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**FaciesGAN: A Conditional GAN Framework for Realistic Facies Scenario Generation as
an Efficient Alternative to Multiple-Point Statistics**

Florianópolis
2026

Yineth Viviana Camacho De Angulo

**FaciesGAN: A Conditional GAN Framework for Realistic Facies Scenario
Generation as an Efficient Alternative to Multiple-Point Statistics**

Master's Dissertation submitted to the Post-graduate Program in Computer Science at the Federal University of Santa Catarina, in fulfillment of the requirements for the degree of Master in Computer Science.

Orientador: Mauro Roisenberg, PhD.

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RESUMO

Fácies são conjuntos de rochas que mostram ambientes sedimentares particulares e possuem um papel essencial na caracterização de reservatórios. A caracterização precisa das fácies — incluindo as propriedades físicas, químicas e biológicas das rochas do reservatório — é essencial para entender os ambientes sedimentares e cruciais para a modelagem do reservatório. Um desafio fundamental na caracterização de reservatórios continua sendo a simulação de realizações de fácies que sejam geologicamente plausíveis e compatíveis com os dados dos poços. Este trabalho apresenta o FaciesGAN, uma nova arquitetura de aprendizado profundo baseada em Redes Generativas Adversariais Condicionais (cGANs). O método utiliza uma estrutura hierárquica de geradores e discriminadores que aprimoram gradualmente as estimativas iniciais em modelos de fácies de alta resolução, assegurando a conformidade com os dados dos poços e os padrões de deposição em cada etapa. O FaciesGAN foi validado com o conjunto de dados limitado da Stanford Earth Science Data, apresentando um desempenho robusto mesmo diante da limitação de dados. A avaliação quantitativa foi realizada utilizando duas métricas principais: o Escalonamento Multidimensional (MDS), que foi aplicado para verificar a preservação das distribuições estatísticas, e o índice de Interseção sobre União (IoU), usado para avaliar a consistência espacial em relação aos dados condicionais de poço. Com base em essas métricas, o modelo atingiu um IoU de 99,96%, quase perfeitamente alinhado com as informações condicionais. Esses resultados comprovaram que o modelo é capaz de criar cenários com alta fidelidade, mantendo as distribuições estatísticas. O FaciesGAN gerou realizações geológicas mais realistas e diversificadas em comparação com uma implementação convencional da Estatística de Múltiplos Pontos (MPS), apresentando uma eficiência computacional significativamente superior. Esses resultados sugerem que métodos baseados em cGAN, como o FaciesGAN, constituem uma direção promissora para modelos de sub-superfície, fornecendo ferramentas robustas para aumento de dados, avaliação melhorada da incerteza e caracterização aprimorada do reservatório.

Palavras-chave: Rede Adversária Generativa Condicional; Fácies; Poços condicionantes; Simulação geoestatística; Inversão sísmica.

ABSTRACT

Facies are groups of rocks that represent particular sedimentary environments and have an essential role in reservoir characterization. Accurate characterization of facies—including the physical, chemical, and biological properties of reservoir rocks—is essential for understanding sedimentary environments and crucial for reservoir modeling. A key challenge in reservoir characterization remains the simulation of facies realizations that are geologically plausible and compatible with well data. This work presents FaciesGAN, a new deep learning architecture based on Conditional Generative Adversarial Networks (cGANs). The method uses a hierarchical structure of generators and discriminators that gradually refine initial estimates into high-resolution facies models, ensuring compliance with well data and deposition patterns at each stage. FaciesGAN was validated with the limited Stanford Earth Science Data dataset, showing robust performance even in the face of data limitations. Quantitative evaluation was performed using two main metrics: Multidimensional Scaling (MDS), which was applied to verify the preservation of statistical distributions, and the Intersection over Union (IoU) index, used to assess spatial consistency in relation to conditional well data. Based on these metrics, the model achieved an IoU of 99.96%, almost perfectly aligned with the conditional information. These results proved that the model is capable of creating high-fidelity scenarios while maintaining statistical distributions. FaciesGAN generated more realistic and diverse geological realizations compared to a conventional Multiple Point Statistics (MPS) implementation, with significantly higher computational efficiency. These results suggest that cGAN-based methods, such as FaciesGAN, are a promising direction for subsurface modeling, providing robust tools for data augmentation, improved uncertainty assessment, and enhanced reservoir characterization.

Keywords: Conditional generative adversarial network; Facies; Conditioning wells; Geostatistical simulations; Seismic inversion.

RESUMO ESTENDIDO

INTRODUÇÃO

Na caracterização de reservatórios, as fácies são definidas como unidades rochosas com atributos específicos que refletem o ambiente deposicional e controlam diretamente as propriedades petrofísicas e a heterogeneidade do reservatório (GRANA; MUKERJI; DOYEN, 2021; SHERIFF; GELDART, 1995). As fácies são fundamentais para a compreensão dos ambientes deposicionais, pois permitem a correlação com dados sísmicos e de poços, desempenhando um papel central nos processos de inversão sísmica (LEE; MUKERJI, 2012; SANGREE; WIDMIER, 1979). Diferenças entre fácies, como maior porosidade em arenitos em relação a folhelhos, auxiliam na identificação de reservatórios e na distinção entre zonas produtivas e não produtivas (SELLEY, 1998). Assim, a incorporação de fácies em algoritmos de inversão sísmica aumenta a consistência entre os modelos e as propriedades petrofísicas estimadas, resultando em modelos mais robustos e confiáveis (GRANA et al., 2012; KEMPER; GUNNING, 2014; VEEKEN, 2007).

A geração de múltiplos cenários de fácies capazes de reproduzir estruturas heterogêneas complexas é essencial para a caracterização e modelagem de reservatórios geológicos e para fluxos de trabalho de inversão sísmica estocástica (PYRCZ; DEUTSCH, 2014; SLATT, 2013). A produção de múltiplas realizações plausíveis do subsuperfície permite capturar e quantificar explicitamente a incerteza geológica, reduzindo vieses associados a interpretações determinísticas únicas e garantindo consistência com o conhecimento geológico e os dados observados (PETROVIC; AIGNER; PONTIGGIA, 2018; GRANA; MUKERJI; DOYEN, 2021). Essa capacidade é particularmente relevante em fluxos de inversão sísmica, nos quais dados de poços, análises de testemunhos e respostas sísmicas devem ser integrados de forma coerente (ROKSANDIĆ, 1978; STREBELLE, 2002).

A geração de cenários de fácies pode ser realizada por metodologias clássicas e modernas que combinam geologia, estatística e inteligência artificial (DEMBICKI, JR., 2017; SLATT, 2013). Entre os métodos clássicos, destaca-se a simulação sequencial por indicadores (SIS), que utiliza indicadores binários para cada fácies e gera cenários condicionados aos dados disponíveis (JOURNAL; JOURNAL, 1989). Apesar de permitir a quantificação da incerteza, o SIS apresenta limitações importantes, como padrões fracamente conectados, dificuldade em representar canais ou falhas, alta sensibilidade ao ajuste do variograma e possíveis inconsistências geológicas quando mal calibrado (AL-MUDHAFAR, 2021; JOURNAL, 1983).

As metodologias modernas incluem técnicas capazes de representar padrões complexos e relações espaciais avançadas. As estatísticas de múltiplos pontos (MPS) representam um avanço significativo, permitindo capturar padrões espaciais a partir de imagens de treinamento e modelar relações entre múltiplas localizações (MARIETHOZ; CAERS, 2014; STREBELLE; JOURNAL, 2001). Essas técnicas são especialmente úteis em áreas com dados limitados, mas dependem fortemente da qualidade das imagens de treinamento e podem apresentar dificul-

dades na incorporação de dados condicionantes sem comprometer a continuidade dos padrões simulados (HARDING et al., 2004; STREBELLE, 2002; ZHANG et al., 2019).

Nesse contexto, as Redes Adversariais Generativas (GANs) surgem como uma abordagem inovadora para a geração de cenários de fácies. As GANs apresentam vantagens em relação aos métodos geoestatísticos tradicionais e às MPS, pois aprendem diretamente a partir dos dados reais, preservando estatísticas de primeira ordem (proporções de fácies) e de segunda ordem (continuidade espacial e geometria dos corpos) (TIAN et al., 2021; FENG; GRANA; MOSEGAARD, 2025). Elas são capazes de capturar padrões espaciais complexos e reduzir a subjetividade no desenho dos modelos, além de atuarem como complemento à simulação estocástica de fácies (BOWLES et al., 2018; GOODFELLOW et al., 2014; MOSSER et al., 2018; SHAHAM; DEKEL; MICHAELI, 2019).

As GANs são compostas por duas redes concorrentes: um gerador, que cria dados sintéticos, e um discriminador, que avalia sua autenticidade em relação aos dados reais (GOODFELLOW et al., 2014; ZHANG, 2019). No contexto de fácies, essas redes podem gerar registros sintéticos que preservam as características estatísticas dos dados reais. As GANs condicionais (cGANs) ampliam essa capacidade ao introduzir informações condicionantes, como dados de poços ou fácies em profundidades específicas, permitindo maior controle sobre o processo de geração e a avaliação de cenários geológicos complexos (CRESWELL et al., 2018; DOUZAS; BACAO, 2018).

Nos últimos anos, diversos estudos demonstraram a eficácia de tecnologias baseadas em GANs para a geração de cenários de fácies (LIU et al., 2022; MIELE; AZEVEDO; GRANA, 2024; FENG; GRANA; MOSEGAARD, 2025). Apesar dos bons resultados, poucos trabalhos incorporam informações duras conhecidas durante a fase de treinamento, como sequências de fácies observadas em poços. Até onde se tem conhecimento, não existem aplicações publicadas voltadas à geração de dados de fácies em seções verticais 2D.

Diante desse cenário, este trabalho tem como objetivo explorar técnicas de IA para a geração de cenários de fácies, com foco em redes cGANs aplicadas a dados de seções verticais 2D condicionadas por dados de poços. Utilizando um conjunto de dados público e limitado, busca-se demonstrar que o método proposto é capaz de lidar com a escassez de dados, integrando informações de poços, distribuições de fácies e dados sísmicos. Dessa forma, pretende-se mostrar que as cGANs podem gerar modelos mais precisos, consistentes e robustos para a caracterização de reservatórios, superando limitações das técnicas tradicionais.

OBJETIVO GERAL

Explorar as Redes Adversárias Generativas Condicionais (cGANs) para a geração de cenários sintéticos de litofacies condicionados a dados de poços.

METODOLOGIA

A metodologia proposta baseia-se na utilização de redes adversárias generativas condicionais (cGANs) para gerar cenários de fácies em seções verticais 2D, respeitando restrições geológicas e dados de poço. O fluxo de trabalho inicia-se com

- **Coleta e Processamento dos Dados:** A seleção do Stanford VI-E dataset, que contém registros de poços e modelos de fácies em sistemas fluviais meandrantess. As fácies originais foram reclassificadas em duas categorias principais — reservatório (canal) e não-reservatório (planície de inundação, barra de ponta e limite) — simplificando a modelagem. A partir desse conjunto, foram extraídas e rotuladas 200 fatias 2D de 256×256 pixels, que funcionaram como base de treinamento e permitiram a conversão de dados tabulares em imagens anotadas com informações de profundidade, utilizadas como condição externa no treinamento.
- **Seleção e implementação da Técnica de Aumento de Dados:** O modelo desenvolvido, denominado FaciesGAN, combina elementos de SinGAN(SHAHAM; DEKEL; MICHAELI, 2019) e WGAN-GP(GULRAJANI et al., 2017) em uma arquitetura multiescala composta por geradores e discriminadores hierárquicos. O processo de geração inicia-se em baixa resolução, condicionado às informações de poço, e é progressivamente refinado em escalas mais altas, permitindo capturar tanto a estrutura global quanto heterogeneidades locais. Cada gerador aprende padrões em diferentes resoluções, enquanto os discriminadores avaliam a plausibilidade em nível de patches de imagem, reforçando consistência espacial. O treinamento alterna entre atualizações do gerador e do discriminador, incorporando penalidade de gradiente, perda de reconstrução e perda mascarada, garantindo estabilidade e fidelidade aos dados condicionantes.
- **Avaliação dos Resultados:** Os resultados do FaciesGAN foram avaliados utilizando uma série de métodos complementares. Inicialmente, aplicou-se Multidimensional Scaling (MDS) para comparar quantitativamente as imagens geradas com as imagens de treinamento. Esse método permitiu analisar padrões de variabilidade espacial e a similaridade entre as fácies geradas e as reais. Para validar o respeito às condições de poços, as imagens geradas foram comparadas com as imagens originais utilizando o índice Intersection Over Union (IoU), que calcula a razão entre a área de interseção e a área de união entre a máscara gerada pelo modelo e a máscara de referência, permitindo avaliar a fidelidade na reprodução da localização e forma das fácies ao redor dos poços. Além disso, foi realizada uma comparação com métodos tradicionais de simulação geológica, utilizando a abordagem de Multiple-Point Statistics (MPS) por meio do algoritmo SNESIM. Essa comparação permitiu analisar a diversidade estrutural, a preservação da continuidade espacial e a fidelidade às condições de poços entre os métodos. Para complementar a análise, avaliou-se a distribuição de classes nas imagens geradas por meio de histogra-

mas de proporção de facies, comparando os resultados do FaciesGAN e do SNESIM com os dados originais. Por fim, considerou-se a eficiência computacional como um critério de avaliação.

RESULTADOS E DISCUSSÃO

Os resultados mostram que o FaciesGAN foi capaz de gerar cenários de fácies realistas e consistentes a partir de um conjunto de treinamento reduzido (200 imagens do Stanford dataset). O modelo foi treinado por 100 épocas em 10 escalas, utilizando WGAN-GP com penalidade de gradiente $\lambda_{gp} = 0.1$, otimizador Adam (taxa de aprendizado $5e^{-5}$, $\beta_1 = 0.5$, filtros 2D de 3×3 e stride de 1×1). No primeiro experimento (avaliação global), o modelo gerou 1000 cenários em apenas 20 segundos, preservando com alta fidelidade a informação de poço imposta (hard conditioning), com IoU médio de 99,96%. A análise por inspeção visual, histogramas de proporção de fácies e MDS confirmou que as distribuições geradas mantêm continuidade espacial, diversidade estrutural e proximidade estatística com os dados de referência, superando o método geoestatístico SNESIM, que exigiu 137 minutos para gerar o mesmo número de cenários e apresentou maior dispersão e desvio em relação às proporções reais.

No segundo experimento (condicionamento específico a poços), foram selecionadas cinco seções 2D verticais condicionadas, para as quais o modelo gerou 100 realizações por poço em cerca de 6 segundos por inferência. As distribuições de fácies nos cenários gerados apresentaram desvios inferiores a 4% em relação às imagens reais, mantendo consistência estatística e estrutural. Visualmente, as realizações mostraram boa variabilidade estocástica, mas sempre respeitando os padrões espaciais observados nos dados originais. Os valores médios de IoU entre 99,37% e 99,66% confirmam a preservação da informação de condicionamento.

CONSIDERAÇÕES FINAIS

O FaciesGAN foi treinado com sucesso com um pequeno número de imagens de facies verticais 2D, o que é vantajoso para cenários com dados limitados e notável por sua capacidade de realizar aumento de dados com um pequeno número de imagens. A principal vantagem do modelo é a capacidade de gerar cenários de fácies 2D sintéticos respeitando informações condicionais conhecidas, isso garante a consistência com as condições reais que representam informações de um poço real perfurado e analisado. O MDS e as estatísticas de proporção de fácies destacaram a capacidade da rede de reproduzir os dados condicionais de forma consistente. Os resultados preservam as distribuições de frequência e as correlações espaciais características das imagens originais, isso garante que as imagens geradas sejam visualmente consistentes e mantenham as propriedades estatísticas e geológicas. Adicionalmente, a comparação com o SNESIM demonstrou que o FaciesGAN fornece uma representação mais precisa da proporção de fácies, reduz a dispersão e o desvio em relação aos dados de referência, mantendo uma maior fidelidade espacial, estatística e maior eficiência computacional, permitindo a geração rápida de

múltiplos cenários em comparação com métodos de simulação sequencial. Essas previsões podem ser integradas aos fluxos de trabalho de caracterização de reservatórios, auxiliando na estimativa de incertezas e melhorando a qualidade dos resultados. Por fim, a metodologia proposta pode ser expandida para incluir etapas subsequentes do fluxo de trabalho, como a incorporação de fácies com propriedades acústicas e/ou elásticas, a geração de dados sísmicos sintéticos e a avaliação de sua consistência com observações sísmicas reais, o que estabelece uma ligação direta com o processo de inversão sísmica.

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LIST OF ABBREVIATIONS AND ACRONYMS

ANN	Artificial Neural Network
cGAN	Conditional Generative Adversarial Network
CPD	Conditional Probability Distribution
DCGAN	Deep Convolutional Generative Adversarial Network
DNN	Deep Neural Network
DL	Deep Learning
GAN	Generative Adversarial Network
IK	Indicator Kriging
IoU	Intersection Over Union
ML	Machine Learning
MSE	Mean Squared Error
MDS	Multidimensional Scaling
MPS	Multiple-Point Statistics
SIS	Sequential Indicator Simulation
SK	Simple Kriging
SNESIM	Single Normal Equation Simulation
TI	Training Image
UK	Universal Kriging

LIST OF SYMBOLS

$\mathcal{L}_{adv}$ or L_{adv}	Adversarial loss
z_N	Coarsest-scale noise
$D(x, y)$	Conditional discriminator output
$G(z, y)$	Conditional generator output
y	Conditional variable
ψ_n	Convolutional block
D_n	Discriminator at scale n
$\mathcal{L}_D^{(n)}$ or L_D	Discriminator loss
D	Discriminator network
$D(G(z))$	Discriminator output (generated sample)
$D(x)$	Discriminator output (real sample)
θ_d	Discriminator parameters
$\{D, \dots, D_N\}$	Discriminator pyramid
k_d	Discriminator steps
r	Downscaling factor
$\ \cdot\ _2$	Euclidean (L2) norm
$\mathbb{E}_{x \sim p_{\text{data}}}[\cdot]$	Expectation over real data
$\mathbb{E}_{z \sim p_z}[\cdot]$	Expectation over latent space
L_{fake}	Fake loss
z_n^*	Fixed noise map
z^*	Fixed reconstruction noise
x_n	Generated image at scale n
x_{n-1}	Generated image from the previous (coarser) scale
G_n	Generator at scale n
G	Generator network
$\{G, \dots, G_N\}$	Generator pyramid
k_g	Generator steps
L_G or $\mathcal{L}_G^{(n)}$	Generator loss
$\mathcal{L}_{GP}$	Gradient penalty loss
λ_{gp}	Gradient penalty weight
θ_g	Generator parameters
$\odot$	Hadamard product
$\hat{x}_n$	Interpolated sample
ε	Interpolation factor
$p_{data}(x, y)$	Joint data distribution
$p_z(z)$	Latent (noise) distribution
z	Latent noise vector

M	Mask
L_{mask}	Masked loss
GETNOISE()	Noise generation function
z_n	Noise map at scale n
Z	Noise pyramid
σ_n	Noise standard deviation
z_n^*	Fixed noise map
$p_{data}(x)$	Real data distribution
x	Real data sample
x_{fake}	Fake image
x_{real}	Real image
x_n	Real image at scale n
L_{real}	Real loss
$\tilde{x}_n^{rec}$ or x_{rec}	Reconstructed image
x_{rec_in}	Reconstruction input
Z_{rec}	Reconstruction noise
α_{rec}	Reconstruction weight
$\mathcal{L}_{rec}$ or L_{rec}	Reconstruction loss
α_{rec} or α	Reconstruction weight
n	Scale index
$\mathcal{L}(G_n, D_n)$	Total loss at scale n
N	Total number of scales
$\mathcal{U}[0, 1]$	Uniform distribution
$\tilde{x}_{n+1}^\uparrow$	Upsampled image from the next coarser scale
$\uparrow^r$ or $U(\cdot)$	Upsampling operator
$V(D, G)$	Value function
α	Weighting factor

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1 INTRODUCTION

In reservoir characterization, facies are defined as rock units with specific attributes that reflect the depositional environment and directly control the petrophysical properties and heterogeneity of the reservoir (GRANA; MUKERJI; DOYEN, 2021; SHERIFF; GELDART, 1995). Facies are essential for understanding depositional environments, as they allow geoscientists to correlate these units with seismic and well data, thus playing a crucial role in the seismic inversion process (LEE; MUKERJI, 2012; SANGREE; WIDMIER, 1979). For example, as sandstone facies show higher porosity than shale facies, this can facilitate the reservoir identification and differentiation between productive and non-productive zones (SELLEY, 1998). From this perspective, the integration of facies into seismic inversion algorithms provides greater consistency between the models and the petrophysical properties obtained, generating more robust and reliable models (GRANA et al., 2012; KEMPER; GUNNING, 2014; VEEKEN, 2007).

The generation of multiple facies scenarios that reproduce complex heterogeneous structures plays a key role in the characterization and modeling of geological reservoirs and the stochastic seismic inversion workflow (PYRCZ; DEUTSCH, 2014; SLATT, 2013). By enabling the generation of multiple plausible realizations of the subsurface, facies scenario modeling provides a rigorous framework to explicitly capture and quantify geological uncertainty (PETROVIC; AIGNER; PONTIGGIA, 2018). This approach reduces the risk of bias introduced by single-deterministic interpretations, and ensures that the resulting reservoir models remain consistent with both geological knowledge and observed field data (GRANA; MUKERJI; DOYEN, 2021; SLATT, 2013). This ability is particularly critical in seismic inversion workflows, where the relationship between well log measurements, core analysis, and seismic responses must be established in a consistent and geologically meaningful way (ROKSANDIĆ, 1978; STREBELLE, 2002).

Facies scenario generation can be carried out using traditional and modern methodologies that combine geology, statistics, and artificial intelligence (DEMBICKI, JR., 2017; SLATT, 2013). Classical methodologies for generating facies scenarios include techniques used in geology and geostatistics. For instance, Sequential indicator simulation (SIS) uses binary indicators for each facies, generating scenarios conditioned on available data (JOURNEL; JOURNEL, 1989). SIS is useful for modeling facies, but has several limitations. It generates loosely connected patterns and simple geological structures (AL-MUDHAFAR, 2021). This makes it difficult to represent channels or faults. SIS is sensitive to variogram fitting, which complicates its use with sparse data. It allows quantification of uncertainty; however, if not accurately calibrated, it can result in geologically inconsistent models (JOURNEL, 1983).

Modern methodologies include techniques that have revolutionized geological modeling by allowing the representation of complex patterns and advanced spatial relationships. Multiple point statistics (MPS) represents a significant advance in this area. MPS allows capturing spatial patterns in geological data while using patterns to model multi-location relationships (MARIETHOZ; CAERS, 2014; STREBELLE; JOURNEL, 2001). These techniques are especially

useful for simulating facies distributions in regions with limited information. They respect spatial distributions observed in training data, such as geological maps or previous simulations (HARDING et al., 2004; STREBELLE, 2002). MPS may have difficulties in constructing representative training images, as it relies on the analyst's expertise. Furthermore, conditioning to real data may be complex to implement without breaking the continuity of the simulated patterns (STREBELLE, 2002; ZHANG et al., 2019).

In this context, Generative Adversarial Networks (GANs) emerge as an innovative methodology for the generation of facies scenarios. GANs offer significant advantages over traditional geostatistical methods and MPS-based simulation. GANs can learn directly from real data, preserving first-order statistical features (facies proportions) and second-order statistical features (spatial continuity and body geometry) (TIAN et al., 2021; FENG; GRANA; MOSEGAARD, 2025). GANs are capable of capturing complex spatial patterns and facies relationships, thereby producing more realistic realizations and reducing the subjectivity inherent in model design (BOWLES et al., 2018; GOODFELLOW et al., 2014; PATEL; WANG; MAO, 2020). Moreover, they open the possibility of training networks as a complement to stochastic facies simulation (MOSSER et al., 2018; SHAHAM; DEKEL; MICHAELI, 2019).

GANs consist of two competing networks: A generator network creates synthetic data, and a discriminator network assesses the authenticity of the generated data in relation to the real data (GOODFELLOW et al., 2014; ZHANG, 2019). In the context of facies, GANs can be used to generate new synthetic records that preserve the statistical characteristics of real data, e.g., facies distributions or the geophysical properties of wells. On the other hand, conditional Generative Adversarial Networks (cGANs) provide a conditional layer to the data generation process. This conditional layer allows the creation of synthetic data based on specific previous information, such as the type of facies in a particular depth range, providing even more control over the generation process (CRESWELL et al., 2018; DOUZAS; BACAO, 2018). This characteristic allows the assessment of large geological scenarios and the validation of hypotheses about reservoir connectivity and quality.

In recent years, the generation of facies scenarios has been studied by several case studies, showcasing advanced technologies' effectiveness. (LIU et al., 2022) proposed an approach for generating 3-dimensional subsurface facies map models based on GAN. The study developed by (MIELE; AZEVEDO; GRANA, 2024) proposed to integrate a GAN with SPADE to predict realistic facies map patterns while respecting local probabilities. It combines with geostatistical methods of sequential simulation to model facies-conditioned rock properties. Furthermore, (FENG; GRANA; MOSEGAARD, 2025) presented a method based on a GAN that proposed an implementation where the GAN is trained using a training facies map image. Research shows good results using facies map data and statistical similarity. However, few studies have incorporated known hard information from the GAN training stage, such as observed facies sequences in wells at specific locations. Within the scope of this study and to the best of our knowledge, no published applications addressing facies data in 2D vertical sections.

Considering the current progress, this work aims to explore advanced techniques for gener-

ating facies scenarios, with a particular focus on cGANs networks. The objective is to evaluate the effectiveness of this technique on 2D vertical section facies data conditioned on well data. Using a public dataset with a limited number of samples, it aims to demonstrate that the proposed method can effectively address one of the main challenges in reservoir characterization: data scarcity. This approach leverages synthetic data and AI training models to improve the integration of well logs, facies distributions, and seismic information, producing scenarios that respect geological conditions and preserve statistical and spatial consistency. Therefore, it is intended to demonstrate how cGANs, by including this information, are able to generate more accurate and robust models for reservoir characterization respecting this additional condition. The cGANs offer an innovative solution to overcome the limitations of traditional techniques, contributing to a more coherent and efficient reservoir modeling.

1.1 RESEARCH PROBLEM

Accurate characterization of oil reservoirs remains a critical challenge in the industry due to the inherent heterogeneity of geological formations and the limited availability of comprehensive data. In this context, the main goal is to quantitatively describe the spatial distribution of geological properties -such as lithofacies, porosity, permeability, and saturation -that guide fluid flow in the reservoir and provide direction for development and production strategies. To address the limitations of available data and the non-unique nature of the inverse problem, traditionally geostatistical models are used to generate multiple scenarios that recognize conditional data and geological knowledge.

Conventional geostatistical methods, such as variograms and two-point statistics, are traditionally divided into estimation and simulation techniques. Kriging, uses spatial continuity to assign weights to neighboring data, although it results in smoothed surfaces that ignore actual heterogeneity. Sequential Indicator Simulation (SIS), seeks to represent this variability in facies models, but often generates discontinuous patterns known as the “salt and pepper” effect. Since these methods represent only second-order correlations, they do not adequately capture complex deposition patterns. In contrast, multiple point statistics (MPS) approaches use training images to reproduce more realistic deposition geometries—such as channels, bars, and lenses—while preserving connectivity and spatial continuity consistent with the deposition environment. However, MPS techniques face limitations such as high computational cost, heavy dependence on training image quality, and challenges in accurately conditioning to well data.

In this scenario, there is a significant need to investigate alternatives based on generative networks, in particular cGANs, as analogous methods to MPS techniques capable of generating multiple geological scenarios. The synthetic generation of scenarios using GAN architectures allows the representation of complex geometric patterns and the preservation of spatial relationships. However, to the best of our knowledge, GAN-based approaches have not yet been applied to the generation of 2D lithofacies scenarios conditioned on well data, and it is essential to understand what extent different strategies based on generative networks affect the consistency,

robustness, and geological fidelity of the generated scenarios, especially in reservoir environments characterized by high structural and depositional complexity.

Therefore, the research problem can be formalized as

Research question

How can Conditional Generative Adversarial Networks be used to generate synthetic lithofacies scenarios conditioned on well data, and can these methods be an alternative to Multiple Point Statistics techniques for geological reservoir modeling?

1.2 RESEARCH OBJECTIVES

Based on the problem and research question presented, the general objective of this work is outlined below. Furthermore, specific objectives are established as concrete targets that must be achieved in order to fulfill the general objective.

General objective

To explore Conditional Generative Adversarial Networks (cGANs) for generating synthetic lithofacies scenarios conditioned on well data.

Specific objectives

- To develop a pre-processing pipeline to structure well data and generate appropriate representations of lithofacies, ensuring consistent conditioning of observed data.
- To implement and train a model based on Conditional cGANs to generate lithofacies scenarios conditioned on well data.
- To validate the effectiveness of cGAN techniques by comparative analysis with Multiple Point Statistics techniques, to quantify the performance of predictive models applied to oil exploration.

1.3 SUMMARY OF METHODOLOGY:

The implementation of Generative Adversarial Networks (GANs) and Conditional Generative Adversarial Networks (cGANs) on oil well logs follows a series of structured steps to ensure the accurate generation of synthetic data and assess its impact on machine learning models.

To address the research question, a structured methodology was developed, comprising several key stages, each designed to contribute specifically to achieving the overarching objective of the research. These stages are shown in Fig. 1 and detailed below.

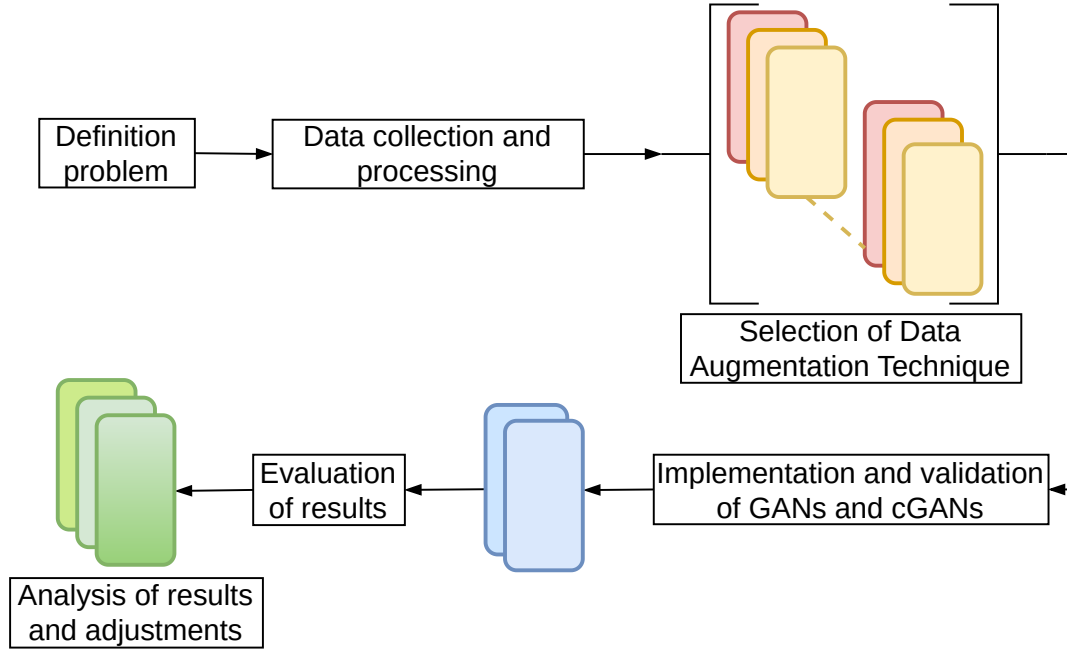


Figure 1 – Workflow of the proposed methodology.

1. Data Collection and Processing

- **Collection of Well Logs and Facies Data:** Initially, it is essential to gather a representative set of oil well logs that include facies data. This data will form the foundation for generating synthetic data using cGANs.
- **Filtering Reservoir and Non-Reservoir Facies:** Once the facies data is collected, the samples must be classified into two categories: reservoir (areas with potential for oil storage) and non-reservoir. This classification is critical for optimizing the generation of synthetic data tailored to each category, thereby enhancing the model's accuracy.
- **Generation of Facies Images:** After filtering, the facies data samples are transformed into images that visually represent the geological characteristics of each facies type (reservoir and non-reservoir). These images will serve as inputs to the generative networks, facilitating the application of data augmentation techniques.
- **Normalization and Standardization:** To ensure effective training of GANs and cGANs, the images and data need to be normalized or standardized. This step ensures that all variables are on a consistent scale, making them suitable for model training.
- **Splitting the Data into Training and Testing Sets:** Finally, the prepared data is divided into training and testing sets. The training set will be used to train the genera-

tive networks, while the testing set will be used to evaluate the model's performance in generating synthetic data.

2. Selection of Data Augmentation Technique

GANs and cGANs will be employed to generate new synthetic data that mirrors the characteristics of real well log data. In this framework, the generator network creates data from a random noise input, while the discriminator network differentiates between real and generated data, continuously enhancing the generation of more realistic outputs. cGANs will be used to generate synthetic data conditionally, allowing for control over the generation process based on specific characteristics, such as facies types or depth intervals. This approach ensures higher precision in data generation, tailored to specific requirements.

3. Implementation of GANs and cGANs

- **Network Architecture Configuration:** The architectures of the GAN and cGAN networks are defined, including the number of layers, neurons, activation functions, and other hyper-parameters. For cGANs, the conditioning factor that will guide the data generation (e.g., facies class or depth) is specified.
- **Network Training:** The GAN and cGAN networks are trained using the training dataset. Throughout this process, the generator improves its ability to produce realistic synthetic data, while the discriminator refines its skill in distinguishing between real and generated data.
- **Hyper-parameter Tuning:** During training, key parameters such as learning rate, batch size, and the number of epochs are adjusted to ensure the generation of high-quality synthetic data.
- **Synthetic Data Generation:** After training, the generator networks create new synthetic data that closely mirrors the properties of real well log data.

4. Evaluation of Results

A comparative evaluation is conducted between the synthetic data generated and the real data, employing techniques such as visual comparison, statistical analysis of distributions, and verification of geological consistency. Additionally, validation is performed to assess the model's performance across different data subsets, determining whether the synthetic data improves the model's generalization ability compared to training with real data alone.

5. Analysis of Results and Adjustments

Based on the obtained results, the impact of the synthetic data generated by GANs and cGANs on the models' accuracy and robustness is analyzed. The effectiveness of data augmentation is evaluated by comparing the model's performance with and without the use of synthetic data.

1.4 CONTRIBUTIONS

The implementation of GANs, and in particular cGANs, contributes to the increase in facies data, generating synthetic datasets that replicate the statistical and geological characteristics of the original data. This provides a realistic and consistent sample of physical and subsurface conditions, providing useful data for predictive modeling. In addition, these techniques mitigate class imbalance, a common problem in geological datasets, by generating more examples of underrepresented facies, minimizing errors, and improving the performance of machine learning models.

Successful implementation of a cGAN involves controlled expansion of the dataset, allowing the lack of real data to be overcome, as a frequent limitation in geological applications is increasing the amount of available data without the need for costly and time-consuming field or laboratory collection processes.

The generation of diverse and representative data improves model generalization, making them more robust for new conditions or previously unobserved scenarios. Thus, in this context, cGAN offers the ability to generate synthetic data conditioned on specific properties or parameters, such as 2D vertical section facies data conditioned on well data, which is especially valuable in facies studies. The network allows the creation of samples that meet precise requirements for analysis and offer remarkable flexibility to adapt to various geological and operational contexts, adjusting to specific needs.

Finally, cGAN plays a key role in improving the quality and quantity of data used in facies characterization, facilitating more accurate analysis and optimizing decision-making in the oil industry. These tools contribute significantly to increasing the effectiveness of projects, consolidating themselves as a key part of the advancement of this discipline.

Publication statement

Part of the results presented in this thesis were previously published in (CAMACHO-DE ANGULO et al., 2025). The present thesis extends those results by providing a more detailed methodological description, additional experiments, and a broader discussion of the findings.

1.5 DOCUMENT STRUCTURE

This document is structured as follows: Chapter 1 presents the introduction, which outlines the general context of the study, the research problem, and the research objectives. Chapter 2 includes the theoretical baseline, providing a review of the literature on lithological facies and the use of artificial intelligence, specifically GANs and cGANs. Chapter 3 presents the literature review. Chapter 4 presents the proposed methodology, and Chapter 6 includes the experiments, results, and discussion. Finally, Chapter 7 emphasizes the conclusions and future works.

2 THEORETICAL FOUNDATION

This chapter presents the theoretical foundation necessary for understanding the problem this research seeks to address. It is structured into two main sections: the theoretical background on lithofacies and the fundamentals of cGANs.

2.1 OIL WELL DATA

The theoretical basis of oil well data constitutes a multidisciplinary framework that integrates principles of geology, geophysics, and reservoir engineering to understand subsurface hydrocarbon systems. In this context, the main data used in the reservoir characterization are summarized in the diagram (Fig 2). This integration allows for the interpretation of subsurface complexity and the establishment of relationships between direct observations, indirect measurements, and descriptions derived from interpretation methods.

Facies classification is central to this process, as it articulates lithological, sedimentological, petrophysical, and seismic attributes in a coherent framework for analysis. On this basis, facies classification is introduced, and in particular, the concept of lithofacies, considered the fundamental unit for describing depositional environments and representing reservoir heterogeneity.

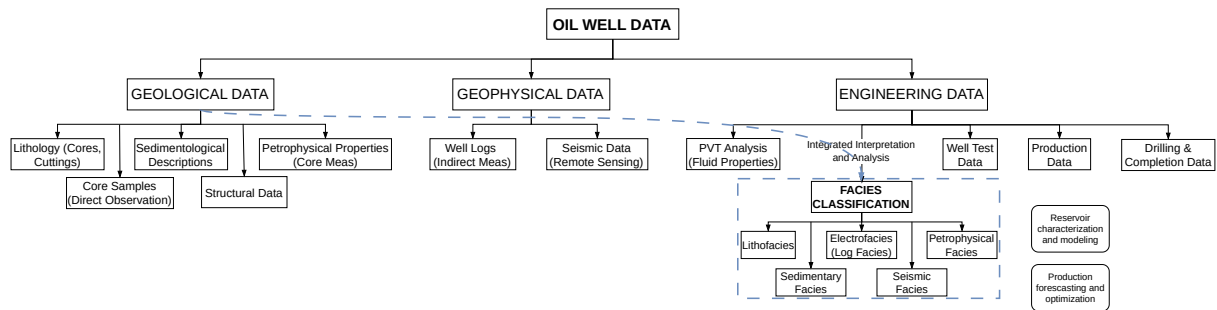


Figure 2 – Overview of the multidisciplinary data framework used in reservoir characterization, illustrating the integration of geological, geophysical, petrophysical, and engineering information. The dashed box emphasizes the facies classification workflow, highlighting lithofacies as a central element linking geological, geophysical, and petrophysical information toward the construction of subsurface models. Image reproduced and adapted from (SERRA; ABBOTT, 1982).

2.1.1 Facies

A facies is understood as a body of rock or sediment that has a distinctive combination of lithological, textural, mineralogical, paleontological, and structural characteristics that differentiate it from adjacent deposits (READING, 2009; GRANA; MUKERJI; DOYEN, 2021). These attributes are the expression of the physical, chemical, and biological conditions that prevailed at the time of deposition and during subsequent diagenetic processes (WALKER, 1992; BOGGS, 1995). Facies analysis seeks not only to describe these observable characteristics, but

also to interpret the depositional environments, sedimentary dynamics, and stratigraphic relationships that control their distribution (GRANA; MUKERJI; DOYEN, 2021; CATUNEANU, 2022).

Facies classification

Facies classification is the most critical aspect of interpreting well oil data, as it directly translates raw measurements into meaningful geological units and environmental interpretations. Its theoretical foundation lies in the principle that specific combinations of geological and geophysical attributes are diagnostic of particular depositional environments and their associated reservoir characteristics (GRANA; MUKERJI; DOYEN, 2021; SELLEY, 1998).

Facies can be classified into different categories depending on the type of data and scale of observation: lithofacies (based on lithological and sedimentological attributes), biofacies (based on fossil assemblages), electrofacies (derived from well log responses), seismic facies (from seismic reflection patterns), and petrophysical facies (from rock property variations) (SANGREE; WIDMIER, 1979). This multidimensional perspective highlights the role of facies as the fundamental unit for linking observational data with depositional models and reservoir characterization.

2.1.1.1 Lithological facies

Lithofacies are subdivisions of the rock record defined by their observable lithological and sedimentological attributes, such as grain size, mineral composition, texture, sedimentary structures, color, and bed geometry. Each lithofacies reflects specific depositional conditions and processes, making them fundamental for reconstructing sedimentary environments and stratigraphic architectures (ROKSANDIĆ, 1978; IMAMVERDIYEV; SUKHOSTAT, 2019).

In sedimentology, lithofacies analysis allows the recognition of recurring patterns in rock successions, which can be grouped into facies associations that represent larger depositional systems (e.g., fluvial channels, deltaic lobes, turbidite systems) (GRANA; MUKERJI; DOYEN, 2021). From a petroleum geology perspective, lithofacies are crucial for reservoir characterization, since variations in lithofacies often control porosity, permeability, and fluid flow behavior within hydrocarbon-bearing formations (READING, 2009).

For practical purposes, lithofacies are typically identified through core descriptions, outcrop studies, and thin section petrography, but can also be inferred indirectly from well log responses when integrated with geological knowledge (CATUNEANU, 2022). Their classification provides the foundation for facies models, reservoir zonation, and ultimately, for static and dynamic reservoir simulation (AHR, 2011).

2.2 CLASSICAL GEOSTATISTICAL METHODS

Geostatistics is a branch of statistics that focuses on the analysis of spatial data, particularly those that exhibit continuity in space but cannot be modeled by simple mathematical functions (KRIGE, 1951). Its central objective is to predict the value of a regionalized variable at unsampled locations while explicitly accounting for spatial dependence.

2.2.1 Kriging

Kriging is a geostatistical technique for spatial estimation or interpolation that provides a linear, unbiased, and optimal estimator (with minimum estimation variance) (KRIGE, 1951). The estimate $Z^*(\mathbf{x}_0)$ at an unsampled point $\mathbf{x}_0$ is given by the weighted average of the sample data $Z(\mathbf{x}_i)$:

$$Z^*(\mathbf{x}_0) = \sum_{i=1}^n \lambda_i Z(\mathbf{x}_i) \quad (2.1)$$

where $Z(\mathbf{x}_i)$ are the observed values at sampled locations $\mathbf{x}_i$, and λ_i are the kriging weights. These weights quantify the contribution of each sampled point to the estimate and are determined based on the spatial correlation structure of the data.

The weights λ_i are not chosen arbitrarily. They are obtained by solving an optimization problem that minimizes the kriging estimation variance σ_K^2 while satisfying the unbiasedness condition. In its most common form, this condition requires that the expected value of the estimate equals the expected value of the true variable at $\mathbf{x}_0$.

For ordinary kriging, this condition leads to the constraint:

$$\sum_{i=1}^n \lambda_i = 1 \quad (2.2)$$

The minimization of σ_K^2 under this constraint is carried out using a Lagrange multiplier μ , resulting in a system of linear equations involving the semivariances between sampled locations $\gamma(\mathbf{x}_i, \mathbf{x}_j)$ and between each sample and the estimation point $\gamma(\mathbf{x}_i, \mathbf{x}_0)$. Solving this system yields both the kriging weights and the estimation variance.

Different kriging variants arise from different assumptions about the mean structure of the regionalized variable.

2.2.2 Simple Kriging (SK)

Simple Kriging is the most theoretically rigorous kriging variant, although it is less frequently applied in practice due to its strong assumptions. In SK, the regionalized variable is assumed to have a known and constant mean:

$$\mathbb{E}[Z(\mathbf{x})] = m \quad \forall \mathbf{x} \quad (2.3)$$

Because the mean is explicitly known, the unbiasedness condition does not require the sum of the weights to equal one. Instead, the estimator is written as:

$$Z^*(\mathbf{x}_0) = m + \sum_{i=1}^n \lambda_i [Z(\mathbf{x}_i) - m] \quad (2.4)$$

In this formulation, any deviation of the weights from unity is compensated by the known mean value. As a result, SK generally yields lower estimation variance than ordinary kriging when the assumption of a known mean is valid (KRIGE; UECKERMANN, 1963). However, this assumption is rarely satisfied in real-world environmental applications.

2.2.3 Universal Kriging (UK)

Universal Kriging is used when the mean of the regionalized variable is not constant but varies smoothly in space, indicating the presence of a deterministic trend (or drift). In such cases, the intrinsic hypothesis of stationarity is violated. The regionalized variable is decomposed into two components:

$$Z(\mathbf{x}) = m(\mathbf{x}) + Y(\mathbf{x}) \quad (2.5)$$

where $m(\mathbf{x})$ represents the deterministic trend, typically modeled as a polynomial function of the spatial coordinates, and $Y(\mathbf{x})$ is a stochastic residual with zero mean and spatial dependence described by a variogram.

Universal kriging simultaneously estimates the trend coefficients and the kriging weights, ensuring unbiasedness with respect to the underlying drift. This makes UK particularly suitable for large-scale environmental variables influenced by systematic spatial gradients, such as topography or climate.

2.3 SIMULATION TECHNIQUES

Geostatistical simulation is a set of stochastic methodologies that aims to generate multiple representations (realizations) of an uncertain spatial field, which honor the descriptive statistics (mean, variance, proportions) and spatial relationships (variogram, patterns) of the input data (PYRCZ; DEUTSCH, 2014). The goal is to quantify spatial uncertainty and heterogeneity by providing a collection of equally probable models rather than a single smoothed estimate (such as Kriging).

2.3.1 Sequential Indicator Simulation (SIS)

SIS is a categorical (or discrete variable) simulation technique based on the two-point concept (variogram) and the sequential decomposition of the multivariate distribution of a random field (JOURNAL; ALABERT, 1989; DEUTSCH, 2006). It is widely used to model the distribution of facies or lithotypes in geosciences.

Sequential simulation

The fundamental premise of sequential simulation (which includes SIS and Sequential Gaussian Simulation - SGS) is the decomposition of the joint probability distribution into a product of conditional distributions (JOURNEL, 1986; DEUTSCH, 2006):

$$P(Z(\mathbf{u}_1), Z(\mathbf{u}_2), \dots, Z(\mathbf{u}_N)) = P(Z(\mathbf{u}_1)) \cdot P(Z(\mathbf{u}_2)|Z(\mathbf{u}_1)) \cdots P(Z(\mathbf{u}_N)|Z(\mathbf{u}_1), \dots, Z(\mathbf{u}_{N-1})) \quad (2.6)$$

Where $Z(\mathbf{u}_i)$ is the value of the random variable at location $\mathbf{u}_i$. By simulating the value at each location $\mathbf{u}_i$ based on all previously conditioned values (original data and simulated values), the algorithm ensures that the two-point statistic (variogram) is reproduced (DEUTSCH, 2006). To avoid simulation artifacts, the grid points are visited in random order (random walk) (ALABERT, 1987).

Indicators and conditional probability

SIS is specifically applied to categorical variables (e.g. lithofacies) or to continuous variables discretized by transformation into indicator variables (JOURNEL, 1983). For a category k , the indicator $I(\mathbf{u}; k)$ is defined as:

$$I(\mathbf{u}; k) = \begin{cases} 1 & \text{if } Z(\mathbf{u}) = k \\ 0 & \text{otherwise} \end{cases} \quad (2.7)$$

The conditional probability that a category k occurs at location $\mathbf{u}$ is estimated using the Simple Indicator Kriging (SIK) technique (DEUTSCH, 2006):

$$P(Z(\mathbf{u}) = k | \text{Conditional Data}) \approx I_{SIK}^*(\mathbf{u}; k) \quad (2.8)$$

The estimate $I_{SIK}^*(\mathbf{u}; k)$ is a probability that varies between 0 and 1 and is used to construct the Conditional Probability Distribution (CPD) at $\mathbf{u}$.

SIS Algorithm Procedure

The SIS algorithm follows these steps (DEUTSCH, 2006; ALABERT, 1987):

1. Data Transformation: Convert categorical (or discretized continuous) data into indicator variables for each category.
2. Variogram Modeling: Calculate and model the indicator variogram for each category.
3. Path Definition: Establish a random path that visits all unsampled nodes in the simulation grid.

4. Sequential Simulation: For each node $\mathbf{u}_i$ on the path:

- Estimate CPD: Use SIK with conditional data (original data + already simulated nodes) within a search neighborhood to estimate probabilities $P(Z(\mathbf{u}_i) = k | \text{Conditional Data})$ for all categories k .
- Ensure Order: The probabilities must be readjusted to ensure that the sum of the probabilities for all categories equals 1 (order of relations).
- Draw a Value: Draw a uniform random number $p \in [0, 1]$ and assign category k to node $\mathbf{u}_i$ based on the acumulative CPD (Inverse CPD - Monte Carlo).
- Conditioning: The simulated value becomes part of the conditional data set for subsequent nodes.

5. Generation of Realizations: Repeat the process from steps 3 to 4 with different sequences of random numbers to generate multiple equiprobable realizations.

2.3.2 Multiple-Point Statistics (MPS)

MPS is a class of stochastic simulation methods developed to overcome the limitation of variograms (two-point statistics) in reproducing complex geometries and higher-order spatial patterns (STREBELLE, 2002; CAERS; ZHANG, 2004).

Higher-order spatial patterns

Unlike variograms, which only consider the relationship between pairs of points, MPS uses spatial patterns involving the configuration of multiple points (three, four, or more) to characterize the discontinuity and connectivity of geological structures (e.g., channels, faults, fractures). This approach aims to model the CPD using the frequency of occurrence of these patterns (STREBELLE; JOURNAL, 2001).

Models based on training images

The central concept of MPS is the use of a Training Image (TI). The TI is a conceptual or analogical model that represents the spatial distribution and expected connectivity of geological features of interest, generally providing statistics for multiple points (patterns) in the system (STREBELLE; JOURNAL, 2001; CAERS; ZHANG, 2004).

The CPD required for sequential (step-by-step) simulation is borrowed (obtained by sampling) from the TI, rather than being estimated from variograms. In formal terms, the CPD at a location $\mathbf{u}$ is conditioned on a data event $d(\mathbf{u})$ which is a configuration of multiple already conditioned points (hard data and simulated nodes) in a predefined neighborhood (template):

$$P(Z(\mathbf{u}) = k | d(\mathbf{u})) \approx \text{Frequency of pattern } d(\mathbf{u}) \text{ in IT} \quad (2.9)$$

Principal MPS algorithms

MPS algorithms are generally based on sequential simulation, but differ in how IT is used to obtain the CPD.

1. SNESIM (Single Normal Equation Simulation): SNESIM (STREBELLE; JOURNAL, 2001) is one of the first and most influential MPS algorithms:
 - Principle: It uses a random sequential simulation approach (similar to SIS).
 - Obtaining the CPD: The CPD for a given node is obtained directly from the TI by searching for occurrences of the conditional pattern (data template) surrounding the node to be simulated.
 - Pattern Tree: To optimize the search, SNESIM stores all IT patterns in a search tree. The depth of the tree and the size of the template define the order of the reproduced multiple point statistics.
 - Limitation: The use of the tree can be memory intensive for large ITs or extensive templates.
2. Direct Sampling: The Direct Sampling (DS) algorithm (MARIETHOZ; CAERS, 2014) is an MPS method that avoids constructing the SNESIM data tree:
 - Principle: It also uses sequential random simulation.
 - Obtaining the CPD: Instead of storing all patterns in a tree, DS randomly traverses the IT a limited number of times to find occurrences of the conditional pattern.
 - Efficiency: The process is generally faster and less memory-intensive than SNESIM, especially for large TIs, as it does not need to preprocess and store the pattern database. The algorithm effectively samples the CPD of the TI directly (MARIETHOZ; CAERS, 2014).
3. Other MPS alternatives: Research into MPS continues to produce variants focused on improvements in efficiency and the ability to model non-stationarity (variation in spatial statistics in the domain).
 - FILTERSIM: Uses multiple filters and neural networks to characterize spatial patterns, representing the patterns through a filter response vector (CAERS; ZHANG, 2004). This approach generally offers greater efficiency.
 - IMPALA: This is a variant that improves the way patterns are compared and sampled in IT. Its goal is to increase fidelity in pattern reproduction in situations with little conditional data (STREBELLE; JOURNAL, 2001).
 - Others: More recent methods such as DeeSse (based on DS) and machine learning approaches aim to optimize pattern reproduction and non-stationarity modeling (STRAUBHAAR; RENARD; CHUGUNOVA, 2020).

2.4 MACHINE LEARNING

The hierarchical structure of the main concepts that define this research, beginning with Machine Learning (ML) and progressively descending to specific deep learning methodologies, is presented in the following diagram (Fig. 3).

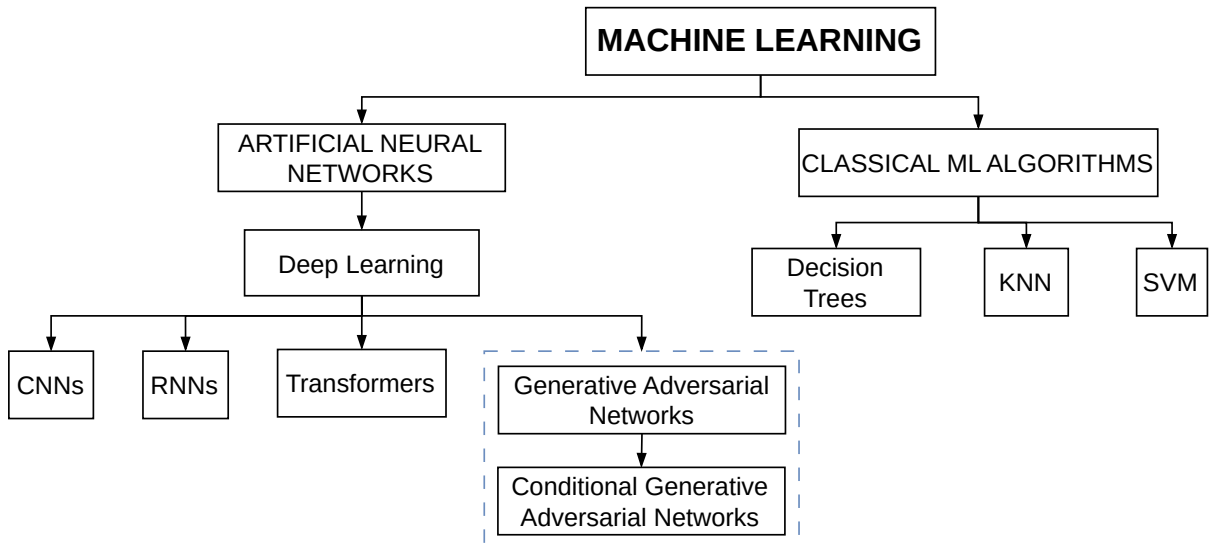


Figure 3 – Hierarchical organization of Machine Learning, showing the relationship between Deep Learning, and classical algorithms and present a representative subset of methods to contextualize some approaches. The dashed box highlights the place of Generative Adversarial Networks (GANs) and their extension, Conditional GANs (cGANs), as central methods within this research. Image reproduced and adapted from (MIRZA, 2014; GOODFELLOW et al., 2014).

ML is a core subfield of artificial intelligence (AI) dedicated to the development of algorithms and models that enable systems to learn from data. This process involves identifying patterns and making inferences without requiring explicit programming for each specific task (MITCHELL, 1997; NAQA; MURPHY, 2015). Rather than relying on predefined rules, machine learning systems use data to iteratively adjust their parameters, enhancing their ability to perform tasks such as classification, prediction, and data generation. This capacity for autonomous learning allows machine learning to tackle complex problems and adapt to new scenarios, making it a powerful tool in a wide range of applications (JORDAN; MITCHELL, 2015).

The central objective of machine learning is to enable systems to generalize from observed data, meaning to apply the knowledge acquired during training to new, unseen data effectively (MITCHELL, 1997). This ability to learn from data and continuously adapt makes machine learning a powerful tool across a wide range of fields, from recommendation systems and medical diagnostics to finance, predictive modeling, and even creative AI systems (MOHAMMED; KHAN; BASHIER, 2016).

The learning process involves pattern extraction and optimization. Initially, algorithms analyze large volumes of data to identify common patterns or features. Then they adjust their internal parameters to represent these patterns as efficiently as possible. This learning process

is continuous, as algorithms can improve their predictions or decisions as more data becomes available (SARKER, 2021b; JAIN; DUIN; MAO, 2000).

Furthermore, machine learning can be classified based on the type of task and the kind of data used in the learning process. The main approaches include:

- **Supervised Learning:** The model learns from labeled data, where each input is associated with a known output. The model aims to learn a function that maps inputs to outputs based on these examples. Common applications include classification and regression tasks
- **Unsupervised Learning:** Instead of labeled data, the model works with unlabeled data, searching for underlying patterns or structures, such as clustering or dimensionality reduction
- **Reinforcement Learning:** The agent interacts with the environment, making decisions based on rewards or penalties. The goal is to maximize a reward function over time, learning through trial and error (KOTSIANTIS; ZAHARAKIS; PINTELAS, 2006).

This ability to learn and adapt makes machine learning particularly useful in scenarios where explicitly programming rules would be difficult or impractical. Instead of manually providing every instruction, machine learning models are trained to autonomously make predictions, decisions, or generate new data based on learning from large datasets (ALLAMANIS et al., 2018; SHINDE; SHAH, 2018).

Among the various approaches within machine learning, deep learning has emerged as a particularly transformative subfield. By deep learning excels at automatically extracting and learning complex features from vast and unstructured datasets, such as images, audio, and text (MICHALSKI, 2013). This capability has enabled groundbreaking advancements in areas like natural language processing, computer vision, and generative modeling (JORDAN; MITCHELL, 2015).

2.4.1 Artificial Neural Networks

Artificial neural networks (ANNs) are the fundamental basis of deep learning. ANNs are computational structures inspired by the biological functioning of the brain, designed to learn and perform specific tasks by interconnecting basic units called artificial neurons (LÓPEZ; LÓPEZ; CROSSA, 2022). Each neuron performs simple mathematical calculations and transmits its output to other neurons through weighted connections. In this context, deep learning uses multilayered neural network architectures, known as deep neural networks (DNNs), which allow hierarchical representations of data to be learned (HASSOUN, 1995).

In an ANN, each neuron receives input, calculates a weighted sum of these, applies an activation function, and then sends the result as output to the next layer (DONGARE et al., 2012). The layers in an ANN are generally organized as follows:

1. **Input layer:** This is the first layer of the network, responsible for receiving raw data (such as pixels in an image, words in a text, or numeric features) and passing it to the following layers. It performs no computations, only serves as a gateway for the data.
2. **Hidden Layers:** These are the inner layers where the main processing occurs. Each hidden layer transforms the input data by applying activation functions and adjustments to the weights and biases. The number of hidden layers and their size (number of neurons) determine the depth of the network. In deep networks, these layers allow complex patterns in the data to be identified, from basic features (such as edges in an image) to abstract concepts (such as the identification of entire objects).
3. **Output layer:** This is the last layer of the network, responsible for generating the final prediction. Depending on the problem, this layer may have a different structure, such as a single neuron for regression, several neurons with softmax activation for multiclass classification, or even more complex structures in data generation problems (ABRAHAM, 2005).

ANN training involves the optimization of weights and biases through a process called backpropagation, combined with optimization algorithms such as gradient descent and its variants (ABDOLRASOL et al., 2021). Backpropagation calculates the error in the output layer and propagates it across the network, iteratively adjusting the weights to minimize the error. This process allows the network to improve its performance over time (LEONARD; KRAMER, 1990).

Deep learning

Deep Learning (DL) a branch of machine learning (ML) centers around the utilization of neural networks (ANNs) especially those incorporating multiple interconnected layers to tackle intricate problems efficiently and effectively (GOODFELLOW, 2016; LECUN; BENGIO; HINTON, 2015). These networks are designed based on the structure and functionality of the brain and comprise layers of nodes (neurons), these nodes alter information to acquire insights, into patterns and representations (SARKER, 2021a). The term "deep", in learning signifies the complexity of these structures with layers that allow for the identification of hierarchical characteristics, from unprocessed information by moving from basic to intricate representations (LECUN; BENGIO; HINTON, 2015).

Deep learning is particularly useful for tasks involving large volumes of unstructured data, where traditional machine learning methods are often insufficient because of deep learning networks' hierarchical structure, these networks are able to automatically learn features directly from the data, eliminating the need for manual feature extraction (ZHANG et al., 2018). This makes them especially effective in areas such as computer vision, natural language processing and audio analysis (SCHMIDHUBER, 2015).

The impact of deep learning has been amplified by advances in computational power (especially with GPUs and TPUs), access to large datasets and sophisticated architectures, such as convolutional neural networks (CNNs) for image processing, recurrent neural networks (RNNs) for sequential data and transformers for natural language tasks (TONG, 2024). In addition, deep learning is the basis for generative models such as Generative Adversarial Networks (GANs) and their conditional variants (cGANs), which enable the creation of highly realistic synthetic data and the solution of complex problems, such as image translation or the generation of visual and textual content (SAXENA; CAO, 2021).

2.4.1.1 *Generative Adversarial Networks (GANs)*

Generative Adversarial Networks (GANs) are a type of model within the Deep Learning domain, introduced by (GOODFELLOW et al., 2014). This approach combines two neural networks working in an adversarial fashion, allowing them to generate highly realistic synthetic data, and is widely used in tasks such as image generation, data quality enhancement, and speech synthesis (RADFORD, 2015).

A GAN consists of two deep neural networks that interact with each other in a competitive framework, known as a zero-sum game.

1. Generator network (Generator): Its goal is to create synthetic data that are as similar as possible to real data. It starts from a random input (usually a noise vector) and learns to generate data similar to the training set.
2. Discriminator Network: Its task is to distinguish between real data (from the training set) and synthetic data (generated by the generator network). It acts as a binary classifier (GOODFELLOW et al., 2014; CRESWELL et al., 2018).

The two networks are trained simultaneously:

- The generator network tries to fool the discriminator.
- The discriminating network improves its ability to detect false data (SALIMANS et al., 2016a).

GANs are based on a theoretical framework that combines the theory of games and deep learning to train two competing neural networks. This mathematical approach allows a generative network to learn to create synthetic data that is indistinguishable from real data from the perspective of a discriminative network (GOODFELLOW et al., 2014). The basis of this process is an objective function that establishes a joint optimization dynamic between the two networks (FIORE et al., 2019; SALIMANS et al., 2016b).

The training of a GAN is based on the minimization of an objective function, formulated as a minimization-maximization problem, in which the generative network (G) tries to minimize the ability of the discriminative network (D) to identify false data:

$$\min_G \max_D V(D, G) = \mathbb{E}_{x \sim p_{\text{data}}(x)} [\log D(x)] + \mathbb{E}_{z \sim p_z(z)} [\log(1 - D(G(z)))] \quad (2.10)$$

where:

- $D(x)$: Probability assigned by the discriminator that a data x is real, i.e. that it comes from the real data distribution.
- $G(z)$: Data generated by the generator network from a noise vector z , taken from a distribution known as $p_z(z)$ (normal or uniform, generally).
- $p_{\text{data}}(x)$: Distribution of the actual data in the training set.
- $p_z(z)$: Distribution of the input noise used to feed the generator network (MESCHER; GEIGER; NOWOZIN, 2018).

The objective of the discriminator(D) is to maximize its ability to distinguish between real data (x) and generated data ($G(z)$), while the generator (G) aims to minimize this ability by continuously improving the quality of its synthetic data to fool the discriminator.

Generative Adversarial Networks (GANs) represent one of the most innovative architectures in deep learning, with impact in multiple areas of artificial intelligence. These networks are noted for their ability to learn complex data distributions and generate synthetic data that are indistinguishable from real data, which has opened up a range of practical and theoretical applications in various domains, consolidating their relevance in research and industry (SALEHI; CHALECHALE; TAGHIZADEH, 2020).

In imaging, GANs have proven to be effective; models such as StyleGAN have been used to generate highly realistic human faces, natural landscapes and virtual scenarios. In medical applications, GANs allow the creation of synthetic images that can be used to train models without exposing sensitive patient data, preserving privacy (WU; STOUFFS; BILJECKI, 2022). Another important application is data augmentation, using GANs to generate additional datasets in situations where available data is limited or unbalanced, improving the generalization of models by providing more diverse examples during training (BOWLES et al., 2018)

The development of GANs has driven the creation of more advanced models, such as Conditional Adversarial Generative Networks (cGANs), which extend their functionality by incorporating additional conditions in the generation process, thus increasing their applicability in specific contexts (MIRZA, 2014).

Generative Model from a Single Natural Image - SinGAN¹

SinGAN (SHAHAM; DEKEL; MICHAELI, 2019) is a generative model able to be learned from a single natural image. The model is trained to capture the internal distribution of

¹ SHAHAM, T. R.; DEKEL, T.; MICHAELI, T. "SinGAN: Learning a generative model from a single natural image". In: Proceedings of the IEEE/CVF International Conference on Computer Vision. [S.l.: s.n.], 2019. p. 4570–4580.

patches within the image and is then trained to generate diverse, high-quality samples that carry the same visual content as the image.

SinGAN contains a pyramid of fully convolutional GANs, each responsible for learning the distribution of patches at a different scale of the image, which allows it to generate new samples of arbitrary size and proportion, which have significant variability but retain the overall structure and fine textures of the training image. In contrast to the other GAN single-image schemes, this approach is not limited to texture images and is not conditional, i.e. it generates samples from noise.

Multi-scale architecture

The model consists of a pyramid of generators $\{G_0, \dots, G_N\}$, based on a pyramid of images $x : \{x_0, \dots, x_N\}$, where x_n is a reduced version of x by a factor r_n , for some $r > 1$. Each generator G_n is responsible for producing image samples that are realistic with respect to the distribution of spots in the corresponding image x_n , this is achieved by means of adversarial training, where G_n learns to fool an associated discriminator D_n , which tries to distinguish the spots in the generated samples from the spots in x_n . The generation of an image sample starts at the coarsest scale and passes sequentially through all the generators up to the finest scale, with noise injected at each scale. All generators and discriminators have the same receptive field and therefore capture structures of decreasing size as they progress through the generation process. At the coarsest scale, generation is purely generative, i.e. G_N maps the spatial Gaussian white noise z_N to an image sample x'_N ,

$$x_N = G_N(z_N) \quad (2.11)$$

The effective receptive field at this level is usually $\sim 1/2$ of the image height, so G_N generates the overall layout of the image and the global structure of the objects. Each of the generators G_n at finer scales ($n < N$) adds details that were not generated by the previous scales. Thus, in addition to the spatial noise z_n , each generator G_n accepts an upsampled version of the coarser scale image, which is

$$x_n = G_n\left(z_n, (x'_{n+1})^{\uparrow r}\right), n < N \quad (2.12)$$

All generators have a similar architecture, as illustrated in figure 4 specifically, the noise z_n is added to the image $(x'_{n+1})^{\uparrow r}$, before being input to a sequence of convolutional layers, this guarantees that the GAN does not disregard the noise, as often happens in conditional schemes involving randomness (ZHU et al., 2017a; ZHU et al., 2017b; MATHIEU; COUPRIE; LECUN, 2015). The function of the convolutional layers is to generate the missing details in $(x'_n + 1)^{\uparrow r}$ (residual learning (HE et al., 2016; ZHANG et al., 2017), i.e. G_n performs the operation

$$x'_n = \left(x'_{(n+1)}\right) \uparrow^r + \psi_n \left(Z_n + \left(x'_{(n+1)}\right) \uparrow^r\right) \quad (2.13)$$

where ψ_n is a full convolutional network with 5 blocks of *conv*(3×3)-*BatchNorm*-*LeakyReLU* (IOFFE; SZEGEDY, 2015). At first, there are 32 kernels per block at the largest scale and this number is increased by a factor of 2 every 4 scales. As the generators are fully convolutional, images of arbitrary size and proportion can be generated at the time of testing (by changing the dimensions of the noise maps).

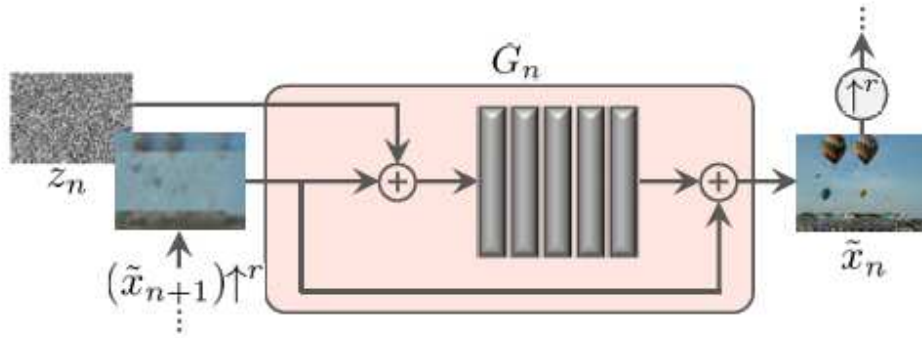


Figure 4 – Single scale generation

Training

Architecture training at various scales is done sequentially, from the coarsest to the finest scale. After each GAN has been trained, it is kept fixed. The training loss for the n th GAN is composed of a contradiction term and a reconstruction term,

$$\min_{G_n} \max_{D_n} \mathcal{L}_{\text{adv}}(G_n, D_n) + \alpha \mathcal{L}_{\text{rec}}(G_n) \quad (2.14)$$

Adversarial loss $\mathcal{L}_{\text{adv}}$ punishes the distance between the distribution of patches in x_n and the distribution of patches in samples generated x'_n . The reconstruction loss $\mathcal{L}_{\text{rec}}$ guarantees the existence of a specific set of noise maps that can produce x_n

Adversarial loss

Each of the generators G_n is coupled to a Markovian discriminator D_n that classifies each of the overlapping *patches* of its input as real or false (LI; WAND, 2016; ISOLA et al., 2017). The WGAN-GP loss (GULRAJANI et al., 2017), which increased training stability, is used, in which the final discrimination score is the average of the *patches* discrimination map. Unlike single-image GANs for textures, loss is defined on the entire image rather than on random crops (a batch of size 1). This allows the network to learn the boundary conditions, an important feature in the configuration. The architecture of G_n is the same as that of the ψ_n network in G_n , so the size of its area (the receptive field of the network) is 11×11 .

Reconstruction loss

The goal is to ensure that a specific set of input noise maps exists to generate the original image x . The specific choice is $\{z_N^{\text{rec}}, z_{N-1}^{\text{rec}}, \dots, z_0^{\text{rec}}\} = \{z^*, 0, \dots, 0\}$, where z^* is a fixed noise map (drawn once and kept fixed during training). Denote by $\tilde{x}_n^{\text{rec}}$ the image generated at the n th scale using these noise maps. Then, for $n < N$,

$$\mathcal{L}_{\text{rec}} = \left\| G_n \left(0, (\tilde{x}_{n+1}^{\text{rec}})^{\uparrow r} \right) - x_n \right\|^2 \quad (2.15)$$

and for $n = N$, $\mathcal{L}_{\text{rec}} = \|G_N(z^*) - x_N\|^2$ is used.

The reconstructed image $\tilde{x}_n^{\text{rec}}$ has another function during training, that is to determine the standard deviation σ_n of the noise z_n at each scale. Specifically, σ_n is considered to be proportional to the root mean square error (RMSE) between $(\tilde{x}_{n+1}^{\text{rec}})^{\uparrow r}$ and x_n , giving an indication of the amount of detail that needs to be added at this scale.

2.4.1.2 Conditional Generative Adversarial Networks (cGANs)

cGANs are an extension of GANs that incorporate additional information (such as labels, images, segmentation maps or text) as an external condition to guide the data generation process (MIRZA, 2014). The objective function of cGANs is defined as:

$$\min_G \max_D V(D, G) = \mathbb{E}_{x, y \sim p_{\text{data}}(x, y)} [\log D(x, y)] + \mathbb{E}_{z \sim p_z(z), p_{\text{data}}(y)} [\log(1 - D(G(z, y), y))] \quad (2.16)$$

where:

- $G(z, y)$: The output generated by the generating network conditional on y
- $D(x, y)$: Probability assigned by the discriminator that x is real, given the context y
- y : Conditional information, such as a label or an input image.

The cGANs work based on an adversarial approach with the inclusion of an additional condition, which guides both the generator and the discriminator in their respective tasks (ALQAH-TANI; KAVAKLI-THORNE; KUMAR, 2021). This condition can represent a wide variety of information, such as class labels, input images, textual descriptions, segmentation maps, among others. This approach allows cGANs to generate more specific data aligned with the features imposed by the condition (GAUTHIER, 2014).

In this context, *the generator* is a network that takes as input a random noise vector (z) and the condition (y) to produce synthetic data ($G(z, y)$). Its object is to generate data that are not only convincingly similar to the real data, but also match the given condition (MIRZA, 2014). For example, if the condition represents a class label, the generator will learn to produce samples that belong to that specific class, while, *the discriminator* is a network that receives both real and generated data along with their respective conditions ($G(z, y), y$) and learns to

distinguish between them, its purpose is to assign a high probability to the real pairs and a low probability to the generated pairs, then, the discriminator acts as a critic that evaluates whether the samples produced by the generator are realistic enough and consistent with the condition (MIRZA, 2014).

The training of a cGAN is structured as a competitive game between the generator and the discriminator:

- The generator tries to minimize the discriminator's ability to identify synthetic data, improving the quality and consistency of its outputs.
- The discriminator aims to maximize its ability to differentiate between real and generated data by continuously refining its predictions (SAXENA; CAO, 2021).

The balance between the two networks is reached when the generator produces data that the discriminator cannot distinguish from real data, obtaining a representation consistent with the condition (PATEL; WANG; MAO, 2020).

cGANs represent a significant advance in the controlled generation of data, because it allows the integration of previous knowledge or specific characteristics in the generation process, making cGANs especially useful in scenarios where it is necessary to guarantee semantic coherence between input and output (DOUZAS; BACAO, 2018).

2.5 MULTIDIMENSIONAL SCALING MODELING (MDS).

MDS is a family of multivariate statistical algorithms designed to find a configuration of n points in a low-dimensional k -space—typically 2D or 3D—such that the Euclidean distances d_{ij} between these points approximate the original high-dimensional dissimilarities δ_{ij} .

2.5.1 Classical Multidimensional Scaling (CMDS)

A fundamental branch of this framework is Classical Multidimensional Scaling (CMDS), frequently termed Principal Coordinate Analysis (PCoA). Drawing on the foundational work of (TORGERSON, 1952) and (GOWER, 1966), CMDS assumes that the initial dissimilarities are Euclidean distances. The method proceeds by "double-centering" a distance matrix D to construct a Gram matrix B ; an eigen-decomposition of B is then performed, allowing the low-dimensional coordinates to be derived directly from the top k eigenvectors and their corresponding eigenvalues.

2.5.2 Metric Multidimensional Scaling (mMDS)

While CMDS offers a direct analytical solution, Metric Multidimensional Scaling (mMDS) approaches the mapping as an iterative optimization task. Formalized by (KRUSKAL, 1964),

mMDS seeks to minimize a specific loss function known as "Stress," which is defined mathematically as $\text{Stress} = \sum_{i < j} w_{ij} (f(\delta_{ij}) - d_{ij}(X))^2$. Through this optimization mechanism, the algorithm iteratively adjusts the positions of points in the target plane to reduce the mismatch between the "true" high-dimensional distances and their projected 2D counterparts.

2.5.3 Non-Metric Multidimensional Scaling (NMDS)

In scenarios where the absolute values of the dissimilarities are less significant than their relative rank order, Non-Metric Multidimensional Scaling (NMDS) provides a specialized alternative. Proposed by (SHEPARD, 1962), this variation utilizes isotonic regression to find a monotonic transformation of the data. This ensures that the final spatial configuration preserves the ordinal relationships of the original data points rather than their exact numeric distances, making it highly effective for qualitative or non-linear datasets.

3 RELATED WORKS

In reservoir characterization, the generation of facies is fundamental to understanding the spatial distribution of lithologies, depositional environments, and petrophysical properties in the subsurface. The accurate modeling of facies contributes to the estimation of heterogeneity, connectivity, porosity, permeability, and fluid saturation of the reservoir, directly influencing reserve evaluation, development planning, and decision-making in exploration and production. As well data provide sparse and one-dimensional observations along boreholes, the generation of facies realizations that respect geological concepts and conditioning data remains a central challenge in subsurface modeling (SELLEY, 1998; DEMBICKI, JR., 2017).

In this context, this study adopted a Systematic Literature Review (SLR) methodology to identify, select, and analyze state-of-the-art deep learning approaches—especially GAN-based methods—applied to the generation, reconstruction, and inversion of geological facies. The review followed this structured protocol. First, a set of keywords was defined through iterative refinement. These included combinations of: “geological facies modeling,” “reservoir characterization,” “deep learning,” “Generative Adversarial Networks,” “GAN,” “cGAN,” “conditional simulation,” “well logs,” and “subsurface modeling.”

The searches were conducted using the main scientific databases used in geoscience and machine learning research: Scopus, Web of Science, IEEE Xplore, ScienceDirect, Springer-Link, and Google Scholar (for complementary retrieval). To ensure coverage of recent advances in deep generative modeling, the temporal scope was limited to publications from 2015 onward, which coincided with the introduction of GAN architectures.

The selection of studies followed two phases. In the initial phase, titles and abstracts were evaluated based on predefined inclusion criteria: (i) the study applies deep learning, GAN architectures, or generative models related to geological, geophysical, or reservoir modeling problems; (ii) the work address facies modeling or pattern generation relevant to subsurface characterization; and (iii) methodological details are provided to enable technical evaluation. The exclusion criteria involved: (i) theoretical articles on GANs without geoscientific application; (ii) studies focused on the synthesis of non-geological images; and (iii) works without reproducible or clearly described methodologies.

This systematic search returned studies covering advances in GANs and methodological advances in inverse problems. To analyze the conceptual influence and evolution of the research, a citation network was constructed from the selected publications (Fig. 5). The resulting figure shows a trajectory that connects some contributions in facies scenario generation with work applying GANs.

The clustering of studies published after 2020 reflects the growth of research that integrates deep generative modeling with subsurface imaging. Based on these observed connections, the studies most closely aligned with the objectives of this research are summarized below, emphasizing their methodological contributions and relevance to the present work.

A relevant study is the work of (RADFORD, 2015) that introduced Deep Convolutional

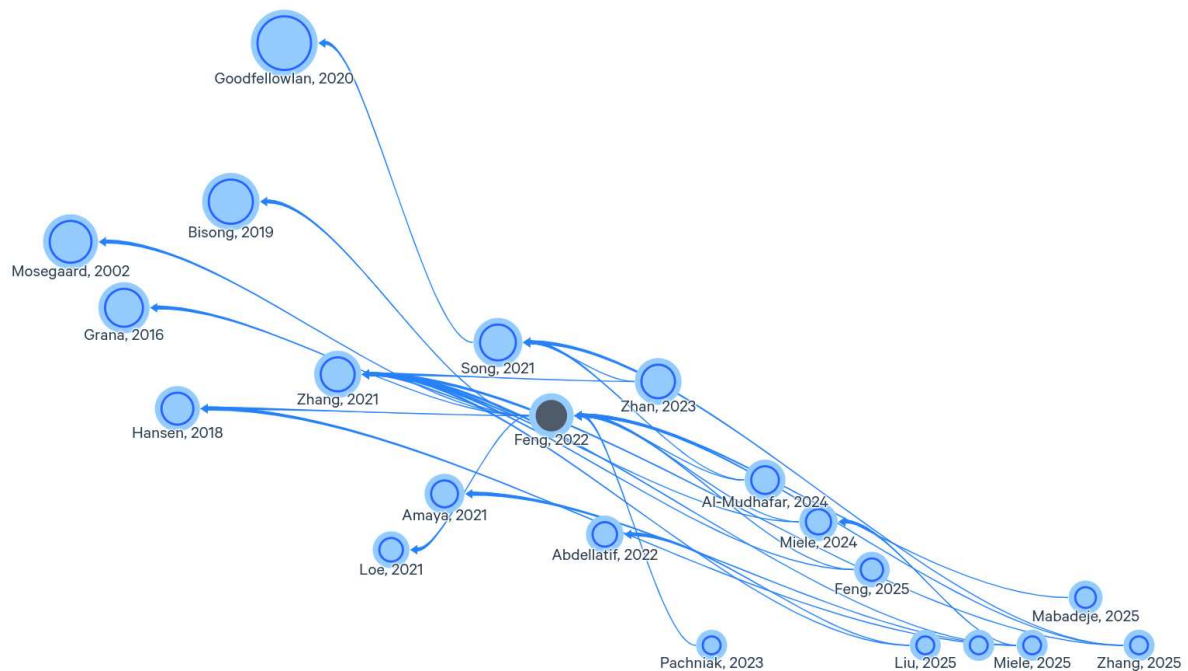


Figure 5 – Citation-based network of the principal studies identified in the systematic search, illustrating the relationships between foundational works (e.g. GAN and geoscience inversion literature) and recent applications involving deep generative models in subsurface characterization.

Generative Adversarial Networks (DCGANs) that improves training stability and the quality of learned representations. DCGANs have influenced the use of GANs in geosciences due to their ability to generate detailed synthetic data. Although focused on other applications, this work laid the foundation for the use of DCGANs in geosciences, enabling the creation of high-quality data in areas with limited datasets, such as well logs.

Similarly, (MOSSER et al., 2018) presented a domain transfer technique for solving direct modeling and seismic inversion problems. They used deep convolutional generative adversarial networks (DCGANs) and a cycle-consistent GAN architecture that learns two functions: from geological model to seismic domain and vice versa. Unlike traditional inversion methods based on wave equations (which are costly and dependent on initial conditions), the method learns from synthetic sets and manages to generalize even in the presence of noise or artifacts in the data. The methodology was tested with synthetic data and the Marmousi2 dataset¹, showing robustness and very fast execution times (seconds on GPU). The authors demonstrated that style transfer and cycle-consistent methods can be effectively applied in seismic inversion, reducing computational cost and preserving geological structures.

(AZEVEDO et al., 2020) explored the use of GANs as a technique for stochastic reconstruction of subsurface models. The central problem they addressed is that, in geostatistical seismic inversion processes, areas with low convergence and high uncertainty often arise, limiting the reliability of the models obtained. To address this difficulty, the authors proposed training a GAN with reference geostatistical realizations so that the network learns the spa-

¹ <http://www.agl.uh.edu/downloads/downloads.htm>

tial structure implicit in the data. Once trained, the GAN is capable of reconstructing discrete properties, such as facies, and continuous properties, such as P-wave velocity, in regions of the model where uncertainty is greater, and can work both with and without additional condition data. The results showed that the reconstructed models adequately preserved statistical moments, covariances, and non-stationary structures, achieving performance comparable to that of classical multipoint simulation methods. This study demonstrated that GANs can be a valuable tool as a post-processing step in seismic inversion, as they improve the representation of areas with greater uncertainty, providing greater robustness to subsurface characterization.

(KIM; BYUN, 2021) presented the challenge of well data imbalance, since some facies classes (such as wet sand) are abundant, while others (such as gas sand) are extremely scarce. This imbalance impairs the performance of machine learning models, which tend to favor majority classes. To address this problem, the study proposed the use of CycleGANs to generate synthetic data that expands minority classes and balances the training set. An iterative synthetic data selection algorithm was also developed, which defines which classes should be augmented and which generated samples should be included, avoiding both noise and overfitting. The method was validated with field data from the Vincent field (Western Australia), showing that the proposed strategy significantly improved facies classification and produced more consistent predictions on surface seismic data. This study demonstrated that the combination of CycleGANs with synthetic data selection criteria is effective in overcoming imbalance in facies classification, increasing the robustness and physical validity of predictions, and represents a promising path for applications in limited data scenarios.

(PAN; TORRES-VERDÍN; PYRCZ, 2021) presented Stochastic Pix2pix, an extension of the pix2pix (cGAN) model designed to generate multiple and diverse realizations of river reservoir models conditioned on seismic and well data. Unlike the classic pix2pix, which tends to produce results with little variability. This approach incorporated a mean square error term in the loss function that measures the fit between the actual conditioning data and the generated data, promoting diversity while respecting the available information. Training is performed with images produced by a rule-based channel simulator, which allows learning the migration and geometry of fluvial facies. The results showed that the method reproduces short- and long-range geological patterns, controls geometric parameters (such as channel width) and successfully conditions the models to field data, avoiding mode collapse problems.

(SONG; MUKERJI; HOU, 2021) presented an improved architecture of progressive adversarial generative networks for the conditional simulation of geological facies. The problem it addresses is that the so-called “post-GAN” approaches require costly searches for latent vectors each time conditioning data is modified, which limits their efficiency and applicability. To overcome this difficulty, GANSim introduced a conditional loss function that forces the generator to directly honor well data or global constraints, such as facies proportions, and also adapts the architecture to receive condition data as input from the start of the generation process. This removed the additional adjustment steps required in other approaches. The results showed that GANSim is capable of generating high-quality realizations conditioned on both global and local

data, or even a combination of both, while maintaining geological consistency and diversity in the simulations. This work provided a flexible and efficient framework for conditional stochastic facies simulation, optimizing the use of GANs in geological modeling applications.

Furthermore, (LIU et al., 2022) investigated the use of RNNs, in particular the bidirectional long short-term memory (Bi-LSTM) variant, for seismic characterization of reservoirs. The objective is to estimate elastic and petrophysical properties, such as velocities, density, porosity, fluid saturation, and lithological facies, from seismic and well data. Traditionally, this task is formulated as an inverse problem based on physical models, which requires calibration, preprocessing, and high computational cost. The proposed approach is data-driven, in which Bi-LSTM directly learns the relationships between seismic traces, which are temporal signals, and reservoir properties, capturing temporal dependencies throughout the series. Two case studies were conducted: the first, with the Stanford VI-E synthetic model (LEE; MUKERJI, 2012), showed that Bi-LSTM was able to predict elastic and petrophysical properties and classify facies with high correlation to the true model, albeit with a loss of high-frequency details; the second, with the Marmousi2 model <http://www.agl.uh.edu/downloads/downloads.htm>, compared Bi-LSTM to the traditional Bayesian Linearized AVO Inversion (BLI) method, revealing that the neural network obtained more accurate results, recovered more details, and was computationally more efficient. The main limitation observed is the need for large training sets, which are not readily available in the early stages of exploration, in addition to the challenge of dealing with seismic imperfections in real data. They concluded that RNNs, and in particular Bi-LSTMs, have great potential to automate and strengthen the seismic characterization process of reservoirs, reducing dependence on physical models and manual steps.

(LIU et al., 2023) proposed that traditional geostatistical methods present difficulties in reproducing complex geological structures due to non-stationarity and high computational cost. Although SinGAN can help in this context, its standard application suffers from training instability, high resource consumption, and the need to manually select the number of stages. To improve this, the authors suggest an upgraded SinGAN that uses spectral normalization to make the discriminator training more stable, a spatial and channel attention mechanism that focuses learning on the most important features, and an adaptive stage selection strategy that automatically decides the number and size of scales, cutting down on manual work. The method is based on multi-stage training from coarse to fine, which preserves both global information and local details, and generates categorical (facies) and continuous (porosity) models from a single training image. The results show that this approach produces diverse, high-quality realizations even in non-stationary geological contexts, outperforming traditional SinGAN in the reproduction of variograms, connectivity functions, and probability distributions. Furthermore, it is more stable, faster, and less memory-intensive. This improved version of SinGAN is a robust tool for unconditional stochastic simulation of geological architectures, capable of handling complexity and non-stationarity without the need for large databases.

(ANTUNES; AZEVEDO; SOARES, 2023) presented a generative seismic inversion methodology to predict geological facies models conditioned simultaneously on well and seismic

data. An architecture based on Pix2Pix GAN is used, modified to accept three inputs: seismic data, well logs, and a noise vector (which introduces stochasticity). The generator had a U-net structure to capture both global and local features, while the discriminator differentiates between real and generated facies, also considering the input data. The results, tested on synthetic 2D examples, showed that the model manages to reproduce spatially coherent and geologically plausible facies, as well as allowing multiple realizations conditioned on the same data. They concluded that training a GAN is a viable alternative to geostatistical methods, especially when only well and seismic data are available.

(HU et al., 2023) addressed the challenge of generating sedimentary facies models from sparse well data. To address this, the authors developed cGAN model that incorporates a deep residual generator and a multiscale discriminator. This framework improves the visual quality of the outputs and introduces a novel element: style codes, which are numerical representations of geological properties (such as the type and orientation of river channels). These codes allow the style of the generated images to be explicitly controlled, beyond simply honoring the well data. The system also included a style classifier that helped the generator learn meaningful geological representations and was scalable when new properties were added. To train and validate the model, the authors constructed a set of realistic geological images based on the morphology of the Yellow River in China, along with an automatic program to generate simulated well data. Experiments showed that the method produces varied, high-fidelity realizations capable of reflecting absolute responses to style codes while maintaining a 94.78% match rate with hard data. The results outperform several reference models in visual quality and quantitative metrics. The work proposed a novel approach to conditional modeling of sedimentary facies, where style codes offer flexible and transparent control over the generated realizations. Although it requires manual annotation of styles in the images, the methodology opens the door to more interpretable and adaptable models in real geological applications.

Recently, (MIELE; AZEVEDO; GRANA, 2024) presented an innovative approach to iterative stochastic seismic inversion that combines GANs with spatially adaptive normalization (SPADE). The central problem lies in the fact that predicting geological heterogeneities and rock properties from seismic data faces non-uniqueness of solutions, high computational costs, and the difficulty of integrating realistic geological patterns into models. To address these limitations, the authors integrate a GAN with SPADE into an iterative geostatistical inversion algorithm. The GAN is capable of generating realistic facies distributions conditioned by local probability maps, while sequential geostatistical simulation methods model continuous properties dependent on these facies, such as acoustic impedance. The methodology consists of training the GAN to learn facies patterns and respect probability maps, then iteratively generating facies and property realizations, calculating synthetic seismic data and comparing it with observed data, and finally updating the probability maps to reduce seismic mismatch in each cycle. The results, validated with synthetic and real data, show that the algorithm accurately predicts both facies patterns and acoustic impedance distributions, even when the patterns differ from the training set. The authors obtained an average correlation of 0.91 between simulated

and observed seismic data, although limitations were identified in the prediction of subseismic structures or those with low impedance contrast. This method improves the accuracy and reliability of predictions of multiple subsurface properties by efficiently integrating deep learning with geostatistical techniques in seismic inversion.

(RANAZZI; LUO; SAMPAIO, 2024) addressed one of the main problems of GANs in geosciences: the need for large datasets to train robust and stable models. The challenge arose because the generation of realistic geological models often involves very limited datasets—few realizations or images—which causes overfitting of the discriminator and unreliable results. To solve this problem, the authors proposed three innovations: i. regularization of the discriminator using zero-centered gradient penalty (R1), which stabilizes training without complex hyperparameter adjustments; ii. an adaptive data augmentation (ADA-PD) technique, which dynamically adjusts the probability of applying geometric transformations such as rotations, translations, or zooms using a proportional-derivative controller instead of the classic scheme; and iii. a new metric called Frechet Reservoir Distance (FRD), adapted to the evaluation of single-band reservoir models, unlike traditional metrics such as FID or Inception Score designed for RGB images. The methodology was tested on two types of datasets: one categorical, with discrete facies generated by SNESIM, and another continuous, based on non-Gaussian permeabilities from the UNISIM-II-H benchmark. Different sizes of training sets were evaluated, ranging from 200 samples to tens of thousands, along with various combinations of data augmentations. The results show that GAN-R1 achieved greater stability and better convergence than previous methods such as WGAN-GP, and that ADA-PD allowed networks to be trained with 200 samples, obtaining results comparable to those of training with 80,000 samples in terms of realism and statistical preservation. The most notable improvements were achieved with geometric and blitting transformations, while noise-based or value-altering augmentations were less effective. In addition, models trained with ADA-PD better reproduced both facies proportions and multimodal permeability distributions. The combination of discriminator regularization and adaptive data augmentation provides an effective framework for training GANs with small datasets, opening the door to practical applications in reservoir modeling and subsurface characterization in contexts where data are scarce or costly to obtain.

(LU et al., 2024) approaches the problem that modeling sedimentary facies using traditional GANs requires large datasets, while in practice only a few training images are available. To solve this limitation, the authors proposed CSinGAN, a variant of SinGAN trained from a single image. Different from classical approaches, CSinGAN does not use conditional data during the training phase, but incorporates conditions (such as well data) into the generation phase using image editing techniques applied to intermediate outputs. The method is based on a multiscale pyramid of GANs from coarse to fine, and during generation, the values at well locations are overwritten with conditional data; subsequently, the network iteratively adjusts the results until geological realism is restored. The results showed that the model performs accurate conditional simulations that respect well data, maintain statistical properties (variograms and connectivity functions), and outperform classical multipoint geostatistical algorithms (Di-

rect Sampling, DisPat) in similarity to training images, as measured by multiscale Wasserstein distance. cSinGAN offers an efficient and low-cost approach to conditional geological modeling when only a training image is available.

(FENG; GRANA; MOSEGAARD, 2025) proposed an innovative approach to simulate geological facies using multiscale GANs trained directly from a single training image, without the need to generate multiple synthetic data using traditional geostatistical methods. The scheme progressively learns from coarse to fine scales, capturing both global structures and local details. In addition, it introduced the gradual deformation method (GDM) to condition the realizations to observed data, thus ensuring that the resulting models respect field measurements such as wells or seismic data. Application cases in the Lena River delta (2D) and the Descalvado aquifer (3D) demonstrated that the method generates diverse, realistic, and geologically consistent realizations, maintaining a balance between variability and reproducibility of the structures.

To consolidate the main methodological characteristics, datasets, and results of the studies most relevant to this research, Table 1 presents a comparative summary of the GAN-based approaches identified in the review. This table highlights the diversity of architectures, the range of geoscientific applications, and the strengths and limitations that inform the design decisions adopted in the present study.

These studies demonstrate that GANs are capable of recreating complex facies map distributions in the subsurface (SHAHAM; DEKEL; MICHAELI, 2019), but they do not ensure that the generated geological data respect the locations or coordinates where facies are already known. Furthermore, studies focus on their use with facies map data. This demonstrates the need to develop methods that effectively integrate conditioning information and the use of 2D vertical section facies datasets to increase the reliability and usefulness of geologic models.

Table 1 – Summary of GAN-based and deep learning methods applied to geoscientific modeling, seismic inversion, and subsurface characterization.

Reference	GAN Model / Architecture	Application / Dataset	Conditional Data	Positive Aspects	Negative Aspects
(RADFORD, 2015)	DCGAN	General computer vision; foundational for later geoscience work	No	Improved stability and representation learning; influential architecture	Not designed for geoscience; no conditioning
(MOSSER et al., 2018)	DCGAN + CycleGAN (domain transfer)	Forward modeling and seismic inversion; Marmousi2	Yes (implicit)	Very fast; robust to noise; preserves geological structures	Domain-transfer approach may generalize poorly to real data
(AZEVEDO et al., 2020)	Standard GAN for stochastic reconstruction	Post-inversion reconstruction of facies and continuous properties	Optional	Preserves statistics, covariances, and non-stationarity; robust in high-uncertainty areas	Requires reference geostatistical realizations; does not replace MPS
(KIM; BYUN, 2021)	CycleGAN + synthetic data selection algorithm	Facies classification in the Vincent field (Australia)	Yes	Mitigates class imbalance; improves classification consistency	Iterative selection may introduce noise; depends on thresholding
(PAN; TORRES-VERDÍN; PYRCZ, 2021)	Stochastic Pix2Pix (cGAN with MSE constraint)	Fluvial reservoir models conditioned to seismic and well data	Yes	Generates diverse realizations; controls geometry; avoids mode collapse	Requires rule-based channel simulator for training
(SONG; MUKERJI; HOU, 2021)	GANSim (progressive GAN with conditional loss)	Conditional facies simulation	Yes	Avoids latent vector search; efficient conditional generation	Architecture may require extensive tuning
(LIU et al., 2022)	Bi-LSTM (RNN, not GAN)	Elastic and petrophysical property inversion; Stanford VI-E, Marmousi2	Yes	High accuracy; outperforms BLI; computationally efficient	Requires large training sets; loses high-frequency details
(LIU et al., 2023)	Improved SinGAN (spectral norm, attention, adaptive stages)	Unconditional simulation from a single training image	No	Stable, faster, memory-efficient; preserves TI statistics and connectivity	Still multi-stage; lacks conditioning to real data
(ANTUNES; AZEVEDO; SOARES, 2023)	Modified Pix2Pix (inputs: seismic, well logs, noise)	Generative seismic inversion (2D synthetic cases)	Yes	Multiple realizations; captures global and local geological features	Evaluated only on synthetic data
(HU et al., 2023)	cGAN + Residual generator + multi-scale discriminator + style codes	Sedimentary facies modeling (Yelow River)	Yes	Explicit control via style codes; 94.78% match with hard data	Requires manual style annotation; specialized dataset
(MIELE; AZEVEDO; GRANA, 2024)	GAN + SPADE integrated into iterative inversion	Seismic inversion (synthetic and real data)	Yes	Correlation 0.91; integrates DL with geostatistics; multi-property prediction	Less accurate for sub-seismic structures; iterative process is costly
(RANAZZI; LUO; SAMPAIO, 2024)	GAN with R1 regularization + ADA-PD augmentation	Facies (SNESIM) and permeability (UNISIM-II-H) datasets	No	Stable with very small datasets; introduces FRD metric; matches performance with 200 samples	Requires tuning ADA-PD; not focused on conditioning
(LU et al., 2024)	CSinGAN (SinGAN with conditional editing at generation time)	Conditional simulation from a single TI; hard data from wells	Yes (during generation)	Works with a single image; preserves TI structure; outperforms DS/DisPat	Conditioning is iterative; may produce artifacts
(FENG; GRANA; MOSEGAARD, 2025)	Multiscale GAN + Gradual Deformation Method (GDM)	Lena River (2D) and Descalvado aquifer (3D)	Yes	Realistic and diverse realizations; balanced variability and conditioning; uses one TI	GDM adds complexity; higher cost for 3D cases

4 METHODOLOGY

This study is focused in the application of a cGAN for facies scenario generation. The methodology follows the workflow shown in Fig. 6 and comprises several interconnected stages.

The process begins with data collection, which involves obtaining the geological information and input images used in the study. The data then undergoes a pre-processing stage, in which operations such as standardization, normalization, cropping, and data format adjustment are performed to ensure consistency and quality in model training. From this processed data, images are generated, forming the visual basis necessary to represent the spatial patterns of the facies.

These images make up the training dataset, highlighted in the figure, in which data labeling is performed. Next, the labeled dataset is used in the model training stage. After training, the model is employed in the inference phase, in which new facies scenarios are generated synthetically, respecting the imposed conditions and maintaining geological consistency.

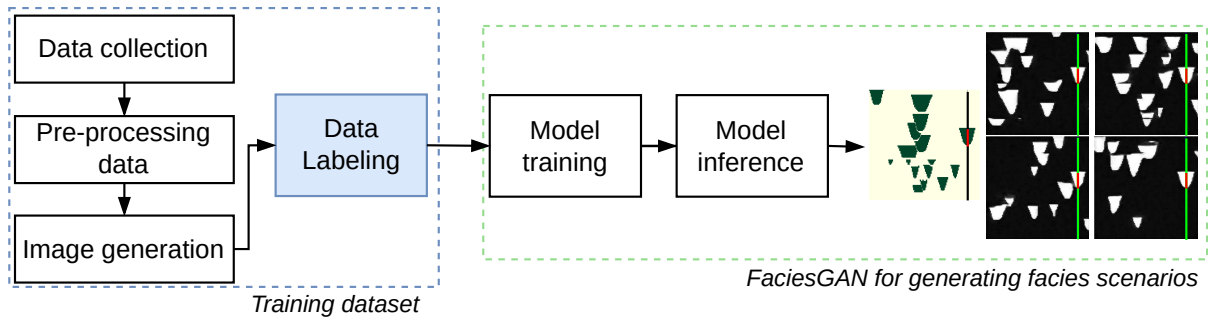


Figure 6 – General proposed methodology for generating facies scenarios.

The interconnected stages are describe below.

4.1 DATA COLLECTION AND PRE-PROCESSING

The dataset was selected based on its relevance to the study objectives. The data was acquired from a public and accessible repository (GitHub) (LEE; MUKERJI, 2012). The format of the raw data was verified to ensure its compatibility with the manipulation and analysis tools (software) that would be used in the study (.mat). The pre-processing step was designed to process the raw data into a clean and structured format, which would be optimized for model training. The original dataset contained several classes or categories, and an initial exploratory analysis was performed to identify and characterize the main classes. To simplify the classification problem and focus on the central interest of the study, the original categories were grouped or reclassified. The classes were simplified into binary categories (e.g., distinction between “reservoir” and “non-reservoir”), prioritizing the categories that represent the phenomenon of interest. This reduction in terms of categories facilitates the interpretation of the model and the analytical focus on the groups of greatest relevance to the research objective.

4.2 IMAGE GENERATION AND DATA LABELING

The goal was to transform the 3D reference model or structured dataset into 2D visual representations, which serve as training images for the neural network. An image processing and visualization environment was set up, using appropriate runtime environment and libraries (Python image processing and visualization library), to create visual representations of the data and facilitate the conversion of tabular data into images. The original model was defined on a grid with specific dimensions and the physical size of the model was calculated based on the cell dimensions. To create 2D images, slices were extracted from the 3D model along a Y-axis. Of the total vertical slices generated, each slice corresponds to a cell position along the slicing axis. Each slice represented a physical section of the model, focusing on a specific layer or total depth. Each extracted physical section was resampled and interpolated to a fixed and uniform pixel array size (e.g., 256×256 pixels). This resampling established the final image resolution in terms of meters per pixel in the horizontal and vertical directions.

The data labeling process ensured that the generated images were suitable for training the cGAN model by providing the condition for each image. Each 2D image was associated with its depth condition. This variable works as a control input for the cGAN generator, allowing it to learn how to produce the right distribution of facies at different vertical levels. Also, vertical polygons (segmentation masks) were designed to delimit the different facies classes within the 2D section. This annotation is important because it represents the conditioning data that the cGAN must learn to respect and replicate. Rather than generate a random image, the cGAN uses these facies/depth annotations to generate new images that are conditioned to follow the same layout and spatial distribution defined by the external label. This ensures that the synthesized images are geologically plausible and consistent with the depth profile and vertical organization of the facies.

4.3 FACIESGAN FOR GENERATING FACIES SCENARIOS

The proposed cGAN called FaciesGAN implements a multistage cGAN. Its architecture is designed to generate geologically realistic facies realizations conditioned on well data.

FaciesGAN is an adaptation of SinGAN (SHAHAM; DEKEL; MICHAELI, 2019). SinGAN is a generative model able to be learned from a single natural image. SinGAN contains a pyramid of fully convolutional GANs. Each responsible for learning the distribution of patches at a different scale of the image. This allows to generate new samples of arbitrary size and proportion. This has significant variability but retain the overall structure and fine textures of the training image (SHAHAM; DEKEL; MICHAELI, 2019).

FaciesGAN model is structured as a hierarchy of generators and discriminators operating at progressively higher resolutions, as shown in Fig. 7. The process begins with a low-resolution generator that produces an initial facies image conditioned on well information (e.g., facies at well locations), and subsequent stages refine this output by adding progressively finer geolog-

ical details. Each generator stage is paired with a corresponding discriminator that evaluates the realism of the generated facies at its specific resolution, while still enforcing consistency with the conditioning data. Conditioning is maintained throughout all stages of the generation pipeline, ensuring that the final high-resolution outputs honor well constraints. This progressive refinement strategy allows the model to capture both large-scale geological structures and small-scale heterogeneities, resulting in high-quality, data-consistent facies simulations.

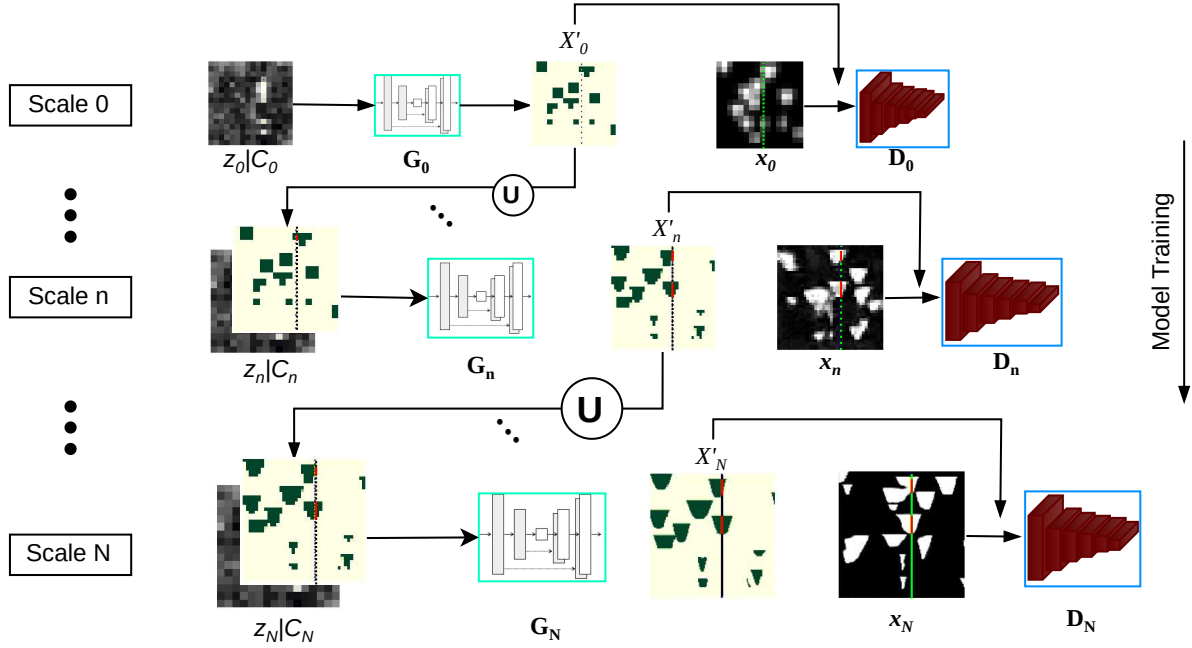


Figure 7 – Schematic representation of the proposed cGAN for generating facies scenarios. The generator and discriminator are trained from coarse to fine scale (0 to N). U corresponds the upsampling.

The pyramid of generator $\{G_0, \dots, G_n, \dots, G_N\}$ is a multi-scale fully convolutional architecture, as shown in Fig. 8. At each scale n , the generator receives as input three components: (i) the upsampled output from the previous coarser scale $U(x'_{n-1})$, which carries large-scale geological context; (ii) a stochastic noise map z_n , which introduces variability; and (iii) conditioning information derived from well data, encoded as additional input channels (masks at well locations). These are passed through a series of 2D convolutional layers with LeakyReLU to produce a residual output. This is added to the up-sampled input to produce the new 2D section facies map x'_n . Each generator G_n is trained to learn the internal structure of the training images at different scales. G_n finer details from the training images are learned sequentially (FENG; GRANA; MOSEGAARD, 2025).

By explicitly injecting the conditioning data at every scale, the generator is forced to honor well constraints throughout the entire generation process, rather than only at the final resolution. Each generator G_n is trained to learn the internal structure of the training image at its corresponding scale, enabling the sequential acquisition of geological features from coarse structural patterns to fine-scale heterogeneities (FENG; GRANA; MOSEGAARD, 2025).

The discriminators $\{D_0, \dots, D_n, \dots, D_N\}$ are implemented as a convolutional PatchGAN classifier, which assesses the realism of local image patches rather than making a single global

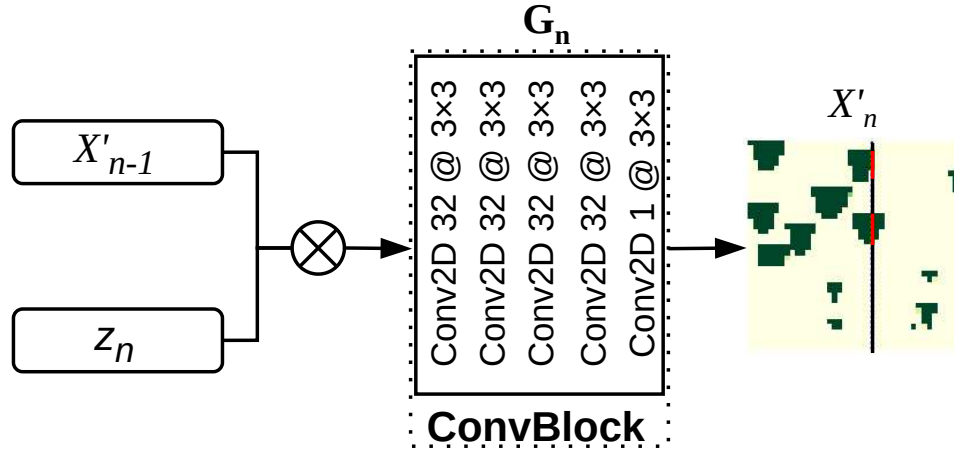


Figure 8 – Network architecture at the n^{th} scale level for the generator.

prediction (Fig. 9). The architecture consists of a sequence of convolutional blocks, each comprising a two-dimensional convolutional layer followed by a LeakyReLU activation function (SHAHAM; DEKEL; MICHAELI, 2019). The number of feature channels is progressively reduced across layers (e.g., from 64 to 1), enabling hierarchical feature extraction at multiple spatial resolutions. Notably, normalization layers (e.g., Batch Normalization) are applied, in order to preserve the raw feature dynamics and stabilize training. The final output is a single-channel feature map in which each spatial location corresponds to the discriminator's assessment of a specific image patch being real or synthetic (FENG; GRANA; MOSEGAARD, 2025).

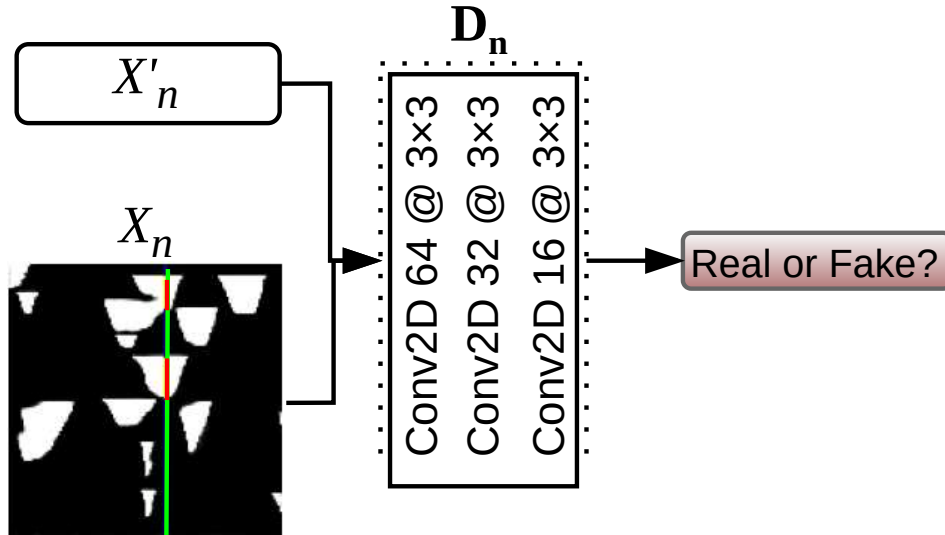


Figure 9 – Network architecture at the n^{th} scale level for the discriminator.

At the n^{th} scale level, an adversarial training process is performed separately: the generator G_n tries to generate fake images x_n to fool the discriminator D_n . The discriminator D_n attempts to distinguish the real image x_n from the fake x'_n (FENG; GRANA; MOSEGAARD, 2025). This multiscale approach captures the large-scale structures present in the geologic models of interest (TRAN, 1994). The formulation for generating an image sample at the n^{th} level is expressed as

the following equation:

$$x'_n = \begin{cases} G_0(z_0), & n = 0, \\ G_n(z_n, U(x'_{n-1})), & 0 < n \leq N \end{cases} \quad (4.1)$$

where U represents the upsampling, based on interpolators in the two-dimensional case.

The loss function at the n^{th} scale level for G_n and D_n is formulated as (SHAHAM; DEKEL; MICHAELI, 2019):

$$\min_{G_n} \max_{D_n} \mathcal{L}(G_n, D_n) = \mathcal{L}_{adv}(G_n, D_n) + \alpha \mathcal{L}_{rec}(G_n) \quad (4.2)$$

which $\mathcal{L}_{adv}$ is the adversarial loss for penalizing the distribution distance between down-sampled image x_n and generated image sample x'_n , α is a weighting factor to balance the two loss functions, and $\mathcal{L}_{rec}$ is the reconstruction loss to ensure that x_n can be reproduced given a specific set of a random noise map.

where $\mathcal{L}_{adv}$ penalizes the distance between the distributions of real images x_n and generated samples x'_n , α is a weighting factor, and $\mathcal{L}_{rec}$ is a reconstruction loss that enforces the ability of the generator to reproduce the reference image given a fixed noise realization.

The discriminator D_n is optimized using the Wasserstein loss with gradient penalty, which provides a smooth and meaningful measure of distributional discrepancy and ensures Lipschitz continuity. This formulation is particularly important for facies modeling, where categorical and multimodal distributions can lead to unstable training under standard GAN losses. The discriminator loss is formulated as:

$$\mathcal{L}_D^{(n)} = -\mathbb{E}_{x_n \sim p_{\text{data}}} [D_n(x_n)] + \mathbb{E}_{z_n} \left[D_n \left(G_n(z_n, \tilde{x}_{n+1}^\uparrow) \right) \right] + \lambda_{gp} \mathbb{E}_{\hat{x}_n} \left[(\|\nabla_{\hat{x}_n} D_n(\hat{x}_n)\|_2 - 1)^2 \right] \quad (4.3)$$

where x_n denotes a real image at scale n , $G_n(z_n, \tilde{x}_{n+1}^\uparrow)$ is the generated image conditioned on noise z_n and the upsampled output from the next coarser scale, and $\hat{x}_n = \varepsilon x_n + (1 - \varepsilon) x'_n$ with $\varepsilon \sim \mathcal{U}[0, 1]$ is the interpolated sample used for the gradient penalty.

The generator G_n is trained with two complementary objectives: (i) an adversarial loss that encourages generated samples to be indistinguishable from real ones at scale n , and (ii) a reconstruction loss that ensures faithful reproduction of the reference image when a fixed noise map z_n^* is used. The generator loss is:

$$\mathcal{L}_G^{(n)} = -\mathbb{E}_{z_n} \left[D_n \left(G_n(z_n, \tilde{x}_{n+1}^\uparrow) \right) \right] + \alpha_n \mathcal{L}_{rec}^{(n)} \quad (4.4)$$

with the reconstruction loss defined as:

$$\mathcal{L}_{rec}^{(n)} = \left\| x_n - G_n(z_n^*, \tilde{x}_{n+1}^\uparrow) \right\|_2^2 \quad (4.5)$$

Here, α_n is a scale-dependent weighting factor balancing adversarial and reconstruction objectives, and $\tilde{x}_{n+1}^\uparrow$ denotes the generated sample from scale $n+1$, upsampled to match the resolution of scale n .

This hierarchical optimization scheme allows the generator to progressively capture global structure at coarse scales and fine details at higher resolutions, while the reconstruction term stabilizes training and preserves fidelity.

4.4 ALGORITHM AND IMPLEMENTATION

FaciesGAN is an architecture designed to generate geologically consistent facies images from a multi-scale noise pyramid. It uses an improved adversarial training framework. FaciesGAN's training loop is composed of two alternating main stages, which involve the update of the discriminator and generator parameters. The algorithm incorporates additional mechanisms, such as gradient penalty, reconstruction, and masking losses, that contribute to improving training stability and fidelity of the generated images. The algorithm with the core procedure for training the FaciesGAN model at a single resolution scale is presented in Algorithm 1.

The FaciesGAN training algorithm works by balancing the generation of new data with the preservation of pre-existing geological constraints. The process begins with the definition of the Conditioning Mask (M), which is a binary matrix of the same size as the image, where the value 1 indicates the presence of known conditional data (such as the exact position of an oil well) and the value 0 indicates empty areas to be filled by the generator. In the geological context, these masks are often structured as vertical polygons, which geometrically represent the vertical continuity of layers or rock bodies (facies) in a cross section, ensuring that the model respects the stratigraphy observed in the field.

In addition, the Reconstruction input (x_{rec_in}) acts as a fidelity anchor; it is a version of the real image (often on a smaller scale) that the model attempts to reconstruct perfectly during training to ensure that the mapping between the latent space and the final image is consistent and not just random. For the dynamics of the functions, the GETNOISE() function is responsible for generating a “noise pyramid,” providing Gaussian noise maps at different resolutions that allow the generator to create textures on both a large scale (main structures) and a small scale (fine details). The stability of this learning is maintained by the CALCULATEGRADIENTPENALTY(D, x_{real}, x_{fake}) function, a regularization technique for WGAN-GP networks that penalizes the discriminator if the norm of its gradients deviates from a unit value, preventing the model from suffering from gradient explosion or mode collapse.

With regard to applied mathematical operators, the symbol $\odot$ represents the Hadamard product, which performs a point-by-point multiplication between matrices; in the algorithm, it is used to isolate the areas of the image that must match the real data ($x_{fake} \odot M$), ensuring that the mask loss (L_{mask}) focuses only where there is conditioning information. The operator $\otimes$, referring to the Kronecker product (or tensor product), is used for dimension expansion or spatial feature mapping, allowing information vectors to be distributed evenly over the height and width dimensions of feature maps during neural layer processing.

Algorithm 1 FaciesGAN Core Training Loop (at a single scale).

Input:

$x_{real} \leftarrow$ Real data
 $x_{cond} \leftarrow$ Conditioning facies image
 $M \leftarrow$ Conditioning mask
 $x_{rec_in} \leftarrow$ Reconstruction input

Models:

$G \leftarrow$ Generator
 $D \leftarrow$ Discriminator

Hyperparameters:

$\lambda_{gp}, \alpha_{rec}, k_d \leftarrow$ Discriminator steps
 $k_g \leftarrow$ Generator steps

/ Step 1: Train Discriminator */*

```

1: for  $j = 1$  to  $k_d$  do
2:   Sample noise pyramid  $Z \leftarrow \text{GETNOISE}()$ 
3:   Generate fake images  $x_{fake} \leftarrow G(Z, x_{cond})$  // Forward pass through Generator
4:    $L_{real} \leftarrow -\mathbb{E}[D(x_{real})]$  // Loss for real data
5:    $L_{fake} \leftarrow \mathbb{E}[D(x_{fake})]$  // Loss for fake data
6:    $L_{gp} \leftarrow \lambda_{gp} \cdot \text{CALCULATEGRADIENTPENALTY}(D, x_{real}, x_{fake})$ 
7:    $L_D \leftarrow L_{real} + L_{fake} + L_{gp}$  // Total Discriminator loss
8:   Update  $D$ 's parameters  $\theta_d$  by ascending the gradient of  $L_D$ 
9: end for
  
```

/ Step 1: Train Generator */*

```

10: for  $j = 1$  to  $k_g$  do
11:   Sample noise pyramid  $Z \leftarrow \text{GETNOISE}()$ 
12:   Generate fake images  $x_{fake} \leftarrow G(Z, x_{cond})$ 
13:    $L_{adv} \leftarrow -\mathbb{E}[D(x_{fake})]$  // Adversarial loss
14:   Sample reconstruction noise  $Z_{rec} \leftarrow \text{GETNOISE}(\text{rec}=\text{True})$ 
15:    $x_{rec} \leftarrow G(Z_{rec}, \text{in\_facie} = x_{rec\_in})$  // Reconstruction pass
16:    $L_{rec} \leftarrow \alpha_{rec} \cdot \text{MSE}(x_{rec}, x_{real})$  // Reconstruction loss
17:    $L_{mask} \leftarrow 100 \cdot \alpha_{rec} \cdot \text{MSE}(x_{fake} \odot M, x_{real} \odot M)$  // Masked loss
18:    $L_G \leftarrow L_{adv} + L_{rec} + L_{mask}$  // Total Generator loss
19:   Update  $G$ 's parameters  $\theta_g$  by descending the gradient of  $L_G$ 
20: end for
  
```

The training hyperparameters were chosen based on the original WGAN-GP and SinGAN models, with empirical adjustments for our specific application. The gradient penalty weight λ_{gp} (referred to as λ in WGAN-GP) was set to 0.1, a value we found stabilized training effectively for the facies data (in contrast to the $\lambda = 10$ used in the original WGAN-GP). The reconstruction weight α_{rec} (referred to as α in Equation 4.2 and α_{in} in Equation 4.4) was set to 10. This value, which is common in SinGAN-based models, was found to be the best balance because it guaranteed that the images that generated remained faithful to the geological structure's underneath, especially at coarser scales. These selected values provided a robust balance

between the adversarial loss and the reconstruction loss, and that's the reason the results are so high-quality and stable.

CODE AVAILABILITY

The code used to train the model, generate synthetic realizations, and evaluate the results presented in this study was developed by the author. To ensure transparency, reproducibility, and reuse of the proposed methodology, the implementation is made available in the FaciesGAN repository.

Hardware requirements: No hardware requirement.

Program language: Python

Software required: Python 3.1x

Program size: 62.9 MB

5 EXPERIMENTS AND RESULTS

This chapter presents the results obtained from the workflow established for the generation and evaluation of facies scenarios. First, the results from the data collection and preprocessing stages are presented, as well as the image generation and labeling processes, which made it possible to structure a coherent and adequate dataset for training the generative models. Subsequently, the results of the two main experiments designed to evaluate the model's performance are detailed.

The first experiment is the global model evaluation, where the consistency, spatial continuity, and statistical representativeness of the scenarios produced by FaciesGAN are analyzed. To provide a robust comparative framework, the same data were used in an implementation of the MPS method, specifically Single Normal Equation Simulation (SNESIM). The comparison between the two methodologies included visual inspection, spatial continuity analysis, and evaluation of facies proportion histograms, with the goal of comparing the ability of each technique to reproduce the geological patterns observed in the original data.

The second experiment focuses on well-specific conditional evaluation, testing the ability of the model to generate realizations that honor local constraints imposed by actual well information. This analysis allows us to evaluate the consistency of the model under specific conditions and its potential applicability in geological modeling contexts where the available information is heterogeneous and localized.

Together, these results allow for a qualitative and quantitative analysis of the ability of the generative model -FaciesGAN- to reproduce representative facies scenarios that are consistent with the original geological data.

5.1 DATA COLLECTION AND PRE-PROCESSING

Stanford Earth Science Data were chosen and downloaded from the repository *GitHub-Stanford-VI-E* (LEE; MUKERJI, 2012). The database contains data from oil well logs, with detailed samples of the different facies found in the reservoirs. From the dataset, the available facies classes were Floodplain(0), Point bar(1), Channel(2), Boundary(3), and the deltaic system (See Fig. 10a). The dataset focused on meandering channels, prioritizing the facies categories directly related to the interest of the study (reservoir and non-reservoir). The remaining facies were reclassified to simplify the categories into 'reservoir' (channel - 1) and 'non-reservoir' (floodplain, point bar and boundary - 0), as shown in Fig. 10b.

5.2 IMAGE GENERATION AND DATA LABELING

An environment was set using the Python image processing and visualization library to create visual representations of the filtered and categorized facies, and the tabular data were converted into images. The 3D Stanford VI reservoir model was employed for the training

data for the deep learning workflow. This reference model is defined on a $150 \times 200 \times 200$ cell grid, with cell dimensions of 25 m in the horizontal (X and Y) directions and 1 m in the vertical (Z) direction. This results in a total physical size of 3750m (X-axis) $\times$ 5000m (Y-axis) $\times$ 200m (Z-axis/depth). The 200 m vertical thickness is composed of three distinct layers (80 m, 40 m, and 80 m). To create the 2D training images, 200 vertical slices (representing X-Z planes) were extracted, corresponding to one slice for each of the 200 cell positions along the Y-axis. Each slice represents the full horizontal (X-axis) distance of 3750 m and the top 80m layer (Layer 1). Subsequently, this physical section of 3750m $\times$ 80m was resampled to a 256×256 pixel matrix. This process gives the final image a resolution of about 14.65 m/pixel in the horizontal direction and about 0.31 m/pixel in the vertical (depth) direction.

A total of 200 divisions in 2D vertical slices were generated and extracted from the 3D facies model. These are used as training images, with 256×256 pixels, and categorized according to the corresponding facies class (See Fig. 10c). Each image was annotated with the depth condition, which was used as an external label to guide the training. In addition, each vertical polygonal masks delineating the distribution of 2D facies provided to the conditional GAN as spatial conditioning inputs. These masks constrain the generator locally, ensuring that the generated facies respect the boundaries imposed within each section. Conditioning is applied independently at the slice level; therefore, no explicit consistency between slices or volumetrically is imposed during training, and consistency between adjacent slices arises only implicitly from the learned facies patterns, rather than from a 3D constraint.

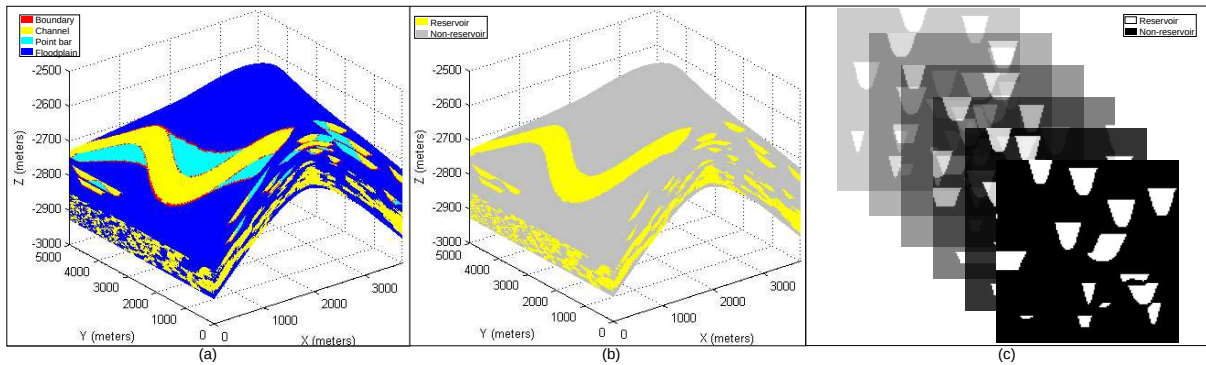


Figure 10 – Stanford Dataset. (a) Multiple sedimentary facies visualization. (b) Facies classification into reservoir (yellow) and non-reservoir (gray). (c) 2D slices examples projection along the depth, differentiating reservoir (white) from non-reservoir (black). Image reproduced and adapted from (LEE; MUKERJI, 2012).

5.3 MODEL TRAINING

FaciesGAN was trained to generate facies scenarios using the Stanford Earth Science dataset. The model was developed with a limited training set of 200 samples. During the inference stage, some samples were analyzed using metrics such as visual inspection, average facies proportion, and multidimensional scaling (MDS) in order to determine consistency and representativeness of the generated scenarios with the original facies.

For a cGAN training, it is essential to have appropriate labels for each image. Label creation process for the images included in the dataset used in the model is shown in Fig. 11. These labels are important because they provide information about the characteristics of each image, allowing the model to learn to generate coherent and realistic images based on specific conditions. In this context, the labels correspond to hard data derived from a simulated well, representing known subsurface information used to condition the facies generation process.

The correctly labeled images were integrated into the dataset and associated specifically with each corresponding image, and the model can use the labels as a conditioner to learn to generate new facies scenarios based on the characteristics and structures defined by the labels. This enables the model to use the labels as conditioning information to learn how to generate new facies scenarios that are consistent with the geological characteristics and structural patterns defined by the conditioning data.

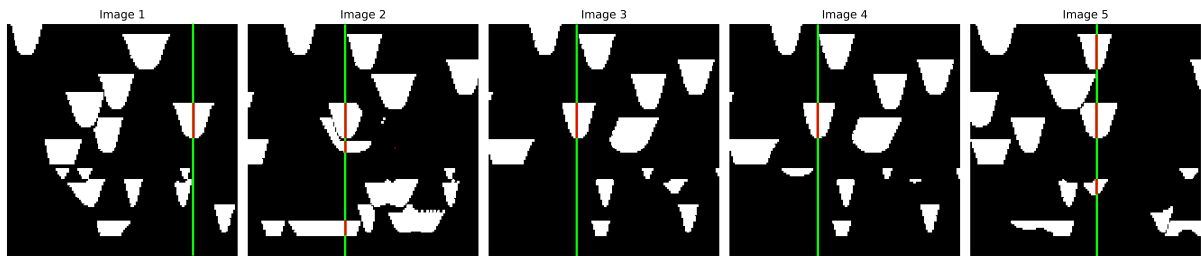


Figure 11 – Conditional label image examples of a 2D fluvial system for cGAN training. Reservoir in white and non-reservoir in black with facies in drilled wells (Conditional trace highlighted in red (reservoir) and green (non-reservoir)). Note: Each image corresponds to a 2D crossline section represented in the pixel domain (256x256 pixels) to a 80 meters deep by 3750 meters wide vertical section.

FaciesGAN was trained for 100 epochs to train per scale and 10 scales, the gradient penalty weight $\lambda_{gp} = 0.1$; and Adam Optimizer with a learning rate of $5e - 5$ and $\beta_1 = 0.5$. The kernel size for 2D filters is 3×3 , with a stride step of 1×1 . The facies size at the coarser scale: 16 pixels for the minimum size and 128 pixels for the maximum size. The model was trained on the complete dataset using a workstation with an Intel(R) Core(TM) i7-8700K CPU @ 3.7GHz (6 cores), an NVIDIA GeForce GTX 1080Ti GPU, and 64 GB of RAM.

5.4 GLOBAL MODEL EVALUATION

In a first test, the model generated 1000 facies scenarios in 20 seconds, and 20 randomly generated conditioning scenarios are shown in Fig. 12. It is observed that the facies configuration of the conditioning trace, highlighted in green, that simulates the function of a real well drilled and analyzed, was closely reproduced in the images generated by FaciesGAN.

The results are promising and show remarkable consistency in reproducing the facies spatial distributions. They provide initial insight into the model's ability to produce images that consistently reflect the expected facies proportions. Visual comparisons with real distributions confirmed that the model is capable of capturing key features of input data and generating consistent variations. Also, the generation time required to generate facies scenarios was short.

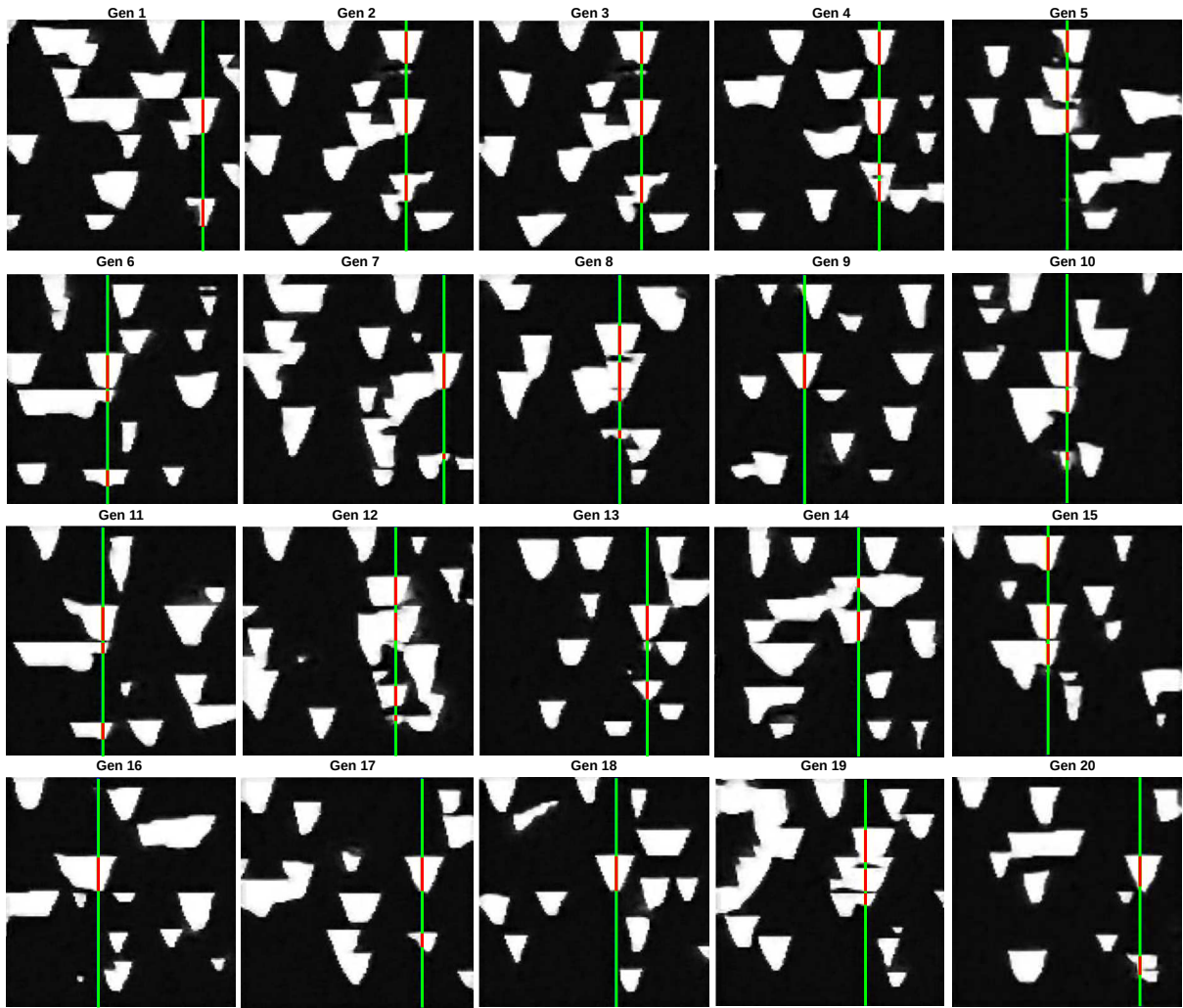


Figure 12 – Twenty randomly selected realizations generated by the proposed cGAN. The generated facies (in white) and the conditioning highlight in red (reservoir) and green (non-reservoir). Note: Each image corresponds to a 2D crossline section represented in the pixel domain (256x256 pixels) to a 80 meters deep by 3750 meters wide vertical section.

This demonstrates the computational efficiency of the proposed approach. The fast generation time allows for practical integration into workflows that require multiple simulations.

MDS was applied to quantitatively evaluate the trained model and to compare patterns of spatial variability between real and generated facies image. MDS is a data analysis and visualization technique that projects high-dimensional data into a lower-dimensional space (typically 2D or 3D) while preserving the relative dissimilarities between samples (COX; COX, 2008).

In this work, MDS was implemented using a gradient-based optimization framework in PyTorch. High-dimensional facies images were first preprocessed by converting them to grayscale and flattening them into one-dimensional vectors. Pairwise dissimilarities between images were then computed using the Euclidean distance, forming a dissimilarity matrix that quantifies the degree of similarity between real and generated facies realizations.

To improve convergence and stability, the low-dimensional configuration was initialized using Principal Coordinates Analysis (PCoA). The MDS embedding was subsequently refined by minimizing a robust stress function based on the $\log(\cosh(\cdot))$ loss, optimized via the Adam

optimizer. This formulation reduces sensitivity to outliers while preserving the overall structure of the distance relationships.

The resulting 2D MDS representation generated overlap well with the training images in two-dimensional space. It indicates a good similarity to the training images, as shown in Fig. 13. The areas where there is a mixture of blue and red indicate that the generated images are very close to the real images. This suggests a good generalization of the generator.

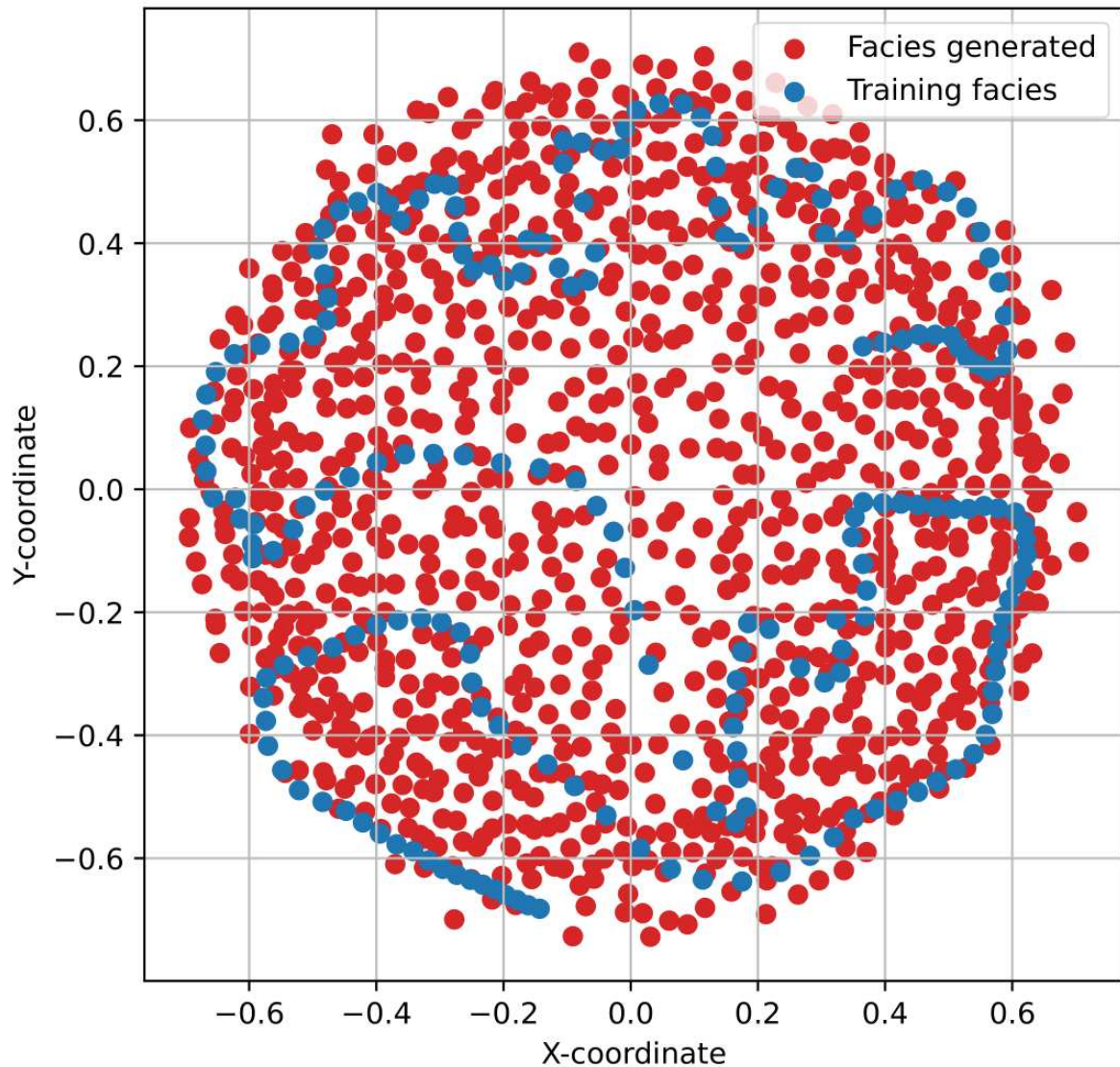


Figure 13 – MDS plot of the training images with conditional realizations.

The generated facies (red) cover well the space of the real facies (blue), indicating the diversity and quality of the generator. The real facies (blue) are closely surrounded by the generated facies, so the generator is interpolating well within the known domain. This indicates a high spatial consistency.

Additionally, to validate how well the known well condition was respected in the Facies-GAN model, a procedure was evaluated linking the problem to a spatial classification task. Specifically, the generated scenarios were compared with their respective original images. It was evaluated how well the location and shape of the facies were reproduced around the actual

wells. The Intersection Over Union (IoU) Index was used as the evaluation metric. This metric is defined as the ratio of the intersection area to the union area between the prediction and the reference data:

$$IoU = \frac{\text{Area of overlap}}{\text{Area of union}} = \frac{GT \cap PD}{GT \cup PD} \quad (5.1)$$

where PD is the prediction mask and GT is the ground truth. In this case, the prediction mask corresponds exclusively to the pixels located along the real well column, i.e., the hard conditioning imposed to FaciesGAN. The hard condition reference data in the original facies image were the ground truth. A total of 1000 images generated from a set of 200 original images were evaluated. The IoU was calculated only over the well-conditioned pixels for each pair of images and the overall mean $IoU = 0.9996$. This indicates a very high fidelity to preserve the well condition. It should be noted that this result is expected, since the FaciesGAN framework explicitly enforces the preservation of hard-conditioning points during generation. Therefore, this metric mainly validates the correct enforcement of the conditioning mechanism, nonetheless, it confirms FaciesGAN's effectiveness as a geological conditioning tool.

5.4.1 SNESIM experiment and comparison

The effectiveness of the FaciesGAN model was validated by comparing its results with synthetic facies scenarios. These scenarios were generated using MPS. The *SNESIM* algorithm (HANSEN; MOSEGAARD; CORDUA, 2018) — a generalized version of the ENESIM approach implemented in MPSlib — was used to generate 1000 facies realizations randomly from the initial 200 facies. In the Python interface to MPSlib, simulation with SNESIM is set up by initializing the MPSlib class with method, specifying the simulation grid, training image, and number of realizations, and then running the simulation in parallel. This approach initializes MPSlib with the generalized SNESIM method, uses the provided training image and grid geometry, and produces the set of conditional MPS simulations. The results of 20 of these simulations are presented in Fig. 14. These results were generated in approximately 137 minutes.

The facies scenarios generated by FaciesGAN (Fig. 12) present a distribution consistent with the expected geology. The scenarios respect the conditions (in green) accurately. These realizations reflect the remarkable ability of the model to capture complex spatial patterns with high diversity between simulations. However, the realizations generated by the *MPS SNESIM* method also preserve the spatial continuity of the facies. Although visually there is less structural variability compared to the FaciesGAN results.

Visual comparison suggested that FaciesGAN reproduces the input conditions and provides better structural diversity in the generated realizations. This makes the proposed methodology a robust alternative for the generation of complex geological scenarios.

To evaluate the general distribution and the balance between the different classes within the dataset, the histogram for reservoir facies proportion is shown in Fig. 15. In addition to

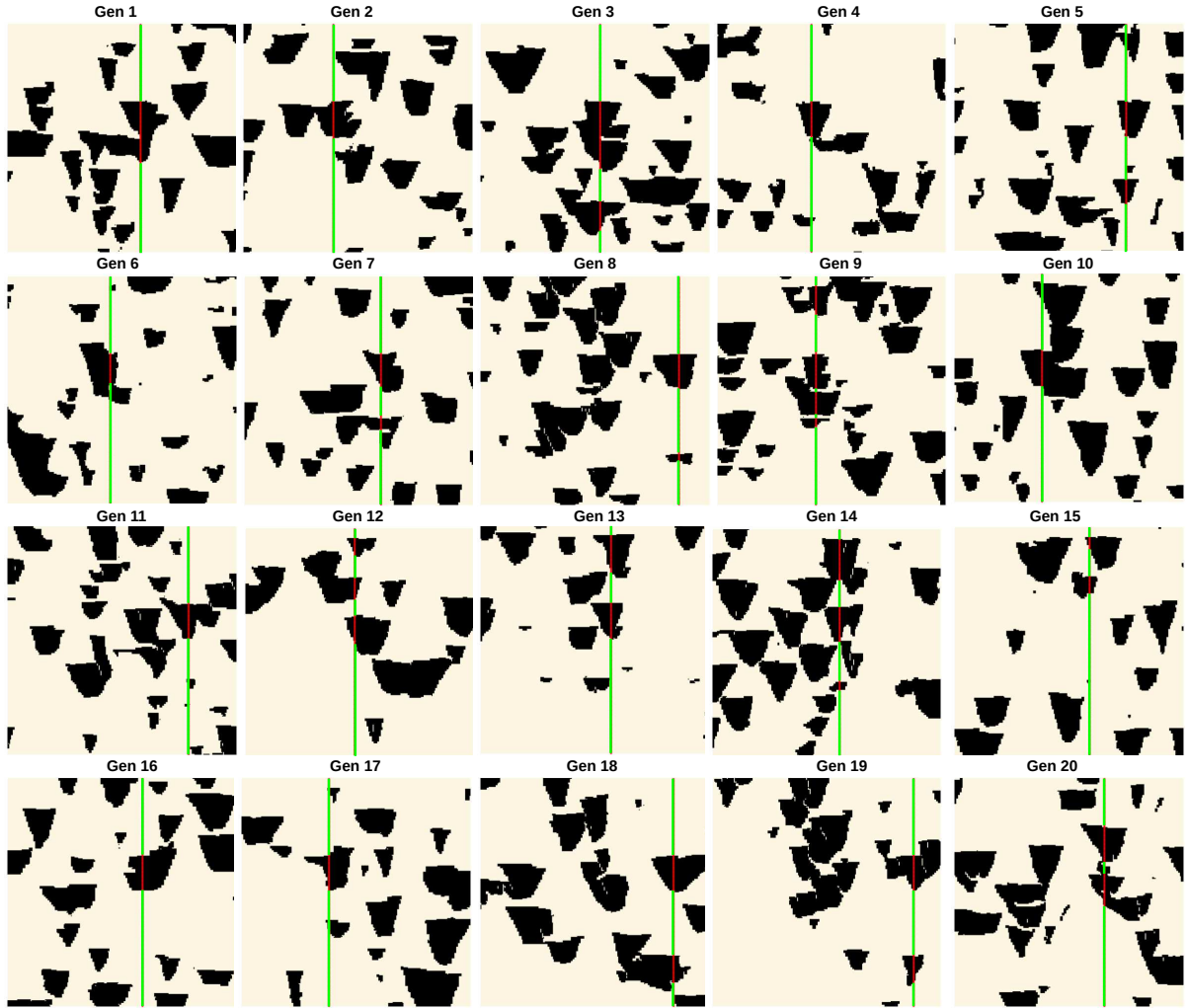


Figure 14 – Twenty randomly selected realizations generated by the MPS SNESIM. The generated facies (in black) and the conditioning highlight in red (reservoir) and green (non-reservoir). Note: Each image corresponds to a 2D crossline section represented in the pixel domain (256x256 pixels) to a 80 meters deep by 3750 meters wide vertical section.

the dataset distribution, the histogram includes the facies proportion results obtained from the FaciesGAN and MPS SNESIM simulations. This allows a comparative analysis of the class balance between the original data and the synthetic realizations with different methods.

The distribution of the dataset (in red) shows a principal peak near 0.16. This value represents the dominant facies ratio in the real data that indicates the dominant facies ratio. The dispersion is moderate, with most of the realizations concentrated between 0.12 and 0.20. The distribution generated by FaciesGAN (green) shows a similar performance compared to the reference, showing a concentration in the same range. However, a minor deviation is observed to higher values, indicating a slight overestimation of the proportion in some simulations.

However, the distribution generated by SNESIM (in blue) is significantly dispersed, covering a broader range, from 0.10 to 0.35. There is also a trend to higher proportions, which implies a lower statistical fidelity compared to the real data. SNESIM has a higher variability than FaciesGAN, which is higher than the reference but is in a controlled interval.

These results indicate that FaciesGAN provides a better approximation of the observed

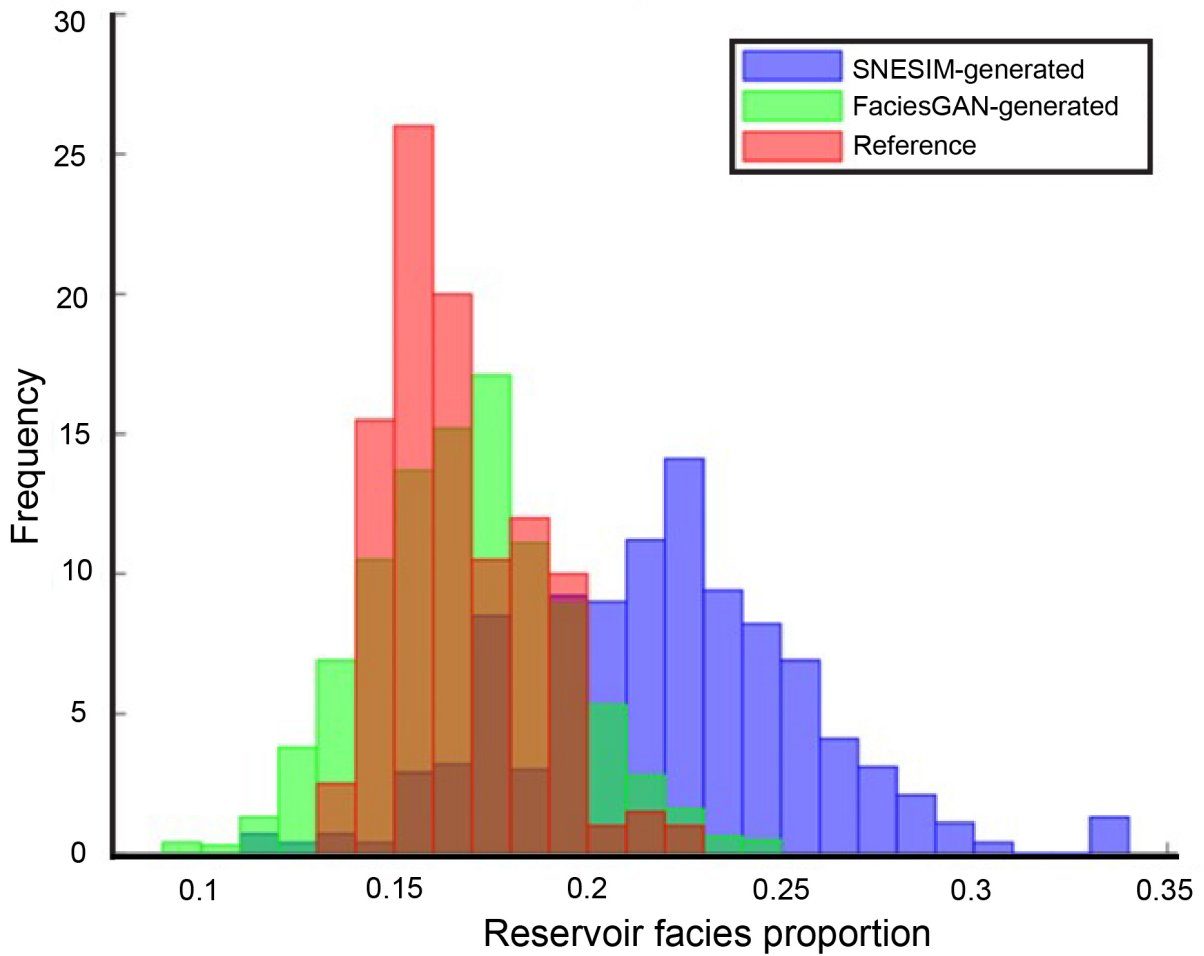


Figure 15 – Comparative histogram of reservoir facies proportion with FaciesGAN and SNESIM.

facies ratio in the reference data compared to SNESIM. SNESIM generated realizations with higher variability that tend to be far from the real statistical performance. This can be a major limitation where facies proportions require accurate preservation. A systematic comparison of these results is presented in Table 2. Additionally, the computational efficiency of FaciesGAN is notably superior: while SNESIM required approximately 137 minutes to generate 1000 realizations, FaciesGAN produced the same number in only 20 seconds. This comparison suggests that FaciesGAN has a greater ability to learn and replicate the distributions observed in the real data. This is made possible by the use of deep learning in its generative process.

With the objective of evaluating how well FaciesGAN reproduces the statistical similarity structure of the reference facies data relative to the SNESIM approach, Fig. 16 presents a two-dimensional MDS projection comparing training (real) facies, SNESIM-generated facies, and FaciesGAN-generated facies. The MDS embedding preserves pairwise dissimilarities between facies realizations, enabling a direct visual comparison of the proximity, overlap, and dispersion of FaciesGAN- and SNESIM-generated facies with respect to the training distribution in a reduced-dimensional space.

Table 2 – Comparative summary of facies proportion performance for Reference data, FaciesGAN, and SNESIM.

Criterion	Reference Data	FaciesGAN	SNESIM
Main peak of facies proportion	~ 0.16	~ 0.16 (well matched)	Shifted to higher values
Concentration range	0.12 - 0.20	0.12 - 0.20 (similar)	0.10 - 0.35 (wide)
Dispersion / variability	Moderate	Low-moderate (controlled)	High (significant)
Statistical fidelity to reference	Baseline	High	Low
Tendency of bias	Balanced	Slight overestimation	Overestimation to higher proportions
Ability to preserve facies proportions	Baseline	Good preservation	Poor preservation
Computational performance	—	1,000 realizations in ~ 20 s	1,000 realizations in ~ 137 min
Overall performance	Baseline	Best fidelity and efficiency	Lowest fidelity and slowest computation

The training facies (red points) compose a compact and continuous manifold that shows the intrinsic variability learned from the original dataset. This manifold is the standard distribution used to test the generative models.

The facies generated by FaciesGAN (the blue points) are very close to the training facies, with a lot of overlap and no clear systematic displacement. The way they are spread out is very similar to the way the training manifold is shaped, which shows that FaciesGAN can accurately capture both the global variability and the local diversity of the real facies. The generated samples fill the inside of the manifold instead of collapsing onto a small subset. This means that there isn't a strong mode collapse and that the model can generalize well beyond the training examples.

In contrast, the SNESIM-generated facies (yellow points) exhibit a noticeably broader dispersion and partial deviation from the core region defined by the training data. While many SNESIM realizations still lie within the global envelope of the real facies, a significant number appear more scattered and isolated, indicating higher variability and less consistency with the original facies distribution. This behavior reflects the stochastic nature of Multiple-Point Statistics, which emphasizes pattern reproduction but does not explicitly constrain the global distribution learned from the data.

5.5 WELL-SPECIFIC CONDITIONAL EVALUATION

In a second test, five conditioning images distributed in two-dimensional space were selected. For each of them, the trained model generated 100 samples in approximately 6 seconds per inference. From each image 5 generated facies scenarios are presented in Fig. 17. The figure shows real facies (left column in green with depth condition in black) and multiple random

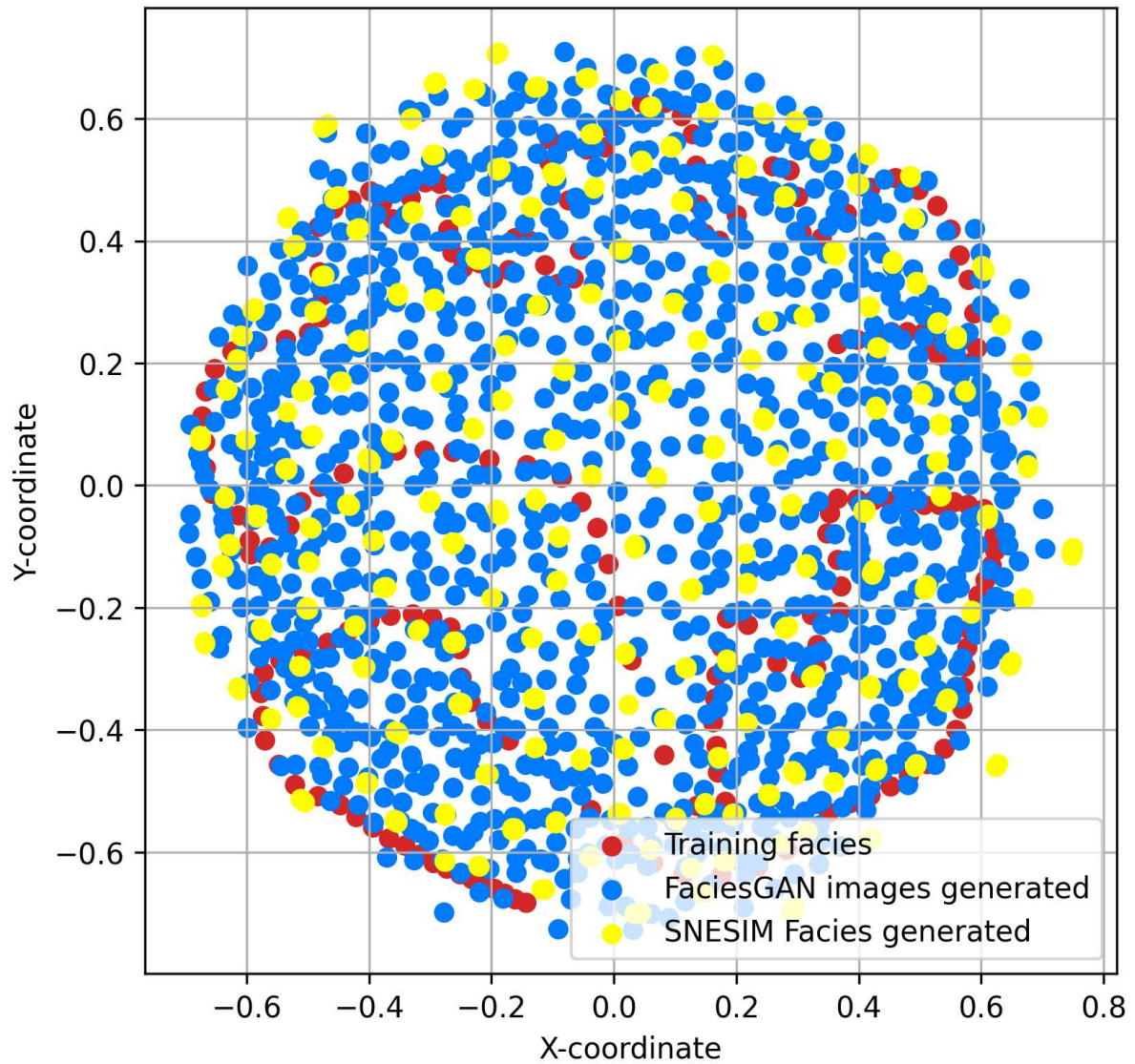


Figure 16 – Two-dimensional MDS projection comparing training facies (red), FaciesGAN-generated facies (blue), and SNESIM-generated facies (yellow), highlighting differences in overlap and dispersion relative to the reference facies distribution.

model-generated realizations (5 columns per well, in black and white) for five different wells, highlighting the depth condition in green.

Overall, the MDS results show that FaciesGAN makes facies realizations that are statistically closer to the training facies distribution than SNESIM does. The closer match between real and FaciesGAN-generated facies suggests that FaciesGAN is better at reproducing the underlying facies variability. SNESIM, on the other hand, tends to show more dispersion and less fidelity in the reduced feature space.

The condition that was included during the FaciesGAN training is held constant. The generated facies surround or overlap with the real facies. This indicates a high spatial consistency between the model realizations and the true data. Although there is variability among the 2D generated facies, most of the realizations retain a structure consistent with the real facies. This fact suggests that the model has adequately learned the underlying spatial patterns.

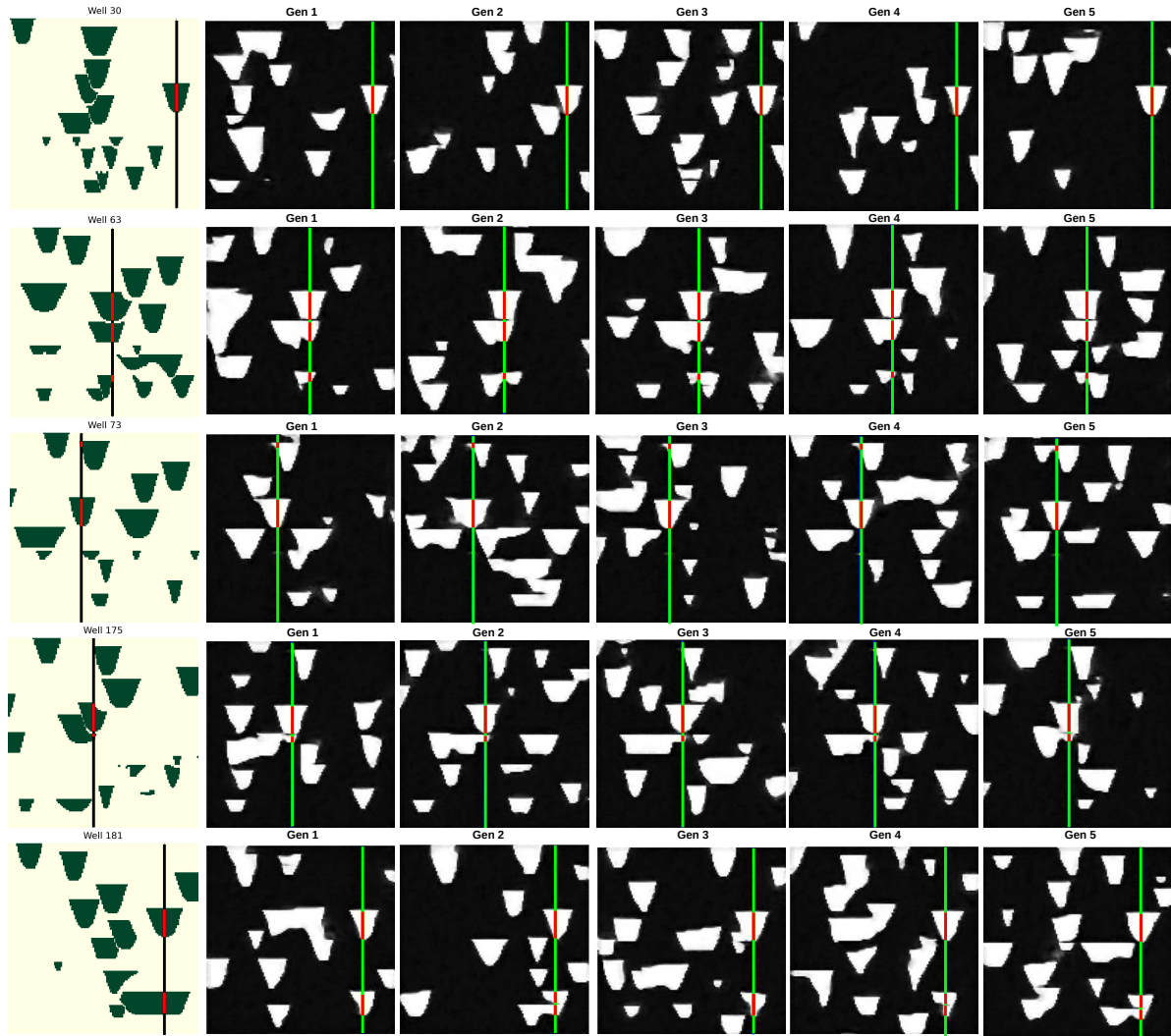


Figure 17 – Real and generated facies for five different wells. The first column shows the real 2D facies logs (in green) with the conditioning in black. The five columns (Gen 1 to Gen 5) display different random realizations generated. The conditioning highlight in red (reservoir) and green (non-reservoir). Note: Each image corresponds to a 2D crossline section represented in the pixel domain (256x256 pixels) to a 80 meters deep by 3750 meters wide vertical section.

The real facies are closely surrounded by the generated facies. This demonstrates the model's ability to preserve spatial structures across different realizations. This consistency highlights the model's reliability in reproducing subsurface geologic patterns, even when they are subject to variability. In this case, the generation of facies scenarios was fast.

The percentages of pixels corresponding to each facies were used to evaluate the statistical consistency between the real images and the images generated by FaciesGAN. The comparison between the percentages observed in the real images and the averages obtained in 100 scenarios generated for the five wells in Fig. 17 are shown in Table 3. This comparison analyzes the model's ability to reproduce the facies distribution realistically. This is important to guarantee that the simulations respect the original geologic characteristics of the data.

In this case, there is a good correspondence between the facies percentages in the real images and in the generated scenarios. For all wells analyzed, the differences between the

real values and the generated averages are less than 4 percentage points. This indicates that FaciesGAN maintains high fidelity in the facies proportion. For example, in well 181, facies 0 represents 72.66% in the real image and 72.71% in the generated average, with a practically insignificant difference. Similar cases are observed in other wells, where the variations between real and generated facies are minimal. The largest deviation is observed in well 63, with facies 1 decreasing by $\sim 3\%$. It could indicate a slight underestimation, although in acceptable ranges for stochastic simulations. In addition, the IoU was calculated for a well-specific analysis to further evaluate the fidelity of facies preservation in the conditioned zones. Table 3 shows the average IoU obtained in each case. The IoU values per well were 99.37% to 99.66%. This indicates an extremely high coherence between the generated scenarios and the original ones in the hard conditioned wells. The results indicate that the percentage distribution of facies remains very close between the actual and generated images. This enhances the FaciesGAN's ability to respect the stratigraphy observed in the real data.

Table 3 – Facies percentage based on image area occupation in the real images compared with the generated scenarios average for five selected wells. Present the percentages and IoU mean of the facies on the hard data.

Well number	Real images		Generated scenarios		IoU
	Facies 0	Facies 1	Facies 0	Facies 1	
30	84.77	15.23	85.16	14.84	99.66
63	69.92	30.08	72.35	27.65	99.42
73	79.30	20.70	80.47	19.53	99.38
175	79.30	20.70	81.35	18.65	99.37
181	72.66	27.34	72.71	27.29	99.43

The generated scenarios were encouraging, as the model demonstrated a high level of consistency and realism in the reproduction of the spatial distributions of the facies from a limited dataset. Furthermore, visual and metrics comparisons with the actual facies distributions demonstrated the model's ability to capture key features of the input data, conditional information and produce significant variations. These results suggest that the FaciesGAN model has generalized well. It provides a robust tool for generating 2D facies scenarios in petroleum applications, even with limited data sets. This could have a significant impact on reservoir analysis and operations planning.

6 DISCUSSION

The results demonstrate that FaciesGAN can generate highly realistic and coherent geological facies scenarios, on a limited training data. The generated realizations accurately represented important geological patterns and spatial continuity, closely following the conditioning information.

The stochastic simulation methodology employed does not aim to identify a single, ‘optimal’ scenario, but rather to quantify geological uncertainty by producing multiple realistic representations of the subsurface that respect the conditioning data. This approach is crucial for risk-based decision-making, as the true facies distribution is unknown. Therefore, rather than determining a single most consistent result, the workflow requires the use of the entire set of simulations (e.g., as inputs to flow models) to estimate a range of possible outcomes (P10, P50, P90). The statistical consistency of this set with the reference model was evaluated using Multidimensional Scaling (MDS) (Fig. 13) and facies proportion histograms (Fig. 15) to ensure that the set of generated scenarios was statistically representative and effectively explored the geological uncertainty space, as detailed in our “Global Model Evaluation” section.

Visual inspection confirmed that FaciesGAN accurately reproduces facies structures and variability, outperforming SNESIM in terms of structural diversity. Quantitative metrics supported this outcome. The average IoU of 99.96% indicates that the well conditioning has been maintained almost perfectly. The facies percentage distributions were also very close to the real data, with differences of less than 4 percentage points in most cases.

The comparative histogram showed that FaciesGAN was better at matching the observed facies proportions than SNESIM, which tended to produce realizations with more variability and statistical deviation. FaciesGAN is also much more efficient in computing, making 1,000 scenarios in 20 seconds, while SNESIM takes 137 minutes.

These results suggest that Deep Generative methodologies, exemplified by FaciesGAN, offer a strong alternative for simulating geological facies. The method shows promise for reservoir modeling workflows that need both accuracy and speed because it can learn spatial patterns and reproduce conditioning information with high accuracy.

The results also showed that FaciesGAN could be used for more than just a single task, since the model maintained highly consistent across different conditioning wells. The stochastic variability was within the acceptable ranges for geostatistics.

FaciesGAN model was trained and validated just on the Stanford Earth Science Data dataset, which simulates a particular meandering channel system. The results are promising, especially under conditions of scarce data; however, additional research is necessary to confirm the model’s applicability to various geological contexts, including turbidite systems, deltas, or carbonate platforms, which could show different spatial patterns and heterogeneities.

Furthermore, the potential of transfer learning is also a promising area for future research. A model pre-trained on a large, diverse set of public geological models could potentially be fine-tuned on a small, field-specific dataset. This approach could make the model even more

useful in real-world reservoir characterization projects where data is almost always scarce.

Overall, the experiments indicate that FaciesGAN can generate realistic, various, and conditionally consistent facies realizations. It is also faster and more statistically accurate than traditional MPS methods. This study mainly compared FaciesGAN with the conventional MPS-SNESIM method; however, subsequent research should evaluate its performance against additional deep generative models. For example, Variational Autoencoders (VAEs) could be explored, although cGANs have demonstrated key advantages in generating scenarios with sharper geological boundaries and a closer match to the underlying statistical distribution of reservoir properties, which are important for realistic facies modeling.

7 CONCLUSIONS

The use of artificial intelligence (AI) techniques to generate facies scenarios is an emerging area that aims to improve the accuracy, interpretability, and reliability of machine learning models used in oil exploration and production. In this context, the present research addressed the issue of restricted data availability by investigating data augmentation methods utilizing conditionally generative adversarial networks (cGANs).

The initial processing workflow that was created successfully turned oil well log data into 2D facies images by cleaning, normalizing, and structuring the data in a series of steps. During this process, the original four facies categories were consolidated into two main groups: "reservoir" and "non-reservoir." This made the structure of the modeling less complex and made it easier to understand the results. This change made it possible to create a balanced and coherent dataset that could be used for conditional data generation and machine learning workflows that came after. So, the preprocessing stage was very important for making sure that the data was consistent, represented the geology well, and fit with the study's first objective.

FaciesGAN was trained on a small number of 2D fluvial facies images, which showed that it could learn complex geological patterns even with little training data. The model was able to make high-quality synthetic facies scenarios that kept the geological continuity and proportions seen in the conditioning data. This result meets the second specific objective, showing that conditional GANs can be a useful way to add more data in geoscientific fields where data is often hard to come by.

Furthermore, the conditional structure of the model facilitated the creation of synthetic realizations that adhered to the data from drilled wells, ensuring alignment with geological reality. Quantitative validations employing multidimensional scaling and facies proportion statistics confirmed that the synthetic data preserved the frequency distributions and spatial correlations of the original images, thereby supporting the third specific objective concerning validation and performance comparison.

FaciesGAN showed a number of advantages over the traditional SNESIM algorithm. For example, it produced a more accurate reproduction of facies proportions, lowered statistical dispersion and skewness, and kept higher spatial and statistical fidelity. Also, the generative process was much faster, which made it possible to quickly create many realizations. This is a big improvement over sequential simulation methods, which usually take a long time.

FaciesGAN is a promising tool to use in reservoir characterization workflows because it is fast and reliable. Synthetic scenarios created can help with facies modeling, improve uncertainty quantification, and make predictions in reservoir studies more accurate. Also, the suggested method could be used in later stages of the workflow, like adding acoustic and/or elastic properties to facies, making synthetic seismic responses, and checking how well they match real seismic data.

In conclusion, the integration of conditional generative models into petroleum geosciences applications represents a substantial advancement towards more adaptable and data-efficient

modeling methodologies. The results of this research validate that cGAN-based data augmentation can improve the quality and reliability of predictive models, facilitating more precise subsurface interpretations and advancing the digital transformation of the oil and gas sector.

7.1 FUTURE WORKS

The development and validation of FaciesGAN have demonstrated its ability to generate conditioned synthetic facies scenarios, preserving the spatial coherence and statistical proportions of the original data. However, the results allow for several lines of research that can enhance its applicability and scope in real hydrocarbon exploration and production contexts. In particular, recent advances in diffusion-based generative models provide a promising complementary or alternative framework for conditional geological modeling.

Firstly, future work will focus on applying the proposed approach to a set of real field data, which will allow the performance of the model to be evaluated under more complex geological and operational conditions. This stage represents the logical and most relevant step for the validation of the robustness of the heterogeneity in real wells and the variability of sedimentary facies in reservoirs.

Secondly, we propose to expand the methodology to integrate physical properties, incorporating the acoustic and elastic characteristics associated with the generated facies. This approach will allow to simulate synthetic seismic responses and compare their consistency with observed seismic data. This reinforces the link between generative models and available geophysical information. In this context, the model will be able to contribute to the geometric characterization of facies and the petrophysical representation of the subsurface.

Thirdly, the synthetic seismic generation process is expected to be combined with seismic inversion flow, establishing a closed cycle between facies modeling, elastic property inversion, and reservoir characterization. This approach will integrate the results generated by FaciesGAN into a broader framework of seismic characterization and uncertainty, allowing for the optimization of petrophysical property estimation and improving the quality of subsurface models.

Finally, it is considered of interest to explore multimodal training and transfer learning strategies that enable the model to be adapted to different geological formations and spatial scales. These extensions could increase the generalization of the method and its applicability to various sedimentary basins. This will strengthen its potential as a decision-making support tool within oil field exploration.

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