}^2$	$B_0^{1/4} = 148 \text{ MeV} +$ $G_V = 0.1 \text{ fm}^2$	$B_0^{1/4} = 165 \text{ MeV}$	$B_0^{1/4} = 165 \text{ MeV} +$ $G_V = 0.05 \text{ fm}^2$	$B_0^{1/4} = 165 \text{ MeV} +$ $G_V = 0.1 \text{ fm}^2$
-	$\mu_{\max} = 936$	$\mu_{\max} = 948$	$\mu_{\max} = 959$	$\mu_{\max} = 1043$	$\mu_{\max} = 1060$	$\mu_{\max} = 1075$
NL3* $\omega\rho$	$\mu_{\max} = 976$	$\mu_{\max} = 1084$	$\mu_{\max} = 1212$	$\mu_{\max} = 1260$	$\mu_{\max} = 1345$	$\mu_{\max} = 1441$
eL3 $\omega\rho$	$\mu_{\max} = 1197$	$\mu_{\max} = 1434$	$\mu_{\max} = 1655$	$\mu_{\max} = 1431$	$\mu_{\max} = 1592$	$\mu_{\max} = 1783$

where the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in [15]. All values are given in MeV

prescriptions for the lepton matter during the phase transition. In the first one, the lepton fraction, defined as the fraction between lepton density and the baryonic density of each phase, is preserved at the point of the phase transition, and in the other one, the lepton density is taken to be the same, and consequently, the same energy density and pressure are obtained, for both phases at the point of the hadron-quark phase transition. In Fig. 3, we compare the two prescriptions.

As the different prescriptions only affect the quark phase, we choose here to present the results from only one of the QHD-based model, namely, the NL3* $\omega\rho$, and vary the MIT-based model parametrizations. As can be seen, the differences are very small. They are greater for the bag pressure value of $B_0^{1/4} = 148 \text{ MeV}$ (blue lines) than for the $B_0^{1/4} = 165 \text{ MeV}$ (red lines) and also decrease with the increase of the G_V constant. But for all parametrizations, the prescription considering the lepton fraction equal at both phases at the transition point (solid lines) is the one where the chemical potentials are always slightly higher. Because of the imposition of flavour conservation and Gibbs' conditions as criteria for the phase transition, there is an increase of the baryon density at the point of the phase transition, so, in the sub-scenario where we impose the lepton fraction conservation, this means also an increase, in the same proportion, of the lepton density, which, in turn, means an increase in the chemical potential. As the contribution to

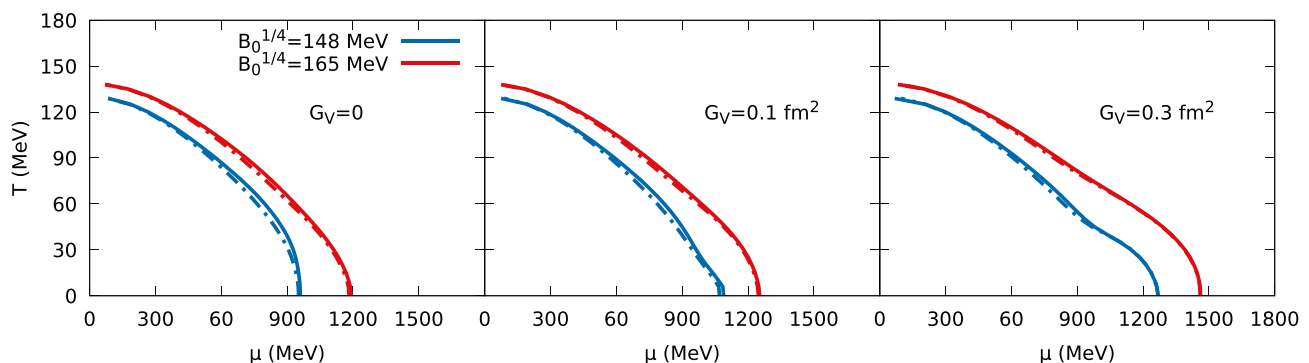
the chemical potential from the leptons is very small, these differences are also very small.

From this sub-scenario, we choose the one where there is a conservation of the lepton fraction at the point of the phase transition because, in this case, the charge neutrality is also preserved. This approach is contrary to what was done in [64].

In the following, we present in Fig. 4 the phase diagrams for stellar matter and compare the results considering flavour conservation (top) and the Maxwell prescription (bottom). The blue lines are the ones where we use the NL3* $\omega\rho$ parametrization, the purple ones are for the eL3 $\omega\rho$ parametrization, and the red ones we use to display, again, also the results obtained using only the MIT-based models where, in this case, the quark matter is always β -stable. Furthermore, the solid lines stand for the results using a bag pressure value $B_0^{1/4} = 148 \text{ MeV}$ and the dashed lines for $B_0^{1/4} = 165 \text{ MeV}$. The value of G_V increases from 0 to 0.3 fm^2 from left to right.

As expected, the general behaviour is the same as observed in Fig. 2 for symmetric matter. For both prescriptions, only the bag pressure value dictates the maximum temperature obtained for $\mu \rightarrow 0$. At low temperatures, the maximum chemical potential depends largely on the bag pressure value, but also on the inclusion or not of the vector field to the bag model and the parametrization chosen for the QHD model.

As done above, we analyse first the region of low chemical potentials. At this region, we have $T_{\max} = 129 \text{ MeV}$ for

**Fig. 3** Comparison between two different prescriptions for the lepton matter in the quark phase for strange stellar matter considering flavour conservation. Here, we present the results for the NL3* $\omega\rho$ and

different parametrizations for the MIT-based model. The solid lines represent the prescription of lepton fraction conservation, and the dashed lines represent the prescription of lepton density conservation

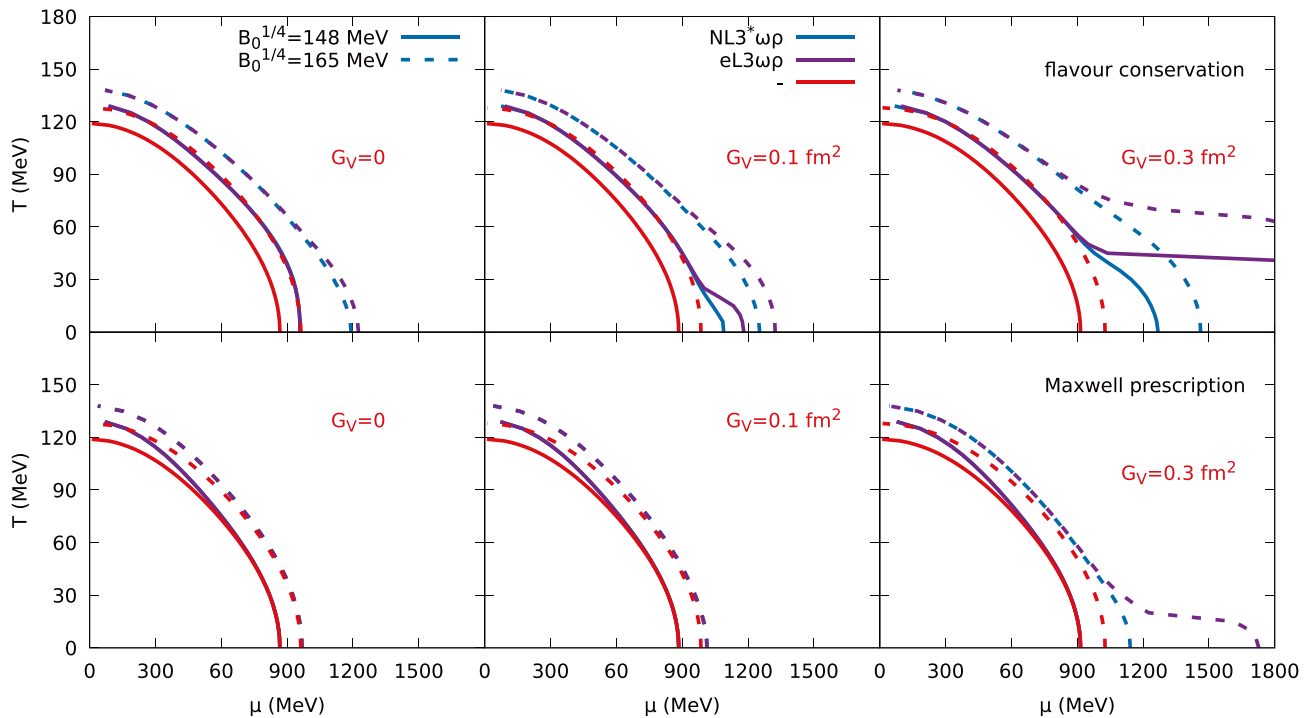


Fig. 4 Phase diagrams for strange stellar matter considering the NL3* $\omega\rho$ and eL3 $\omega\rho$ parametrizations for the hadronic matter and different MIT bag-based models for two temperature-dependent bag $B(T)$ values for

the quark matter. At the top, we use the prescription of the flavour conservation, and at the bottom, we use the Maxwell

$B_0^{1/4} = 148$ MeV and $T_{max} = 138$ MeV for $B_0^{1/4} = 165$ MeV no matter the QHD parametrization, if we add the vector channel to the MIT model or not or even if we use the prescription of Maxwell or of flavour conservation, which is different from what happened in Fig. 2, where the results for T_{max} are more or less the same, i.e., around $T_{max} = 168$ MeV, no matter the value of B_0 , but that is because we used the symmetric matter to chose the values of T_0 in Eq. (4) exactly so that the maximum temperatures were around this value

Now, we analyse the region at low temperatures. Here, as for the symmetric matter, the results are different for each combination of parametrizations, and they also change depending on the prescription used, either flavour conservation or Maxwell construction. The maximum chemical potentials obtained for each combination of parametrizations are presented in Table 5.

Let us first scrutinise the effect of the different MIT bag-based model parametrizations at this region of low temperatures. We notice that the effect of G_V here, in stellar matter, causes smaller changes in the μ_{max} , which allows us to increase the values up to 0.3 fm^2 . Nevertheless, the qualitative results are the same: the higher the values for G_V , the higher are the results for μ_{max} . We can still compare the results between symmetric matter and stellar matter when

using the same parametrizations, namely, when $G_V = 0$ and $G_V = 0.1 \text{ fm}^2$. In doing so, we see very clearly that there is a reduction in the values of chemical potentials at $T = 0$ here, being this reduction even greater when we use the Maxwell prescription. This reduction also grows with the value of G_V and $B_0^{1/4}$ and is also greater when using the eL3 $\omega\rho$ than when using the NL3* $\omega\rho$ parametrization.

Being that the quark EoSs for symmetric matter are always stiffer than the ones for stellar matter and that the ones for stellar matter resulting from the flavour conservation prescription are stiffer than the β -stable and charge neutral ones, in general, stiffer quark EoSs favour the hadron phase. The same is true when we compare the results for different values of G_V , as higher values of G_V result in higher values for μ_{max} and also increase the stiffness of the EoS, the same behaviour found in symmetric matter. The increase of the bag pressure value $B_0^{1/4}$ also results in higher values of μ_{max} , but it softens the EoS, as we already pointed out in [54] for symmetric matter. So, to summarise, stiffer quark EoSs always favour the hadron phase, unless we change the value of the bag constant $B_0^{1/4}$ and then the opposite is true, a softer quark EoS favours the hadron phase.

An interesting aspect to analyse when comparing the results at low temperatures obtained from the two different prescriptions is their behaviour in relation to the results

Table 5 Chemical potentials ($\mu_{\max}$) at $T = 0$ for all combinations of parametrizations considering strange **stellar** matter. The first line corresponds to the results using the MIT-based bag models only, where

Models	$B_0^{1/4} = 148 \text{ MeV}$	$B_0^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$B_0^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$B_0^{1/4} = 165 \text{ MeV}$	$B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$B_0^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$
-	$\mu_{\max} = 867$	$\mu_{\max} = 884$	$\mu_{\max} = 915$	$\mu_{\max} = 962$	$\mu_{\max} = 986$	$\mu_{\max} = 1027$
flavour conservation						
NL3* $\omega\rho$	$\mu_{\max} = 960$	$\mu_{\max} = 1089$	$\mu_{\max} = 1268$	$\mu_{\max} = 1192$	$\mu_{\max} = 1253$	$\mu_{\max} = 1462$
eL3 $\omega\rho$	$\mu_{\max} = 959$	$\mu_{\max} = 1181$	no crossing	$\mu_{\max} = 1225$	$\mu_{\max} = 1324$	no crossing
Maxwell prescription						
NL3* $\omega\rho$	$\mu_{\max} = 867$	$\mu_{\max} = 884$	$\mu_{\max} = 915$	$\mu_{\max} = 967$	$\mu_{\max} = 1012$	$\mu_{\max} = 1142$
eL3 $\omega\rho$	$\mu_{\max} = 867$	$\mu_{\max} = 884$	$\mu_{\max} = 915$	$\mu_{\max} = 967$	$\mu_{\max} = 1013$	$\mu_{\max} = 1727$

the phase transition criterion is just the value of the chemical potential where the pressure goes to zero, as done in [15]. All values are given in MeV

obtained via MIT bag-based model alone. As it is very clear from Fig. 4 and Table 5, when we use the prescription of flavour conservation, the results are quite different, whereas when using the Maxwell prescription the diagrams generally converge to the same $\mu_{\max}$ s. First, we have to point out that the diagrams obtained via MIT bag-based model alone (red lines) presented at the top and at the bottom of Fig. 4 are the same, i.e., of β -stable and charge neutral quark and lepton matter. Now, when we use a QHD+MIT models, with the flavour conservation, the hadronic phase completely determines the quark phase through the bond given by Eq. (10). The quark matter at the phase transition point is not yet β -stable, and, adding to that, in this case, we have P_0 s that are not so close to zero, so it is not surprising that the results for this prescription are different from the ones obtain only with the MIT-based model. When using the Maxwell prescription, however, we have a quark matter at the phase transition point that is β -stable, and when Gibbs' conditions at this region are met at very low pressures ($P_0 \leq 5 \text{ MeV/fm}^3$), we get $\mu_{\max}$ s very similar to the ones obtained via MIT alone. When the P_0 s start to get higher, also, the $\mu_{\max}$ s grow.

As for the effects of the different QHD model parametrizations at this region of low temperatures, we can see that they are not so preponderant here as for the symmetric matter and are the smallest when using the Maxwell prescription. But, whenever there is a difference, the $\mu_{\max}$ s for the eL3 $\omega\rho$ are the highest, and in some cases, we were not able to find crossings at $T = 0$, as, for example, the case with $G_V = 0.3 \text{ fm}^2$ and this QHD parametrization. The reason for this behaviour is the fact that the onset of the hyperons softens the EoS in the hadronic phase, while the G_V monotonically stiffens the EoS of the quark phase. At very high density, we expect that the quark phase becomes non-interacting; therefore, the contribution of the Dirac sea can no longer be neglected.

To finalize the analysis of the results in this region of low temperatures, we point out that we obtain results for $\mu_{\max}$ within a range of 1000 MeV from the $\mu = 1200 \text{ MeV}$ indicated in [32] for β -stable matter and various combinations

of parametrizations when using the prescription of flavour conservation, namely, for NL3* $\omega\rho$ in combination with $B_0^{1/4} = 148 + G_V = 0.3 \text{ fm}^2$, $B_0^{1/4} = 165 + G_V = 0$ and $B_0^{1/4} = 165 + G_V = 0.1 \text{ fm}^2$ and for eL3 $\omega\rho$ in combination with $B_0^{1/4} = 148 + G_V = 0.1 \text{ fm}^2$ and $B_0^{1/4} = 165 + G_V = 0$, but only for one combination of parametrizations when using the Maxwell prescription, namely, for NL3* $\omega\rho$ in combination with $B_0^{1/4} = 165 + G_V = 0.3 \text{ fm}^2$. So, considering the MIT parametrizations, only two, the ones with $B_0^{1/4} = 148$, are within the stability window and are able to describe stable strange matter [16]. However, in order to construct hybrid stars, parametrizations outside the stability window are preferable to avoid that once the phase transition starts, the entire star converts to a strange quark star. In [58], we constructed hybrid stars using the original L3 $\omega\rho$ parametrization (a slightly different parametrization than the one used here) for the QHD model and the MIT-based model.

We could improve the results at this region by adding also a quartic term to the MIT vector model, as done in [15, 16]. Using the very small value of $b_4 = 2$, we are able to obtain crossings at $T=0$ for the two cases pointed out in Table 5 where we did not obtain results without this term, although the values for the $\mu_{\max}$, and also the pressure, would still be very high. Choosing higher values for b_4 , we can adjust the values of $\mu_{\max}$ without altering the values of $T_{\max}$, and the higher the value of G_V , the higher the affect of this term, as already shown in [15].

5.3 Testing Different Hypotheses

In this section, we take a better look at the differences occurring due to the different matter hypothesis and prescription imposed along this work. In order to do so, we choose only two different combinations of parametrizations and plot all results in Fig. 5. We also include the results for stellar matter with no strange matter, i.e., with no hyperons and strange quarks (dashed lines).

As already pointed out above and is more evident here, the maximum temperature for strange stellar matter depends

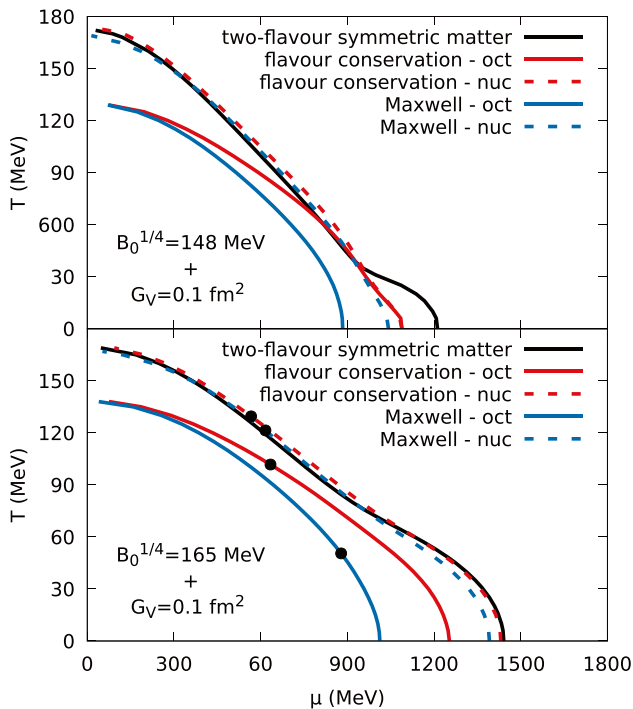


Fig. 5 Phase diagrams for all different matter hypothesis and prescriptions for phase transition used in this work considering the $NL3^*\omega\rho$ parametrization for the hadronic phase and $B_0^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ (top) and $B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ (bottom) for the quark phase. We also include the results for stellar matter with no strange matter (dashed lines). The points presented on the bottom diagrams correspond to the ones where the latent heat L_S goes to zero (see Section 6) and may correspond to the critical end points

only on the bag pressure value and is the same for both prescriptions, the one with flavour conservation (solid red line) and the one with Maxwell construction (solid blue line). It is also more evident here that at low temperatures, the prescription of flavour conservation favours the hadronic phase. What is completely new here, though, are the diagrams for stellar matter with no hyperons in the hadronic matter and no strange quark in the quark matter. We chose only to present those results for this two combination of parametrizations because its general behaviour follows the ones analysed until

now, i.e., the different parametrizations have the same affect to this matter as for the previous cases.

As could be expected, the results for the diagrams for stellar matter without strangeness are very close to the ones for two-flavour symmetric matter. This is because the contribution to the chemical potential coming from the leptons, only present in the stellar matter, is very small.

At very low temperatures, we have obtained an interesting feature for the flavour conservation prescription. For the $NL3^*\omega\rho + B_0^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$ parametrization, the phase transition for the case where the strange matter is allowed to appear occurs at chemical potentials where there is no hyperon onset yet, so, at this region, the results for stellar matter with (solid red line) and without (dashed red lines) strange matter are the same. And at higher temperatures, where the hyperons start to appear at the point of the phase transition, the two lines start to differentiate, and the line without strange matter approximates itself from the line of two-flavour symmetric matter. For the other parametrization, i.e., $NL3^*\omega\rho + B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$, however, the phase transition for strange matter occurs at chemical potentials where there are already hyperons present, and so, the results for stellar matter with and without strange matter are different throughout the entire phase diagram. As for the Maxwell prescription, despite the phase transition for strange matter at low temperatures for both parametrizations occurring at chemical potentials where there are no hyperons, the results for strange matter (solid blue lines) and no strange matter (dashed blue lines) are always very different. This happens because for strange quark stellar matter, the strange quarks are always present, as can be seen in [16].

In Table 6, we present the results for the maximum chemical potentials at the phase transition for stellar matter without strange matter. There, we also include the results when using only the MIT-based model, and, as can be noted, its results are only similar to the ones when combining models when we use $B_0^{1/4} = 148 \text{ MeV}$ and $G_V = 0$. This is because in this cases, $P_0 \rightarrow 0$ and for all the other parametrizations, the phase transition happens at increasing pressures. Yet about the results for stellar matter without

Table 6 Chemical potentials (μ_{max}) at $T = 0$ for all combinations of parametrizations considering **stellar matter without strange matter**. All values are given in MeV

Models	$B_0^{1/4} = 148 \text{ MeV}$	$B_0^{1/4} = 148 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$B_0^{1/4} = 148 \text{ MeV} + G_V = 0.3 \text{ fm}^2$	$B_0^{1/4} = 165 \text{ MeV}$	$B_0^{1/4} = 165 \text{ MeV} + G_V = 0.1 \text{ fm}^2$	$B_0^{1/4} = 165 \text{ MeV} + G_V = 0.3 \text{ fm}^2$
-	$\mu_{max} = 953$	$\mu_{max} = 976$	$\mu_{max} = 1017$	$\mu_{max} = 1063$	$\mu_{max} = 1094$	$\mu_{max} = 1149$
flavour conservation						
$NL3^*\omega\rho$	$\mu_{max} = 960$	$\mu_{max} = 1089$	$\mu_{max} = 1750$	$\mu_{max} = 1259$	$\mu_{max} = 1431$	$\mu_{max} = 1947$
$eL3\omega\rho$	$\mu_{max} = 959$	$\mu_{max} = 1568$	no crossing	$\mu_{max} = 1405$	$\mu_{max} = 1751$	no crossing
Maxwell prescription						
$NL3^*\omega\rho$	$\mu_{max} = 955$	$\mu_{max} = 1042$	$\mu_{max} = 1671$	$\mu_{max} = 1229$	$\mu_{max} = 1391$	$\mu_{max} = 1886$
$eL3\omega\rho$	$\mu_{max} = 957$	$\mu_{max} = 1450$	no crossing	$\mu_{max} = 1350$	$\mu_{max} = 1681$	no crossing

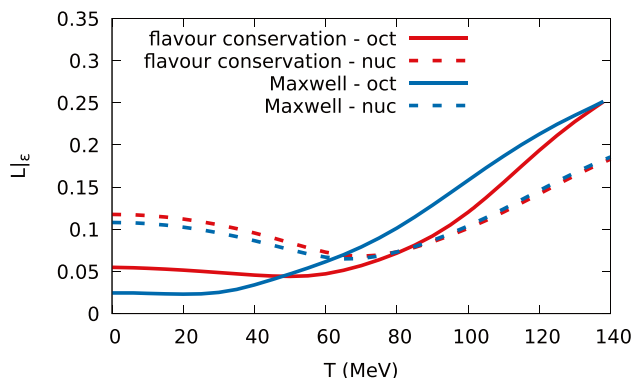


Fig. 6 Results for latent energy for different matter hypothesis and prescriptions for phase transition used in this work considering the NL3* $\omega\rho$ parametrization for the hadronic phase and $B_0^{1/4} = 165$ MeV + $G_V = 0.1$ fm² for the quark phase

strange matter using only the MIT-based model, its T_{max} s reach around only $T=140$ MeV, i.e., very below the ones obtained for the same matter but combining models. That is because when combining models, the crossing of the $\mu \times P$ occurs at pressures considerably high and higher than for strange matter.

6 Latent Heat and Latent Energy

The latent heat is an important quantity in the study of phase transitions, and we investigate two different expressions found in the literature. We first take a look at the relativistic latent heat $L|_\epsilon$, as given in [65, 66]

$$L|_\epsilon = P^H \frac{\epsilon^Q - \epsilon^H}{\epsilon^Q \epsilon^H}, \quad (13)$$

where ϵ^Q and ϵ^H are the energy densities at the point of the phase transition for the quark and hadronic matter,

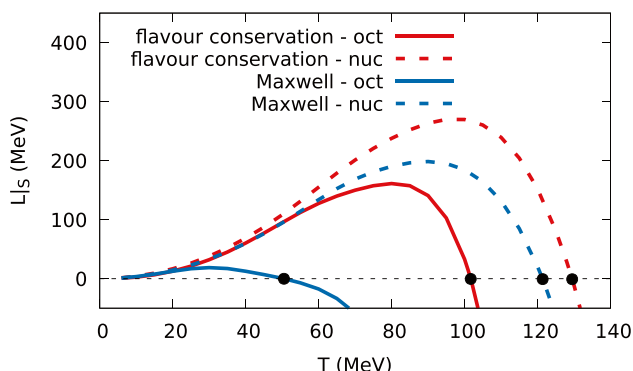


Fig. 7 Results for latent heat for different matter hypothesis and prescriptions for phase transition used in this work considering the NL3* $\omega\rho$ parametrization for the hadronic phase and $B_0^{1/4} = 165$ MeV + $G_V = 0.1$ fm² for the quark phase

respectively. We call it latent energy next, since it yields results that are not zero at zero temperature. Notice that this quantity has no dimension. According to [65], when the latent energy becomes zero, the critical end point, where the first-order phase transition turns into a crossover, is reached.

We also analyse the usual expression found in textbooks and investigated in [67] as well:

$$L|_S = (S^Q - S^H)T, \quad (14)$$

where S^Q and S^H are the entropy densities, defined in Eq. (15), at the point of the phase transition for the quark and hadronic matter, respectively.

$$S = -\gamma \sum_i \int \frac{d^3k}{(2\pi)^3} \left[f_{i+} \ln \left(\frac{f_{i+}}{1-f_{i+}} \right) + \ln(1-f_{i+}) + f_{i-} \ln \left(\frac{f_{i-}}{1-f_{i-}} \right) + \ln(1-f_{i-}) \right], \quad (15)$$

where γ is degeneracy of spin (and color in the case of the quarks), and $f_{i\pm}$ are the Fermi-Dirac distribution functions of the particles and anti-particles, respectively.

In Figs. 6 and 7, we present the results obtained with the latent energy and latent heat, respectively, as a function of the temperature at the point of the hadron-quark phase transition. We choose only one parametrization since they yield qualitatively similar results. One can see that our results for the latent energy at zero temperature are of the same order of magnitude as the ones obtained in [64], but never reach the values shown in [66] and are always positive. As far as the latent heat is concerned, the behaviour we find is more similar to the result shown in [67], and we do find a maximum point, but at much lower temperatures. Our curves then cross the zero value, which, according to [65], is an indication of the end of the first-order phase transition. Assuming this interpretation is correct, our curves depicted in the QCD phase diagrams are only valid until the temperatures where $L|_S$ is zero and the corresponding values are shown as dots also in Fig. 5.

7 Conclusions

In this work, we sought to reproduce the QCD phase diagram using two effective models. For the description of quark matter, a simple relativistic model, the MIT bag model, and a modification thereof were used, the latter being new to this kind of procedure. And for the description of hadronic matter, a model of quantum hadrodynamics was used, the Walecka Model with non-linear terms.

Gibbs' conditions were used to establish the crossing points of the pressure as a function of the chemical potentials obtained in both phases.

Some restrictions were imposed when choosing the models mentioned above. We used temperature-dependent bag $B(T)$ values with the same parametrizations already used in our previous work [15], with one set of constants within the limits the stability window, i.e., $B_0^{1/4} = 148$ MeV and another set outside this window, i.e., $B_0^{1/4} = 165$ MeV, both sets with G_V varying from zero to $G_V = 0.3$ fm². As for the QHD-based model, we used the NL3* $\omega\rho$ and eL3 $\omega\rho$ parametrizations, which satisfy several nuclear and astrophysical properties.

We first obtained the phase diagrams for two-flavour symmetric matter. As the parametrizations for the $B(T)$ were adjusted so that the T_{max} would satisfy the restraint imposed by the LQCD and freeze-out results; the T_{max} s are all around 168 MeV, but only the results for $B_0^{1/4} = 165$ MeV fit the Cleymans line entirely inside the confined (hadron) phase. The results for low temperatures depend on all parametrizations, as already concluded in [54]. The influences of the different MIT-based models follow the conclusions of [15], but here, the results are more sensible to the different values of G_V s. Only a few combination of parametrizations resulted in maximum chemical potentials μ_{max} within the range $1050 \leq \mu \leq 1400$ MeV. Ultimately, only NL3* $\omega\rho + B_0^{1/4} = 165$ MeV and NL3* $\omega\rho + B_0^{1/4} = 165$ MeV + $G_V = 0.05$ fm² satisfy all restrains we imposed.

We also calculated the phase diagrams for stellar matter using two different prescriptions, one where we impose flavour conservation at the point of the phase transition, a prescription never applied to finite temperatures before, and the other where both phases are β -stable and charge neutral, being this last one the Maxwell prescription. Between the two prescriptions, the flavour conservation results in higher chemical potentials at low temperatures, but for both prescriptions, the T_{max} s are the same and depend only on the value of B_0 . Comparing this results to the symmetric matter, we conclude that the stellar matter favours the quark phase, being that the T_{max} s and μ_{max} s are higher for symmetric matter.

In all cases, a higher bag pressure and G_V value increases the value of the chemical potentials at low temperatures, but this increase is lower for the stellar matter in the flavour conservation prescription and even lower in the Maxwell prescription than in the two-flavour symmetric matter. As for the QHD model parametrization, the eL3 $\omega\rho$ always favours the hadron phase when compared to the NL3* $\omega\rho$ one.

We also discussed the results for stellar matter without strange matter and concluded that those results are very similar to the ones for two-flavour symmetric matter.

Finally, at the last section, we discussed briefly the results for latent heat and showed that for stellar matter with strange matter, it always grows with the temperature. For two-flavour symmetric matter and stellar matter without strange matter, though, the behaviour is more of a sinusoid.

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Declarations

Conflict of Interest The authors declare no competing interests.

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Hypermassive quark cores

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ABSTRACT

Using a quantum hadrodynamics (QHD) and MIT-based models, we construct hybrid stars within the Maxwell criteria of hadron-quark phase transition. We are able to produce a hybrid star with maximum mass of $2.15 M_{\odot}$. Furthermore, a $2.03 M_{\odot}$ star with a quark core corresponding to more than 80 per cent of both, its total mass and radius, is also possible.

Key words: equation of state – stars: formation – stars: interiors – stars: massive – stars: neutron.

1 INTRODUCTION

Since the pioneer observation of PSR J1614–2230 (Demorest et al. 2010) with a mass of $1.97 \pm 0.04 M_{\odot}$, over a decade ago, we are aware of the existence of two solar masses neutron stars. More recently, PSR J0348+0432, with a mass of $2.01 \pm 0.04 M_{\odot}$ (Antoniadis et al. 2013), and PSR J0740+6620 with a mass of $2.14^{+0.10}_{-0.09} M_{\odot}$ (Cromartie et al. 2020) were observed. Last year, not only the mass measurement of the PSR J0740+6620 was refined by the *Neutron Star Interior Composition Explorer (NICER)* telescope to the range of $2.072^{+0.067}_{-0.066} M_{\odot}$, but the radius of PSR J0740+6620 was also constrained to be lying between 11.41 and 13.70 km (Riley et al. 2021).

As the quantum chromodynamics (QCD) phase diagram became widely accepted, the possibility that neutron stars could, in fact, contain both a hadronic and a quark phase, started to be considered (Ivanenko & Kurdgelaidze 1965; Menezes 2021). In the same way that asymptotic freedom allows matter to become deconfined when the density increases at low temperatures, a similar behaviour can take place in the interior of neutron stars.

As the density increases towards the core of the star, quarks can become more energetically favorable than baryons, and, ultimately, the core of a neutron star may be composed of deconfined quarks. If the entire star does not convert itself into a quark star, as suggested by the Bodmer–Witten conjecture (Bodmer 1971; Witten 1984), the final composition is a quark core surrounded by a hadronic layer. This is what is generally called a hybrid star.

In this work, we construct hybrid stars using a quantum hadrodynamics (QHD) based model, the so-called Walecka Model (Serot & Walecka 1986) with non-linear terms (Boguta & Bodmer 1977), to construct the hadronic equation of state (EoS) and MIT-based models to construct the quark EoS. As for the MIT based models, we use the original MIT bag model, as introduced in Chodos et al. (1974a,b), and explore two variations thereof, with a vector channel and with the vector channel altogether with the Dirac sea contribution, as introduced in Lopes, Biesdorf & Menezes (2021a,b).

For the hadronic model, we use the parametrization presented in Lopes (2022), which fulfills most of the experimental constraints at the saturation density and also produces massive neutron stars, even when hyperons are present, fulfilling the constraints of the

PSR J0740+6620 (Riley et al. 2021). For the quark model we use parametrizations that reproduce unstable strange matter, as we do not want the entire star to convert into a quark star, which could happen had we used stable strange matter.

We utilize the Maxwell construction to define the hadron-quark phase transition and to make the connection between the hadronic and the quark EoS. Finally, the TOV equations (Oppenheimer & Volkoff 1939) are solved to generate the macroscopic properties of the hybrid stars.

Even though various studies (Gerstung, Kaiser & Weise 2020; Lopes & Menezes 2021) show that hyperons seem inevitable, based on the fact that the hyperon onset in the stellar matter is still an open issue, we dedicate a section to study hybrid stars without hyperons.

We then use the Nambu Jona-Lasinio (NJL) model (Nambu & Jona-Lasinio 1961) to construct the quark EoS and make a comparison with the results obtained with the extended versions of the MIT bag model. We also investigate the differences between the hybrid stars with the parametrization of Lopes (2022), and with the traditional GM1 (Glendenning 2000), as presented in Lopes & Menezes (2021).

To finish, we analyse two important physical quantities, the dimensionless tidal parameter and the speed of sound. We compare the results for the tidal parameter to the recent constraints presented in the literature coming from LIGO/VIRGO gravitational wave observatories and the GW170817 event (Abbott et al. 2018). As for the speed of sound, we obtain its square value and study its relation with the size and mass of the quark core in hybrid stars as proposed in Annala et al. (2020).

2 HADRONIC EOS

The non-linear $\sigma\omega\rho$ QHD model has the following Lagrangian density (Glendenning 2000):

$$\begin{aligned} \mathcal{L}_{\text{QHD}} = & \sum_B \bar{\psi}_B [\gamma^\mu (i\partial_\mu - g_{B\omega}\omega_\mu - g_{B\rho}\tau_B \cdot \rho_\mu) + \\ & -(M_B - g_{B\sigma}\sigma)] \psi_B - U(\sigma) + \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\nu^2 \omega_\mu \omega^\mu + \\ & + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu - \frac{1}{4} P^{\mu\nu} \cdot P_{\mu\nu}, \end{aligned} \quad (1)$$

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Table 1. Parameters of the model utilized in this work and their prediction for the symmetric nuclear matter properties at the saturation density; the parametrization is taken from Lopes (2022) and the phenomenological constraints are taken from Dutra et al. (2014) and Oertel et al. (2017).

	Parameters		Constraints	This model
$(g_{N\sigma}/m_\sigma)^2$	12.108 fm ²	n_0 (fm ⁻³)	0.148–0.170	0.156
$(g_{N\omega}/m_\omega)^2$	7.132 fm ²	M^*/M	0.6–0.8	0.69
$(g_{N\rho}/m_\rho)^2$	4.801 fm ²	K (MeV)	220–260	256
κ	0.004 138	S_0 (MeV)	28.6–34.4	31.2
λ	−0.003 90	L (MeV)	36–86.8	74
$\Lambda_{\omega\rho}$	0.0185	B/A (MeV)	15.8–16.5	16.2

in natural units. The sum on B runs over the entire baryon octet: nucleons (N) and hyperons (Y). ψ_B is the baryonic Dirac field. σ , ω_μ , and ρ_μ are the mesonic fields. The g s are the Yukawa coupling constants that simulate the strong interaction, M_B is the baryon mass, m_σ , m_ω , and m_ρ are the masses of the σ , ω , and ρ mesons, respectively. The antisymmetric mesonic field strength tensors are given by their usual expressions as presented in Glendenning (2000). τ are the Pauli matrices. The $U(\sigma)$ is the self-interaction term introduced in Boguta & Bodmer (1977) to fix the incompressibility and is given by

$$U(\sigma) = \frac{\kappa M_N (g_\sigma \sigma)^3}{3} + \frac{\lambda (g_\sigma \sigma)^4}{4}. \quad (2)$$

Now, besides the traditional non-linear $\sigma\omega\rho$ QHD lagrangian density, we introduce two additional terms. The first one is the strangeness hidden ϕ vector meson, which couples only with the hyperons (Y), not affecting the properties of symmetric matter:

$$\mathcal{L}_\phi = g_{Y\phi} \bar{\psi}_Y (\gamma^\mu \phi_\mu) \psi_Y + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu - \frac{1}{4} \Phi^{\mu\nu} \Phi_{\mu\nu}. \quad (3)$$

As pointed out in Lopes & Menezes (2021), Cavagnoli & Menezes (2005), Lopes & Menezes (2020), and Weissenborn, Chatterjee & Schaffner-Bielich (2012), this vector channel is crucial to obtain massive hyperonic neutron stars. The second one is a non-linear $\omega\rho$ coupling term (Fattoyev et al. 2010):

$$\mathcal{L}_{\omega\rho} = \Lambda_{\omega\rho} (g_{N\rho}^2 \rho^\mu \cdot \rho_\mu) (g_\omega^2 \omega^\mu \omega_\mu), \quad (4)$$

which is necessary to correct the slope of the symmetry energy (L) and has strong influence on the radii and tidal deformation of the neutron stars (Cavagnoli, Menezes & Providencia 2011; Dexheimer et al. 2019).

In order to produce β stable matter, with zero net charge, we also add leptons as a free Fermi gas.

The detailed calculations of the EoS for symmetric nuclear matter, as well as for β -stable matter in the QHD formalism are well documented and can be easily found in the literature (Serot 1992; Glendenning 2000; Menezes 2021).

The parametrization utilized in this work, retired from Lopes (2022), fulfills most of the experimental constraints at the saturation density and also produces massive neutron stars, reaching the mass of the PSR J0740+6620 even when hyperons are present. Its values, as well as the predictions of this model for the symmetric nuclear matter are presented in Table 1. The nuclear constraints at the saturation density are also in Table 1 and were taken from two extensive review articles, Dutra et al. (2014) and Oertel et al. (2017). Furthermore, the masses of the particles we use are the physical ones. The meson masses are $m_\omega = 783$, $m_\rho = 770$, $m_\phi = 1020$, and $m_\sigma = 512$ MeV; the baryon octet masses are $M_N = 939$, $M_\Lambda = 1116$, $M_\Sigma = 1193$,

and $M_\Xi = 1318$ MeV; and the lepton masses are $m_e = 0.51$ and $m_\mu = 105.6$ MeV.

When hyperons are present, it is crucial to define the strength of the hyperon-meson coupling constants. The only well-known parameter is the Λ^0 hyperon potential depth, $U_\Lambda = -28$ MeV. Here, we follow Lopes (2022) and Lopes & Menezes (2014) and assume that both vector and scalar mesons are constrained by SU(3) symmetry group (De Swart 1963; Miyatsu, Cheoun & Saito 2013). In this case, given a value of α_v , the value of α_s is automatically determined to reproduce $U_\Lambda = -28$ MeV. In order to produce very stiff EoS, we use here $\alpha_v = 0.50$, which implies $\alpha_s = 0.911$ (Lopes 2022). Thereby, the ratios g_{YB}/g_{NB} are (Lopes & Menezes 2014; Lopes & Menezes 2021; Lopes 2022)

$$\begin{aligned} g_{\Lambda\omega}/g_{N\omega} &= 0.714; & g_{\Lambda\phi}/g_{N\omega} &= -0.808; & g_{\Lambda\rho}/g_{N\rho} &= 0; \\ g_{\Sigma\omega}/g_{N\omega} &= 1.0; & g_{\Sigma\phi}/g_{N\omega} &= -0.404 & g_{\Sigma\rho}/g_{N\rho} &= 1.0; \\ g_{\Xi\omega}/g_{N\omega} &= 0.571; & g_{\Xi\phi}/g_{N\omega} &= -1.01; & g_{\Xi\rho}/g_{N\rho} &= 0; \\ g_{\Lambda\sigma}/g_{N\sigma} &= 0.646; & g_{\Sigma\sigma}/g_{N\sigma} &= 0.690; & g_{\Xi\sigma}/g_{N\sigma} &= 0.314. \end{aligned} \quad (5)$$

3 QUARK EOS

We compare here three variations of the MIT bag model. The original MIT bag model, as introduced in Chodos et al. (1974a,b), and two variations, with a vector channel and with the vector channel altogether with the Dirac sea contribution, as introduced in Lopes et al. (2021a,b).

3.1 Original MIT bag model

The Lagrangian density of the original MIT bag model, as introduced in Johnson (1978), reads

$$\mathcal{L}_{MIT} = \sum_{u,d,s} \{ \bar{\psi}_q [i\gamma^\mu \partial_\mu - m_q] \psi_q - B \} \Theta(\bar{\psi}_q \psi_q), \quad (6)$$

where ψ_q is the Dirac field of the quark q , m_q are the quark masses running over u , d , and s , whose values are 4, 4, and 95 MeV, respectively (Tanabashi et al. 2018). The Bag B plays the role of the strong force, and it is the vacuum pressure constant. $\Theta(\bar{\psi}_q \psi_q)$ is the Heaviside step function to assure that the quarks exist only confined to the bag. The way to obtain the EoS in the original Bag model is well discussed in Lopes et al. (2021a,b) and Johnson (1978).

The free parameter B is not completely arbitrary if one is interested in the absolute stability of the quark matter. In this case, the energy per baryon (E/B) must be lower than 930 MeV at zero pressure and zero temperature. If the B value is too low, the $u-d$ quark matter is more stable than the hadronic matter ($E/B < 930$ MeV), therefore, the universe as we know would not exist. If the B is too high both the $u-d$ matter and the $u-d-s$ quark matter are unstable ($E/B > 930$ MeV). However, for a small range of values, called stability window, the $u-d$ matter is unstable, while the $u-d-s$ matter is stable. In this case, the universe as we know is only meta-stable. This is the so-called Bodmer–Witten conjecture. To more details, see Torres & Menezes (2013), Backes et al. (2021), Lopes et al. (2021a,b), and the references therein.

The values of the Bag pressure that fulfill the stability window are ultimately dependent on the s quark mass. For the value used in this work, $m_s = 95$ MeV, this corresponds to $148 < B^{1/4} < 159$ MeV.

Now, as in this work we want to construct a hybrid star, we need to use an unstable $u-d-s$ matter. If the $u-d-s$ matter is stable, as soon as the core of the star converts into the quark phase, the entire star may convert into a quark star in a finite amount of time (Olinto 1987). Therefore, here we use $B^{1/4} > 159$ MeV.

Another important quantity is the baryon chemical potential at the interface, μ_S , which is the chemical potential value where the quark phase becomes energetically favorable, and a hadron-quark phase transition occurs. Its value is strongly model dependent, and it is defined at the point where the quark chemical potential equals the baryon chemical potential at the same pressure. Chemical equilibrium at a sharp interface is given by

$$\mu_S^H = \mu_S^Q \quad \text{and} \quad p_H = p_Q, \quad (7)$$

where μ_S for the hadrons and quarks are

$$\mu_S^H = \frac{(\sum_H \mu_H n_H + \sum_l \mu_l^H n_l^H)}{\sum_H n_H},$$

$$\mu_S^Q = \frac{3 \left(\sum_Q \mu_Q n_Q + \sum_l \mu_l^Q n_l^Q \right)}{\sum_Q n_Q}. \quad (8)$$

The use of equation (7) as the criteria to define hadron-quark phase transition is called Maxwell construction. In the so-called Maxwell construction, the hadron and quark phases are spatially separated and there is no mixed-phase. The Maxwell construction is not the only possible way to study quark-hadron phase transition. Another one is the so-called Gibbs condition, where there is a mixed phase and the quarks and hadrons can coexist. Some authors (Maruyama et al. 2007; Paoli & Menezes 2010) performed studies on hybrid stars with both, Maxwell and Gibbs construction and concluded that there is no significant difference on the macroscopic properties of the hybrid stars. On the other hand, some studies (Yasutake et al. 2014; Maslov et al. 2019) point out that the presence of a mixed phase can affect some neutron star properties, especially those with intermediate masses, although the value of the maximum mass is hardly affected. Moreover, a recent work (Lugones & Grunfeld 2021) suggests that the Maxwell construction is favoured if the surface tension between quarks and hadrons is high enough, therefore, we use this construction.

There is no experimental evidence about the value of the baryon chemical potential at the interface at zero temperature. Thus, we follow Lopes et al. (2021a) and only analyse models where the baryon chemical potential at the interface lies within $1050 < \mu_S < 1400$ MeV. These values are not aleatory. An inferior limit of 1050 MeV was pointed out in Fukushima & Hatsuda (2011) using the Polyakov loop formalism. The upper limit is based on the discussion of Annala et al. (2020), where the authors point out that quark matter inside massive neutron stars is not only possible but probable. Using the chiral bag model, with $B^{1/4} = 170$ MeV, Klähn, Fischer & Hempel (2017) found a value of μ_S around 1250 MeV. We assume a maximum value of $\mu_S = 1400$ MeV as a less restrictive estimate.

In Fig. 1, we plot the pressure as a function of the chemical potential for the baryonic matter (QHD), as well as for different Bag pressure values in the original MIT bag model. When the QHD curve crosses the MIT one, we have the Maxwell criteria of equation (7). The baryon chemical potential at the interface, as well as the pressure and the energy density (in both phases) are displayed in Table 2. As can be seen, for $B^{1/4} = 165$ MeV, the baryon chemical potential at the interface is below 1050 MeV; therefore, this result will not be analysed any further. Notice that μ_S grows with the Bag pressure value.

Now, we construct a hybrid EoS with different values of the Bag for the original MIT model. The EoS will be hadronic for values below the baryon chemical potential at the interface, and a quark EoS is the preferential one for values above it. Then, we use this EoS as an input and solve the TOV equations (Oppenheimer & Volkoff 1939) to obtain the macroscopic properties of the hybrid stars. The

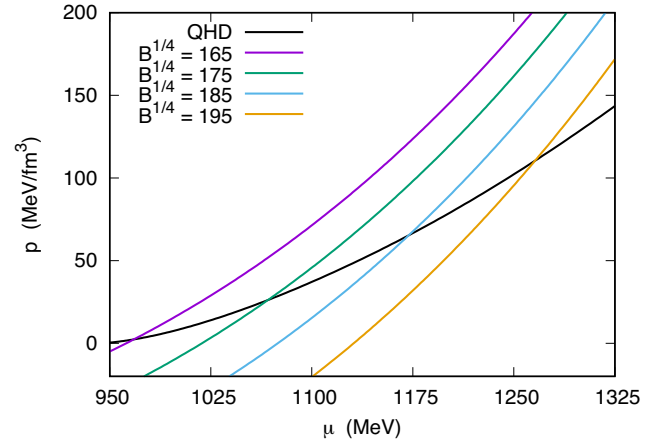


Figure 1. (Colour online) The pressure as a function of the chemical potential for different values of the Bag. The crossing points indicate the baryon chemical potential at the interface, where the quark phase becomes energetically favorable.

Table 2. Baryon chemical potential at the interface and pressure at the phase transition from hadron (H) to quark (Q) for different values of the Bag.

$B^{1/4}$ (MeV)	$\mu_S^H = \mu_S^Q$ (MeV)	$p^H = p^Q$ (MeV/fm ³)	ϵ_H	ϵ_Q
165	968	2	121	407
175	1067	26	319	596
185	1172	65	440	832
195	1266	110	558	1112

Note. We also show the energy density (in MeV/fm³) at both phases at the baryon chemical potential at the interface.

BPS EoS is used to simulate the neutron star crust (Baym, Pethick & Sutherland 1971). We also define the quantity $M_{\min}$ as the minimum mass star that has a quark core. Stars with masses below $M_{\min}$ are purely hadronic. The results are presented in Fig. 2, and summarized in Table 3.

At first glance, it may seem that the maximum mass of a hybrid star grows with the Bag pressure value. But it is not what really happens. For values of $B^{1/4} > 175$ MeV, the central energy density lies in the window between the quark and hadron EoS. This region does not physically exist in the Maxwell construction, and the phases are spatially separated. Only with $B^{1/4} = 175$ MeV, we have a dynamically stable hybrid star, i.e., a star family that satisfies $\partial M / \partial \epsilon_0 > 0$ (Glendenning 2000), with a maximum mass of $1.40 M_\odot$, which is way below the observational limit of the PSR J070+6620 (Riley et al. 2021). Therefore, with the chosen QHD parametrization we cannot explain massive pulsars as a hybrid star within the original MIT bag model.

3.2 Vector MIT bag model

We can overcome the issue of not obtaining massive hybrid stars by introducing a vector channel to the original MIT bag model, similar to the vector channel in QHD models, as was done in Lopes et al. (2021a,b). To this end, we add the following Lagrangian density to the original one presented in equation (6):

$$\mathcal{L}_V = \left\{ \sum_{u,d,s} \bar{\psi}_q g_{qv} \psi_q - \frac{1}{2} m_V^2 V^\mu V_\mu \right\} \Theta (\bar{\psi}_q \psi_q), \quad (9)$$

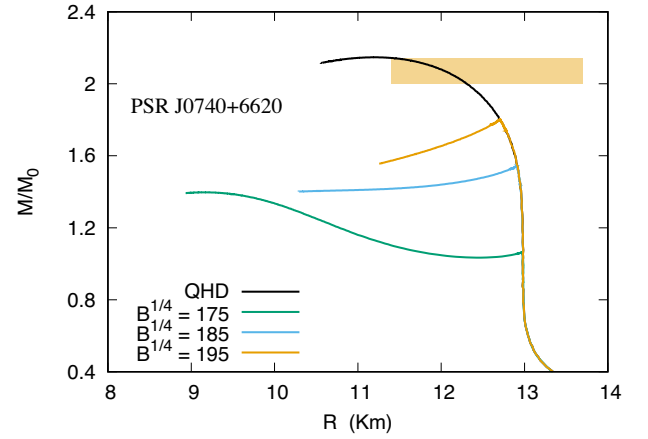
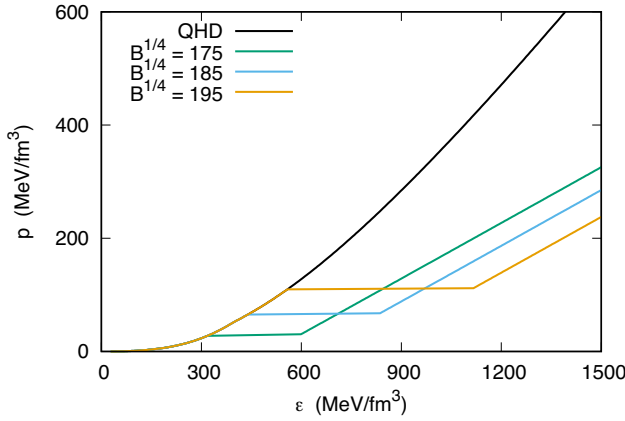


Figure 2. (Colour online) EoS (left-hand panel) and the TOV solutions (right-hand panel) for different values of the Bag. The hatched area represents the uncertainty about both, the radius and mass of the PSR J0740+6620 (Riley et al. 2021). For the original Bag model, no dynamically stable hybrid star reaches the mass of the PSR J0740+6620.

Table 3. Hybrid star properties for different values of the Bag.

Bag ^{1/4} (MeV)	$M/M_{\odot}$	R (km)	ϵ_c	$M_{\min}/M_{\odot}$	$R_{1.4}$ (km)
175	1.40	9.15	2170	1.05	9.15
185	1.54	12.90	444	1.54	12.96
195	1.80	12.70	868	1.80	12.96
QHD	2.15	11.20	1279	–	12.96

Note. Only $B^{1/4} = 175$ MeV produces dynamically stable hybrid stars.

where the quark interaction is mediated by the vector channel V_{μ} , analogous to the ω meson in QHD. Indeed, in this work we consider that the vector channel is the ω meson itself. The inclusion of vector channels in the MIT bag model is not new, as can be seen in Gomes et al. (2019a), Gomes, Char & Schramm (2019b), Franzon, Gomes & Schramm (2016), Klähn & Fischer (2015), and Wei et al. (2019).

Unfortunately, in the previous papers, the authors missed the mass term of the vector channel. As discussed in Lopes et al. (2021b), in mean field approximation, the vector field becomes zero if the mass is zero. The construction of the EoS within the vector MIT bag model in mean field approximation is analogous to the QHD one, and it is well discussed in Lopes et al. (2021a,b).

It is interesting to notice that both the QHD and the vector MIT models use the exchange of massive mesons to simulate the strong force, indicating an unified interaction scheme. Moreover, as we know the quantum numbers related to the isospin, hypercharge for the baryons, quarks, and mesons, we can also use the same criteria here as we used in the QHD to fix the relative strength of the quarks with the ω meson. In other words, we also use here the SU(3) symmetry group. The detailed calculation is presented in the appendix of Lopes et al. (2021b). We have

$$g_{sV}/g_{uV} = g_{sV}/g_{dV} = 0.40. \quad (10)$$

Another important quantity is the G_V defined as

$$G_V \doteq (g_{uV}/m_V)^2, \quad (11)$$

and is related to the strength of the vector field. The higher the value of G_V , higher the vector field interaction and stiffer is the EoS. It is clear from equation (11) that, to keep thermodynamic consistence, the mass term cannot be zero, at least at mean field approximation. Also, in the vector MIT bag model, the stability window now depends on the value of the G_V . The higher the value of G_V , the lower the

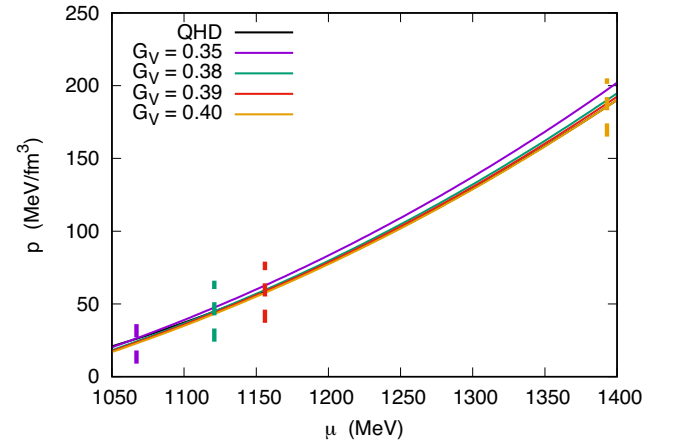


Figure 3. (Colour online) The pressure as a function of the chemical potential for different values of G_V . The dotted vertical lines indicate the position of the baryon chemical potential at the interface.

maximum value of B that satisfies the Bodmer–Witten conjecture. Indeed, as shown in Lopes et al. (2021b), for $G_V = 0.3 \text{ fm}^2$, the stability window lies within $139 < B^{1/4} < 150$ MeV. Here, we use $B^{1/4} = 158$ MeV, ensuring an unstable $u - d - s$ matter ($E/B > 930$ MeV).

We plot the pressure as function of the chemical potential for the vector MIT bag model within different values of G_V in Fig. 3. Unlike Fig. 2 for the original MIT, here the crossing of the hadron-quark curves are not so easily identifiable. This is because the slope of the curves in both phases are very similar. To help identify the baryon chemical potential at the interface, we plot dotted vertical lines in Fig. 3. The baryon chemical potential at the interface, the pressure and the energy density for each value of G_V are displayed in Table 4. As can be seen, the higher the value of G_V , the higher the μ_S . We also see that G_V must lie within $0.35 < G_V < 0.40 \text{ fm}^2$ so that the baryon chemical potential at the interface falls within $1050 < \mu_S < 1400$ MeV.

We plot in Fig. 4 the EoS and the corresponding TOV solutions for the hybrid stars within the vector MIT bag model. Compared with the original MIT bag model, we see that the EoS are much stiffer within the vector channel. We also see that the gap in the energy

Table 4. Chemical potential and pressure at phase transition for hadron (H) to quark (Q) for different values of G_V .

G_V (fm ²)	$\mu_S^H = \mu_S^Q$ (MeV)	$p^H = p^Q$ (MeV/fm ³)	ϵ_H	ϵ_Q
0.35	1063	25	303	364
0.38	1120	45	375	407
0.39	1156	58	415	440
0.40	1394	186	720	735

Note. We also show the energy density (in MeV/fm³) at both phases at the baryon chemical potential at the interface.

density is significantly smaller here than in the original one. Another point is the fact that varying G_V from 0.35 to 0.40 fm², although it causes a large effect on the value of the baryon chemical potential at the interface, it does not affect too much the stiffness of the EoS. All these facts are reflected in the mass–radius relation. For instance, the maximum mass of a hybrid star varies from 2.03 to 2.10 $M_\odot$, a difference around 3 per cent, but the $M_{\min}$, the minimum mass value for which the hybrid star is formed, varies from 0.98 to 1.99 $M_\odot$, a difference greater than 100 per cent. Indeed, for G_V up to values of 0.38 fm², the hadron–quark phase transition takes place below the hyperon threshold. The first hyperon onset happens around $n = 0.42$ fm^{−3}, and at a neutron chemical potential of $\mu_n = 1131$ MeV. A baryon chemical potential at the interface below 1131 MeV can, therefore, be faced as a possible solution to the so-called hyperon puzzle. As displayed in Fig. 4, if the hadron–quark phase transition happens at low density, the radius of the canonical 1.4 $M_\odot$ can be significantly lower than its pure hadronic counter-part. This fact is in agreement with the discussion presented in Lopes & Menezes (2018), about the neutron stars compactification due to the onset of new degrees of freedom. We can also see that all models discussed in this section not only fulfill the mass constraint of the PSR J0740+6620, $2.072^{+0.067}_{-0.062} M_\odot$, but also the curves of all neutron star families crosses the radius uncertainty (Riley et al. 2021). All the main results are summarized in Table 5.

Besides the constraint regarding the mass and the radius of the PSR J0740+6620, the *NICER* telescope has constrained the radius of the canonical 1.4- $M_\odot$ star. For instance, in Riley et al. (2019), the authors constrain the radius from 11.52 to 13.85 km, while in Miller et al. (2019), the radius is constrained from 11.96 to 14.24 km. As

Table 5. Hybrid star properties for different values of the G_V .

G_V (fm ²)	$M/M_\odot$	R (km)	ϵ_c	$M_{\min}/M_\odot$	$R_{1.4}$ (km)
0.35	2.03	11.15	1282	0.98	12.54
0.38	2.07	11.36	1235	1.31	12.91
0.39	2.08	11.44	1217	1.47	12.96
0.40	2.10	11.51	1195	1.99	12.96
QHD	2.15	11.20	1279	–	12.96

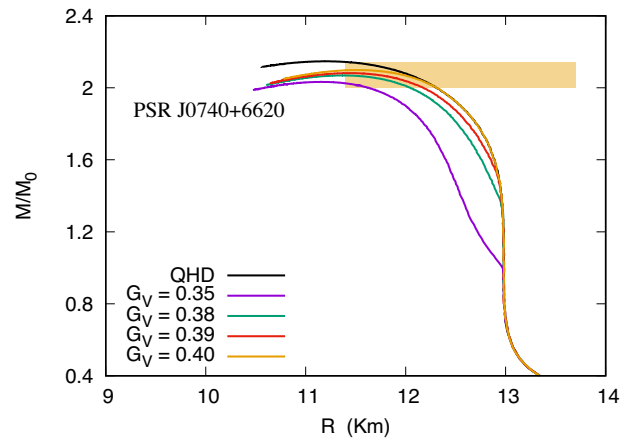
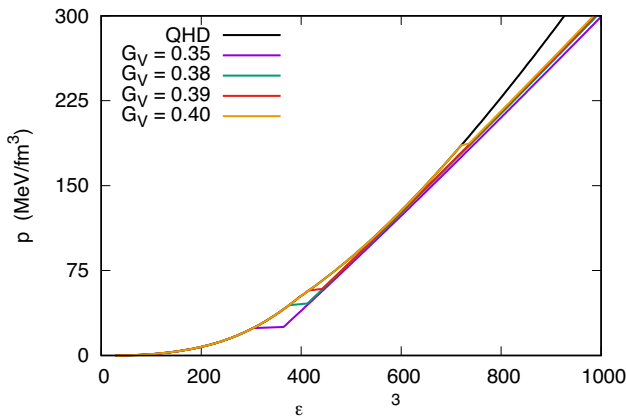
Table 6. Masses and radii of the quark core and their proportional contribution for the maximally massive star within different hybrid stars models.

G_V (fm ²)	0.35	0.38	0.39	0.40
$M_Q/M_\odot$	1.65	1.42	1.30	0.50
R_Q (km)	9.06	8.44	8.08	5.30
$M_Q/M_{\max}$ (per cent)	81	69	63	23
R_Q/R_{total} (per cent)	81	74	70	46

can be seen from the results shown in Table 5, they agree with the *NICER* results for both hadronic and hybrid stars.

Now we confront our results in the light of a very recent study about the nature of massive stars. In Annala et al. (2020), the authors suggest that massive neutron stars have sizable quark core. More than that, they argue that hybrid stars are not only possible, but also probable. They found that for a two solar masses star with a radius of 12 km, the quark core can reach 0.8 $M_\odot$ (40 per cent) and 7 km (58 per cent) of the total composition of the star.

We estimate the mass and size of the quark core in the most massive hybrid star of each model presented in Table 5. To accomplish that we solve the TOV equations for the quark EoS from the energy density corresponding to the critical pressure displayed in Table 4 up to the energy density at the maximum mass shown in Table 5. The results are presented in absolute and relative values in Table 6. As can be seen, within the vector MIT bag model, the quark core can be even more significant than within the pQCD used in Annala et al. (2020). Indeed, for $G_V = 0.35$ fm², we are able to produce hybrid stars with a hypermassive quark core of 1.65 $M_\odot$ and a radius larger than 9 km. In relative values, this corresponds to more than 80 per cent of both the total mass and radius of the hybrid star. It is also worthy to point out that such hybrid star with the hypermassive quark core still fulfills the constraints of the PSR J0740+6620 (Riley et al. 2021).

**Figure 4.** (Colour online) EoS (left-hand panel) and TOV solutions (right-hand panel) for different values of G_V . All the curves crosses the hatched area representing the uncertain about both, the radius and mass of the PSR J070+6620 (Riley et al. 2021).

We can also compare our results with other hybrid stars studies found in the literature. For instance, our results are close to those presented in Gomes et al. (2019b), although both the hadronic and the quark models are different. Here, we use an extended version of the QHD, with the parametrization taken from Lopes (2022), while Gomes et al. (2019b) use a many-body model with a derivative coupling. Both models predict the same maximum mass of $2.15 M_\odot$. The main macroscopic difference is the radius of the canonical star: 12.96 km here versus 14.44 km in Gomes et al. (2019b). Both studies use the vector MIT bag model, although in Gomes et al. (2019b) the mass term, important to keep the thermodynamic consistency, is missing. Within the set with the larger quark core, in Maxwell construction, we found a maximum mass of $2.03 M_\odot$ with an $M_{\min} = 0.98 M_\odot$, while Gomes et al. (2019b) found a maximum mass of $1.99 M_\odot$ with an $M_{\min} = 1.03 M_\odot$.

In Klähn & Fischer (2015), the authors use the so-called tdBAG, which is, ultimately, based on the NJL models for the quark phase, and the TM1 parametrization of the QHD for the hadron phase. They found that only with two flavour $u - d$ quark matter the phase transition is possible. And, unlike in this work, as well as in Gomes et al. (2019b), the hybrid star has a very similar radius with respect to the pure hadronic one.

3.3 Dirac sea contribution

The vector channel in the mean field approximation takes into account only the valence quarks. This scenario is called ‘no sea approximation’, once the Dirac sea of quarks is completely ignored (Furnstahl, Serot & Tang 1997). As the vector field is borrowed directly from QHD, the vector MIT bag model also becomes renormalizable (Serot 1992). However, instead of transforming the mean field approximation (MFA) into a more complex relativistic Hartree or Hartree–Fock approximation, we can take the Dirac sea into account throughout modifications on the effective Lagrangian density as done in Furnstahl et al. (1997). Here, we follow Lopes et al. (2021b), and introduce a quartic contribution for the vector field as a correction for the EoS at high density that will mimic the Dirac sea contribution. Therefore, we add the Lagrangian of equation (12) to the Lagrangians of equations (5) and (6):

$$\mathcal{L}_{\text{Dirac}} = b_4 \frac{(g^2 V_\mu V^\mu)^2}{4}. \quad (12)$$

where we define $g = g_{uV}$.

As pointed out in Lopes et al. (2021a,b) and Furnstahl et al. (1997), the quartic term introduced in equation (12) softens the EoS at high density, which allows us to use higher values of G_V without crossing the limit $\mu = 1400$ MeV. Indeed, in this section, we use $G_V = 0.8 \text{ fm}^2$ and keep $B^{1/4} = 158$ MeV, as in the previous one. The construction of an EoS with the term that mimics the Dirac sea contribution is discussed in Lopes et al. (2021b), Lopes et al. (2021a), and Furnstahl et al. (1997). In Fig. 5, we plot the pressure as a function of the chemical potential for three different values of b_4 . The vertical dotted lines are also plotted to help identify the baryon chemical potential at the interface. The results are also shown in Table 7. As can be seen, there is an already expected relation between the μ_S and the value of b_4 . The higher the b_4 value, the softer is the quark EoS and, therefore, the lower is the baryon chemical potential at the interface. Even with the high value $G_V = 0.8 \text{ fm}^2$, we can choose values of b_4 able to produce the baryon chemical potential at the interface in the range within $1050 < \mu_S < 1400$ MeV.

Now, in Fig. 6, we plot the EoS and the TOV solutions for different values of b_4 . The main results are also displayed in Tables 7 and 8.

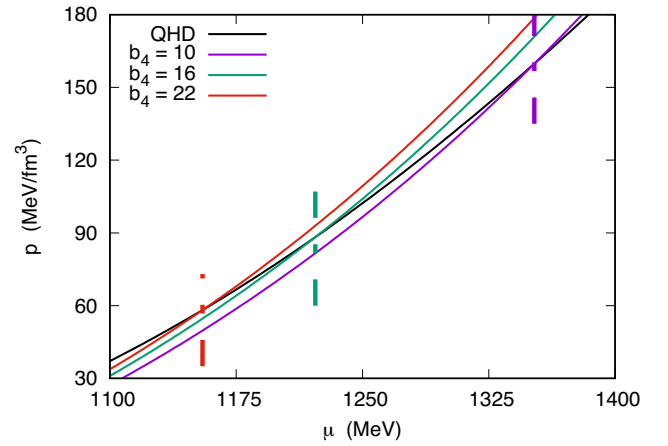


Figure 5. (Colour online) The pressure as a function of the chemical potential for different values of b_4 with $G_V = 0.8 \text{ fm}^2$. The dotted vertical lines indicate the position of the baryon chemical potential at the interface.

Table 7. Chemical potential and pressure at phase transition for hadron (H) to quark (Q) for different values of b_4 .

b_4	$\mu_S^H = \mu_S^Q$ (MeV)	$p^H = p^Q$ (MeV/fm ³)	ϵ_H	ϵ_Q
10	1352	159	667	780
16	1222	88	502	579
22	1154	58	415	493

Note. We also show the energy density (in MeV/fm³) at both phases at the baryon chemical potential at the interface.

As we can see, the Dirac sea contribution makes the gap in the energy density larger than in the previous section, without the quartic term. Also, we can see that the gap depends on b_4 . The higher the value of b_4 , the smaller the gap. With respect to the TOV solutions, we see that, despite the high value of G_V , with the quartic term in the quark phase, no hybrid star reaches the observed limit of the PSR J0740+6620 (Riley et al. 2021). Due to this drawback, we do not calculate the relative values of the quark core. It is, nevertheless, worth pointing out that, for all utilized values of b_4 , we are able to produce a dynamically stable hybrid branch. Also, unlike the vector case, here, the maximum mass strongly depends on the value of b_4 . The maximum mass can vary from 1.82 to $1.98 M_\odot$, a difference of about 9 per cent. Furthermore, within the Dirac sea approximation, no canonical star has a quark phase. As being purely hadronic, the radii of the canonical stars – 12.96 km – agree with the *NICER* results (Miller et al. 2019; Riley et al. 2019).

4 NO HYPERONS

One of the open issues in nuclear astrophysics is the content of the inner core of massive neutron stars. Because of the Pauli principle, as the number density increases, the Fermi energy of the nucleons exceeds the mass of the heaviest baryons, and the conversion of some nucleons into hyperons becomes energetically favorable. It is also well known that the hyperon onset softens the EoS. In some cases, the softening of the EoS pushes the maximum mass below the observational limit of massive neutron stars. This possible conflict between theory and observation is called hyperon puzzle. Among others, two extensive studies about the hyperon threshold (Dapo, Schaefer & Wambach 2010; Lopes & Menezes 2021) show that

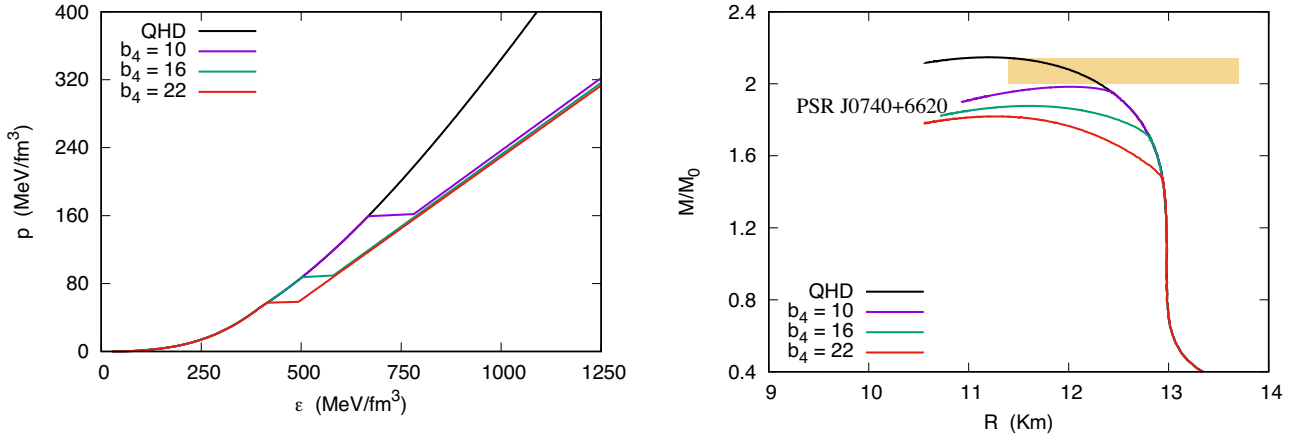


Figure 6. (Colour online) EoS (left-hand panel) and TOV solutions (right-hand panel) for different values of b_4 . No hybrid star reaches the mass of the PSR J0740+6620, not even with very a high value of $G_V = 0.8 \text{ fm}^2$.

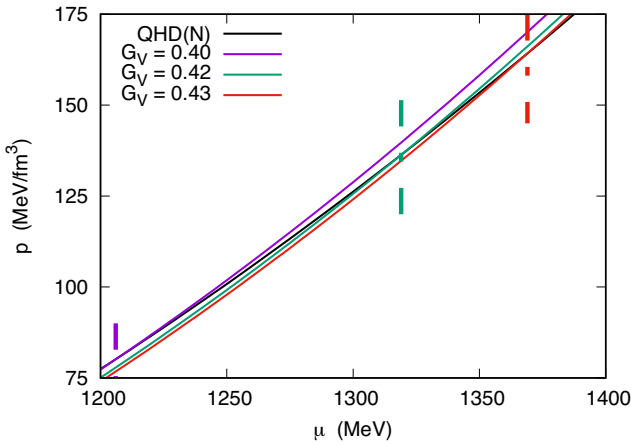


Figure 7. (Colour online) The pressure as a function of the chemical potential for different values of G_V without hyperons. The dotted vertical lines indicate the position of the baryon chemical potential at the interface.

hyperons are, ultimately, inevitable. This is the reason why, up to now, we have always considered the EoS with hyperons.

Nevertheless, as pointed out, the hyperon onset in stellar matter is still an open issue. Moreover, a recent study on non-relativistic models shows that the use of strongly repulsive three-body forces can suppress the hyperon threshold (Gerstung et al. 2020). Therefore, we dedicate this section to study hybrid stars without hyperons. But we have to keep in mind that, for the QHD model we use in this paper, the hyperon suppression is obtained simply by not taking them into account.

As the vector MIT bag model without the Dirac sea contribution predicts the hybrid stars with the highest maximum mass, as well as the biggest quark core, we only use this set in this section. Also, as discussed in previous sections, with $G_V < 0.39 \text{ fm}^2$, the hadron-quark phase transition happens before the hyperon onset. Therefore, it would not make any difference to use these values. So, we restrict our analysis next to values of $G_V > 0.39$. We plot in Fig. 7 the pressure as a function of the chemical potential. We also plot the dotted lines to help the identification of the baryon chemical potential at the interface. The results are also presented in Table 9. We can see that, without hyperons, we can use higher values of G_V while keeping $1050 < \mu_S < 1400 \text{ MeV}$.

Table 8. Hybrid star properties for different values of the b_4 with $G_V = 0.8 \text{ fm}^2$.

b_4	$M/M_\odot$	$R \text{ (km)}$	ϵ_c	$M_{\min}/M_\odot$	$R_{1.4} \text{ (km)}$
10	1.98	12.01	1035	1.95	12.96
16	1.88	11.60	1170	1.69	12.96
22	1.82	11.26	1278	1.46	12.96
QHD	2.15	11.20	1279	–	12.96

Table 9. Chemical potential and pressure at phase transition from hadron (H) to quark (Q) for different values of G_V without hyperons.

$G_V \text{ (fm}^2\text{)}$	$\mu_S^H = \mu_S^Q \text{ (MeV)}$	$p^H = p^Q \text{ (MeV/fm}^3\text{)}$	ϵ_H	ϵ_Q
0.40	1206	80	466	491
0.42	1319	136	572	614
0.43	1369	164	415	440

Note. We also show the energy density (in MeV/fm^3) at both phases at the baryon chemical potential at the interface.

In Fig. 8, we display the EoS and the mass-radius relation obtained with the TOV solutions. The results are also presented in Table 10. We see that the energy gap here is even smaller than in the hyperonic case. Also, the gap grows with the increase of G_V . We also see that all models are able to reproduce the PSR J0740+6620 constraint. As in the case of hyperonic matter, the maximum mass is not very sensitive to the G_V value. Here, the maximum mass grows $0.05 M_\odot$, only 2 per cent approximately. There are some coincidences here, when compared with the hyperonic case. For instance, the maximum mass with hyperons but no quarks is $2.15 M_\odot$, which is the same value of the maximum mass found with quarks but no hyperons. Even more impressive are the properties of the maximum mass with $G_V = 0.40 \text{ fm}^2$. With and without hyperons, this value is $2.10 M_\odot$. The radius of the maximum mass is also equal to 11.51 km in both cases. Even the central energy density differs only by 1 MeV/fm^3 . This indicates that, for the models and parametrizations used in this work, it is the quark EoS over the hadron EoS that is the more important term in hybrid stars, a result also obtained with the NJL model (Lopes & Menezes 2021). The main difference here is the minimum mass that produces the hybrid branch. With hyperons we have $M_{\min} = 1.99 M_\odot$, while without hyperons, we have $M_\odot = 1.67 M_\odot$. This is easily

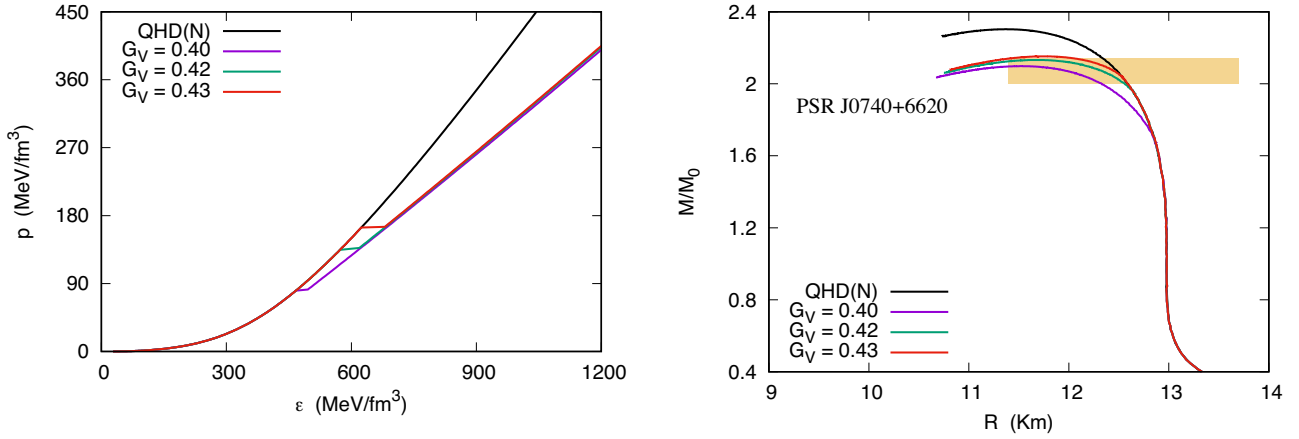


Figure 8. (Colour online) EoS (left-hand panel) and TOV solution (right-hand panel) for different values of G_V without hyperons. A maximum mass of $2.15 M_\odot$ is possible.

Table 10. Hybrid star properties for different values of the G_V without hyperons.

G_V (fm ²)	$M/M_\odot$	R (km)	ϵ_c	$M_{\min}/M_\odot$	$R_{1.4}$ (km)
0.40	2.10	11.51	1196	1.67	12.96
0.42	2.13	11.70	1130	1.95	12.96
0.43	2.15	11.75	1123	2.04	12.96
QHD(N)	2.30	11.36	1220	–	12.96

Table 11. Masses and radii of the quark core and their proportional contribution for the maximally massive star within different hybrid stars models without hyperons.

G_V (fm ²)	0.40	0.42	0.43
$M_Q/M_\odot$	1.10	0.72	0.55
R_Q (km)	7.48	6.21	5.60
$M_Q/M_{\max}$ (per cent)	52	33	25
R_Q/R_{total} (per cent)	65	53	48

explained by the differences in the baryon chemical potential at the interface. Nevertheless, the hadron EoS could play the major role as compared with the quark EoS if the phase transition would happen at very high densities (Lugones, Mariani & Sandoval 2021).

Now we estimate the size and mass of the quark core in hybrid stars without hyperons. The results are presented in Table 11. The quark core is still pretty significant, reaching over 50 per cent for $G_V = 0.40$ fm². This value is even larger than what is predicted by the pQCD presented in Annala et al. (2020). Increasing G_V makes the quark core smaller, reaching 25 per cent for $G_V = 0.43$ fm². Nevertheless, in all cases, we have a quark core with masses above $0.50 M_\odot$ and a radius above 5 km.

5 COMPARISON WITH THE NJL MODEL

For the sake of completeness, we construct also a hybrid star with another quark model, the so-called NJL model (Nambu & Jona-Lasinio 1961). Indeed, here we are able to make a double comparison. We not only compare the differences of the NJL model with the extended versions of the MIT model discussed in the previous section, but we also compare the differences between the hybrid stars

with the parametrization of Lopes (2022), and with the traditional GM1 (Glendenning 2000), as presented in Lopes & Menezes (2021).

The NJL and the MIT bag models and its extensions are effective models with complementary features. For instance, the MIT bag model is able to explain the confinement due to the bag, which introduces a vacuum pressure in the Lagrangian, while the NJL model lacks confinement. On the opposite side, the NJL model presents chiral symmetry, which is a symmetry predicted by the QCD, while it is exactly the Bag term that breaks the chiral symmetry in the MIT bag model. Another difference is the interaction. In the vector MIT, as in the QHD, the strong force is simulated by the exchange of massive mesons, therefore these models are renormalizable. In the NJL model, there is no mediator. The interaction is a direct quark-quark point-like scheme. This makes the NJL model a non-renormalizable model, and a cutoff is needed to obtain physical results. To make a direct comparison with Lopes & Menezes (2021), we use here an SU(3) version of the NJL, whose Lagrangian includes a scalar, pseudo-scalar, and the 't Hooft six-fermion interaction. Its Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\text{NJL}} = & \bar{\psi}_q [\gamma^\mu (i\partial_\mu - m_q) \psi_q] + \\ & G_S \sum_i^8 \left[(\bar{\psi}_q \lambda_a \psi_q)^2 + (\bar{\psi}_q \gamma_5 \lambda_a \psi_q)^2 \right] + \\ & -K \left\{ \det [\bar{\psi}_q (1 + \gamma_5) \psi_q] + \det [\bar{\psi}_q (1 - \gamma_5) \psi_q] \right\}, \end{aligned} \quad (13)$$

where ψ_q are the quark Dirac fields, with three flavours, $m_q = \text{diag}(m_u, m_d, m_s)$ are the current quark masses, λ_a are the eight Gell-Mann flavour matrices, and G_S and K are dimensionfull coupling constants. Here, as in Lopes & Menezes (2021), we use the so-called HK parametrization (Hatsuda & Kunihiro 1994). Another term that can be summed to the Lagrangian is a vector channel:

$$\mathcal{L}_{\text{NJLV}} = G_V (\bar{\psi}_q \gamma^\mu \psi_q)^2; \quad (14)$$

where G_V is treated as a free parameter. The detailed calculation of the EoS for the SU(3) NJL model with and without the vector term can be found in Lopes & Menezes (2021), Hatsuda & Kunihiro (1994), Buballa (2005), and the references therein.

We plot in Fig. 9 the pressure as a function of the chemical potential for the SU(3) NJL model with and without the vector term. Also, we plot the QHD model with and without hyperons. The results are also displayed in Table 12. To differentiate models with and without hyperons in the QHD and with and without the vector

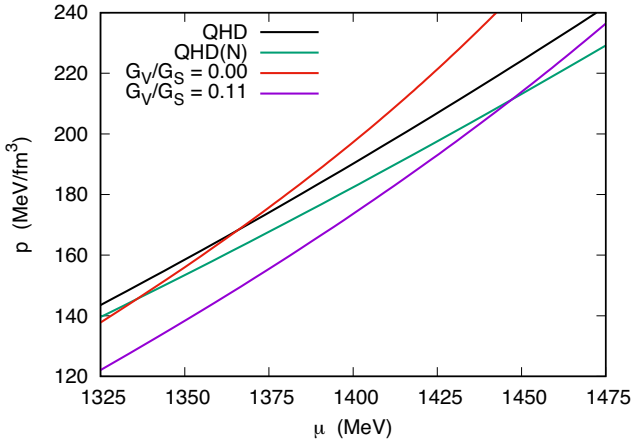


Figure 9. (Colour online) The pressure as a function of the chemical potential with and without G_V for the NJL model and with and without hyperons for the QHD model.

Table 12. Chemical potential and pressure at phase transition from hadron (H) to quark (Q) for the above discussed models of the QHD and the NJL.

	$\mu_S^H = \mu_S^Q$ (MeV)	$p^H = p^Q$ (MeV/fm ³)	ϵ_H	ϵ_Q
B1	1446	210	699	1029
B0	1332	143	588	817
A0	1364	167	682	921

Notes. We also show the energy density (in MeV/fm³) at both phases at the baryon chemical potential at the interface. Here A indicates QHD with hyperons and B indicates QHD without hyperons. The number 0 indicates $G_V/G_S = 0.00$ and the number 1 indicates $G_V/G_S = 0.11$.

term in the NJL model, we use the following scheme: A indicates QHD with hyperons and B indicates QHD without hyperons. The number 0 indicates $G_V/G_S = 0.00$ and the number 1 indicates $G_V/G_S = 0.11$.

As can be seen, for the hyperonic QHD model, only the NJL without the vector channel allows phase transition. Nevertheless, even for the pure nucleonic EoS, for the NJL with the vector channel, the baryon chemical potential at the interface is already above 1400 MeV. Also, we can notice, that for the HK parametrization, the baryon chemical potential at the interface is always very high, above 1300 MeV. This fact contrasts with the MIT bag models, where, in its original form, it produces a baryon chemical potential at the interface of only 968 for a Bag pressure value of $B^{1/4} = 165$ MeV. When we compare the parametrization of the QHD used in this work with the GM1 used in Lopes & Menezes (2021), we see that the μ_S for the GM1 is always smaller for the same value of $\alpha_v = 0.50$: 1285 (GM1) versus 1364 MeV (here) without the vector term and 1437 (GM1) versus 1660 MeV (here) for $G_V/G_S = 0.11$.

We display in Fig. 10 the EoS and the mass–radius relation via TOV solutions for different configurations of the QHD and the NJL model. We see that the gap in the energy density is significantly larger in the NJL than in the vector MIT bag model in both configurations: with and without the Dirac sea contribution. We also see that the parametrization from Table 1 predicts a significantly larger gap when compared with the GM1 model (Lopes & Menezes 2021).

When we look at the mass–radius relation and in Table 13, it seems that only the B1 configuration is able to fulfill the constraints of the PSR J070+6620, but it is not the case. When we compare Table 12

with Table 13, we see that only the B0 configuration produces a dynamically stable hybrid branch. In all other configurations, the maximum mass falls in the gap region. This is in contrast with the vector MIT bag model with and without the Dirac sea contribution, where always a dynamically stable hybrid branch is formed. This is also in contrast with the results within GM1 model, where most (although not all) parametrizations predict a dynamically stable branch (Lopes & Menezes 2021).

We estimate next the quark core size and mass for the B0 configuration, despite the fact that it does not fulfill the PSR J0740+6620 constraint, and display the results in Table 14. We can see that the results are very timid. The quark core mass does not reach 2 per cent, in total contrast with the results obtained with the extended versions of the MIT bag model where the quark core mass varies from 23 to 81 per cent. On the other hand, the results are just a little smaller than those presented in Lopes & Menezes (2021) for the GM1 parametrization, where the quark core mass represents 3.3 per cent (for $\alpha_v = 0.50$).

As we have done in Section 3, we now compare the results for the NJL with other results presented in the literature. In Ranea-Sandoval et al. (2016), the authors use the same GM1 for the hadron phase, but with different hyperon-meson coupling constants; and use the same HK parametrization for the NJL model with and without the vector term. However, instead of using the Maxwell construction as the criteria for hadron-quark phase transition, they use the so-called CSS parametrization, which produces a critical speed of sound, instead of a baryon chemical potential at the interface. They found that for vector channel varying from 0 within $< G_V/G_S < 0.30$ the phase transition is always possible. However, the quark core masses are even smaller than what we found in this work, their values varying within $10^{-5} < M_Q/M_\odot < 0.011$.

Another work that uses the GM1 model for the hadronic matter and the NJL model with and without vector coupling is presented in Lenzi & Lugones (2012) with similar results to those obtained in Lopes & Menezes (2021). Increasing the value of G_V/G_S increases the maximum mass but causes a reduction of the dynamically stable hybrid branch. Also, for $G_V/G_S > 0.15$, no dynamically stable hybrid branch exists, which again agrees with Lopes & Menezes (2021).

6 THE DIMENSIONLESS TIDAL PARAMETER AND THE SPEED OF SOUND

Before we finish our work, we analyse two important physical quantities: the first one is the dimensionless tidal parameter Λ , and compare it to the recent constraints presented in the literature coming from LIGO/VIRGO gravitational wave observatories and the GW170817 event (Abbott et al. 2018); and the second one is the square of the speed of sound, v_s^2 , and its relation with the size and mass of the quark core in hybrid stars as presented in Annala et al. (2020). As the models that produce higher maximum masses, as well as larger values of quark core masses and radii, are the models within the vector MIT bag model without the Dirac sea contribution, we restrict our analyses here to the parameters presented in Table 5.

If we put an extended body in an inhomogeneous external field it will experience different forces throughout its extent. This is called a tidal interaction. The tidal deformability of a compact object is a single parameter λ that quantifies how easily the object is deformed when subject to an external tidal field. A larger tidal deformability indicates that the object is easily deformable. On the opposite side, a compact object with a smaller tidal deformability parameter is more compact and it is more difficult to deform it.

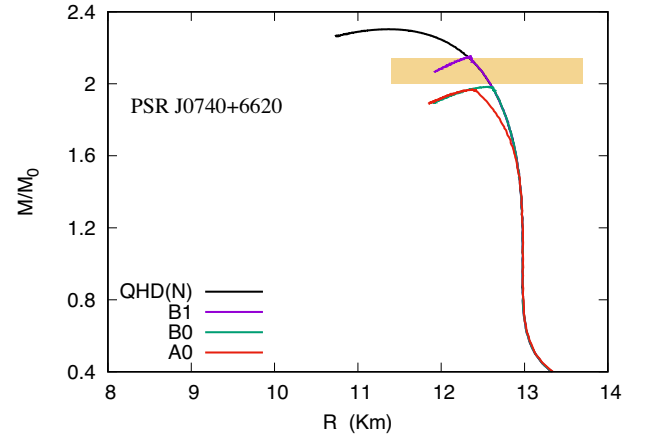
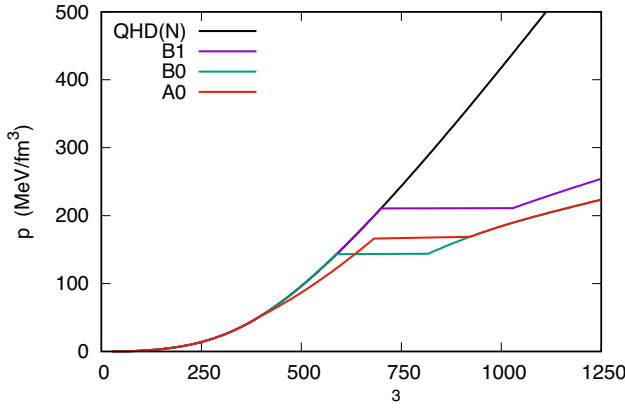


Figure 10. (Colour online) EoS (left-hand panel) and TOV solution (right-hand panel) for different values of G_V for the NJL model and with and without hyperons for the QHD model. Here A indicates QHD with hyperons and B indicates QHD without hyperons. The number 0 indicates $G_V/G_S = 0.00$ and the number 1 indicates $G_V/G_S = 0.11$.

Table 13. Hybrid stars properties for different values of the G_V/G_S , with and without hyperons.

–	$M/M_\odot$	R (km)	ϵ_c	$M_{\min}/M_\odot$	$R_{1.4}$ (km)
B1	2.15	12.35	947	12.15	12.96
B0	1.98	12.56	888	1.96	12.96
A0	1.96	12.37	767	2.04	12.96
QHD(N)	2.30	11.36	1220	–	12.96

Table 14. Masses and radii of the quark core and their proportional contribution for the maximally massive star.

–	B0
$M_Q/M_\odot$	0.031
R_Q (km)	2.13
$M_Q/M_{\max}$ (per cent)	1.55
R_Q/R_{total} (per cent)	17

Note. Within NJL, only the B0 configuration produces dynamically stable hybrid branches.

The tidal deformability is defined as the ratio between the induced quadrupole $Q_{i,j}$ and the tidal field $\mathcal{E}_{i,j}$ that causes the perturbation:

$$\lambda = -\frac{Q_{i,k}}{\mathcal{E}_{i,j}}. \quad (15)$$

However, in the literature, the dimensionless tidal deformability parameter (Λ) is more commonly found and it is defined as

$$\Lambda \doteq \frac{\lambda}{M^5} = \frac{2k_2}{3C^5}, \quad (16)$$

where M is the compact object mass and $C = GM/R$ is its compactness. The parameter k_2 is called Love number and is related to the metric perturbation. A complete discussion about the Love number and its calculation is both very extensive and well documented in the literature. Therefore, it is out of the scope of this work. We refer the interested reader to Abbott et al. (2018), Flores et al. (2020), Chatziioannou (2020), Hinderer (2008), and to references therein. We plot the tidal dimensionless parameter for three different values of G_V , as well as for the pure hadronic model with hyperons (QHD) in Fig. 11, and show its value for the canonical mass ($\Lambda_{1.4}$) in Table 15.

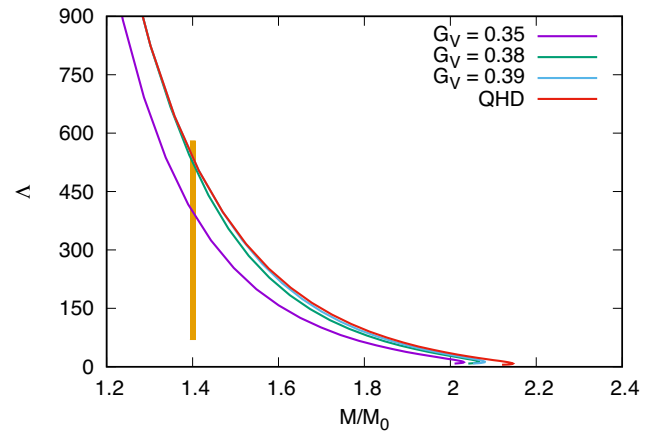


Figure 11. (Colour online) Tidal dimensionless parameter as a function of the star mass for different values of G_V as presented in Table 5. $G_V = 0.40 \text{ fm}^2$ is not shown as its curve is almost identical to the QHD model.

Table 15. Tidal dimensionless parameter for the canonical $1.4\text{-}M_\odot$ star and the speed of sound of quark matter at the baryon chemical potential at the interface.

G_V (fm ²)	0.35	0.38	0.39	0.40	QHD
$\Lambda_{1.4}$	395	520	534	534	534
v_s^2	0.40	0.42	0.42	0.45	–

In Abbott et al. (2018), the authors constraint the tidal parameter for the canonical mass in the range within $70 < \Lambda_{1.4} < 580$. As can be seen, all our models fulfill this constraint. This is a direct consequence of the chosen QHD parametrization. Also, for $G_V = 0.35 \text{ fm}^2$, the tidal parameter is significantly smaller, as it also has a smaller radius for the canonical star. However, this is not the main reason, as pointed out in Lopes et al. (2021b) and Hinderer (2008), the tidal parameter is, in fact, more dependent of the microscopic k_2 than of the macroscopic compactness. For $G_V = 0.39 \text{ fm}^2$ and above, the $\Lambda_{1.4}$ is already equal to a pure hadronic EoS, as the minimum mass that corresponds to a hybrid star with this parametrization is already above $1.40 M_\odot$.

Now we turn our attention to the speed of sound. As pointed out in Annala et al. (2020), the speed of sound of the quark matter is

closely related to the mass and radius of the quark core in hybrid stars. The authors found that, in their own words, ‘if the conformal bound ($v_s^2 < 1/3$) is not strongly violated, massive neutron stars are predicted to have sizable quark-matter cores’. Indeed, for speed of sound that satisfies the conformal bound, a quark core mass above $0.8 M_\odot$ arises (see fig. 6 from Annala et al. 2020). We then calculate the square of the speed of sound of the quark phase and display the corresponding value at the baryon chemical potential at the interface for different values of G_V in Table 15.

As can be seen, indeed, the higher the value of the speed of sound, the smaller the quark core in the hybrid star. However, unlike Annala et al. (2020), the results here are not so drastic. For instance, for all the values of G_V we have a speed of sound that violates the conformal limit. Moreover, while the differences in the speed of sound are about 10 per cent, the quark core mass differences reach over 100 per cent. Nevertheless, we agree with Annala et al. (2020) about the fact that a quark core in massive neutron star is not only possible, but probable.

Before finishing, it is worth pointing out that we have only analysed stars that satisfy the criterium $\partial M/\partial \epsilon_0 > 0$. This is needed in all EoS discussed in this work. However, had we assumed a slow phase transition, this criteria would neither be necessary nor sufficient. A complete discussion about this subject is far beyond the scope of this work, but we refer the interested reader to Pereira, Flores & Lugones (2018) and the references therein.

7 CONCLUSIONS

In this work, we studied hybrid stars, with the quark phase described by the extended versions of the MIT bag model via Maxwell construction. Using the QHD parametrization from Lopes (2022), we were able to construct hybrid stars with masses above $2.0 M_\odot$ with a quark core reaching more than $1.6 M_\odot$, while the dimensionless tidal parameter for the canonical mass still obeys $\Lambda_{1.4} < 580$. The main remarks are resumed as follows:

- (i) With the chosen QHD parametrization for hadronic matter and the original MIT bag model for quark matter, no dynamically stable hybrid star reaches the PSR J07040+6620 inferior mass limit observation (Riley et al. 2021).
- (ii) Within the vector MIT bag model, we were able to produce very massive hybrid stars, reaching $2.10 M_\odot$, whose quark core mass and radius can both account for more than 80 per cent of the total quantities of the hybrid star.
- (iii) The value of the vector channel G_V has little influence on the maximum mass of the hybrid star, but has strong influence on the size of the quark core.
- (iv) Within the parametrizations used in this work, the radius of the canonical star is 12.96 km for a pure hadronic star, and varies from 12.54 to 12.91 km in the hybrid case. All these values are in agreement with both *NICER* results (Miller et al. 2019; Riley et al. 2019).
- (v) For all models that are able to fulfill the mass constraint of the PSR J0740+6620, the radius constraint inferred by the *NICER* telescope was also satisfied.
- (vi) When we took a term that mimics the Dirac quark sea contribution into account, the maximum mass became below the inferior limit of the PSR J07040 + 6620. Unlike the previous case, the b_4 parameter influences both, the size of the core and the maximum mass.
- (vii) Despite the fact that hyperons seem inevitable, as pointed out in Lopes & Menezes (2021) and Āapo et al. (2010), we studied

hybrid stars with no hyperon content and were able to produce even more massive hybrid stars, reaching $2.15 M_\odot$.

(viii) For $G_V = 0.40 \text{ fm}^2$, the maximum mass and the corresponding radius of the hybrid star are independent of the presence or absence of hyperons. The main differences are the baryon chemical potential at the interface and the minimum hybrid star mass, $M_{\min}$. This indicates that within the models chosen in this work is the quark EoS over the hadronic one, which determines the maximum mass of a hybrid star. Similar results have already been discussed for the NJL model (Lopes & Menezes 2021).

(ix) Compared with the NJL model, we saw that the vector MIT bag model produces more massive hybrid stars and larger quark cores. Moreover, within the chosen QHD parametrization combined with the NJL model, only the configuration without hyperons can produce a dynamically stable hybrid branch.

(x) For low values of the baryon chemical potential at the interface, the radius of the canonical star and the corresponding tidal parameter are significantly reduced.

(xi) We did not find a drastic dependence of the quark core on the speed of sound as suggested in Annala et al. (2020). Indeed, we were able to produce hypermassive quark cores still violating the so-called conformal limit ($1/3$). Nevertheless, we agree with the authors about the nature of massive compact objects being hybrid stars.

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DATA AVAILABILITY

All data that support the findings of this study are included within this paper.

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